Modeling Consumer Decision Making Process with Triangular Norms

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Abstract. Consumer decision making processes are conditioned by various forces. Recognized premises are being constantly reevaluated and future decisions are made in connection with previous ones. Therefore, authors propose an approach to decision making modeling based on pairs of vectors describing attitudes towards certain attributes influencing consumer's decision. As a result, possible is a description of preferences based on multiple evaluations of one feature. Methodological approach allows to reevaluate opinions, which even though were taken in past, but still influence current decisions. In the article we discuss how triangular norms can be applied to decision making modeling based on information encouraging the choice gathered in paired vectors. Developed methodology is based on combining different known triangular norms for given pairs of vectors representing various consumers, who are facing the same decision. The choice of preferences evaluation was performed so that they would represent different real-life situations. Results of applied processing based on combinations of triangular norms are compared. Drawn are conclusions about various processing properties of aggregation operators. Suggested is, in which cases, certain triangular norms describe real-life processes accurately. Shown and described are also examples of operators, which are not suitable at all.

1 Introduction

Decision making processes are driven by various needs. Needs determine our actions, direct us towards reducing motivational tension. As we described in [6], proposed were several hierarchies of how humans proceed with satisfying their needs. A very interesting perspective on the grounds of decision making processes was presented by K. Lewin, [8]. This article continues the discussion started in [5] of how the theory of psychophysical field can be applied to consumer decision making modeling using fuzzy sets and their generalizations.

First, we would like to recall basic notions of how a decision making process can be perceived as a process inside the psychophysical field and how multi-valued logic operators can be applied to model it. In section 3 we discuss various triangular norms, which are applied in section 4 to modeling. Presented is a case study of one particular choice. Decisions are based on fuzzy information, which in fact is very common. Developed methodological approach allows us to reevaluate consumer's opinions, which even though had been taken in past, but still influence current decisions. We compare aggregation possibilities incorporated in different triangular norms and suggest, which cases describe real-life processes most accurately.

2 Preliminaries

Psychophysical field is an abstract term, describing decision making process. It is formed as a combination of three factors:

1. The field (which can be perceived as a set of conditions). Field contains all motivational stimuli arising when subject thinks about the decision. Conditions of the decision are bounded by available knowledge and previous experiences. The field includes not only premises speaking for or against given choice, but also all conscious and subconscious factors, which may influence the decision.
2. Processes, which describe a way of how one makes decisions. It is directly connected with individual's behavioral patterns. Processes are conditioned by customer's cognitive abilities, rationality and susceptibility to behavioral biases.
3. The decision - called also *the gestalt* - it is the outcome of the decision making process.

In this article we would like to focus on how triangular norms can be applied to decision making modeling. We would restrict our discussion to positive premises only, though our future research will include bipolar information aggregation techniques. Due to this restriction, we would be able to model only some fraction of real-life processes, but presented techniques can be also adapted for cases based on both positive and negative premises.

The field (all conditioning factors) will be grouped into two sets: premises and priorities. These are all recognized motivational stimuli, which ground our decisions. Processes are represented by triangular norms, which we apply. The decision (gestalt) is the output of our computations.

As introduced nomenclature may raise some questions, at first, we would like to explain used notions. While speaking about decision making process, we would describe perceived stimuli in the terms of premises and priorities. A premise, in our understanding, is received and decoded information influencing given decision. We assume that one can evaluate the importance of a premise as a number from the range $[0, 1]$, where the greater is the value of the premise, the stronger is positive influence of the premise on the decision. This allows us to apply a flexible scale of evaluation. For each customer we discuss the same set of premises (the decision is made about the same product), but with different evaluations.

While computing the result of the decision making process we introduce aggregating operators, which allow to obtain a decision as a number from the $[0, 1]$ range. The greater is the output of our computations, the more convinced is given customer about the decision. What did we gain? First of all, discussed models describe real life situations better. We are able to base and compute imprecise information. Even though we may calculate the decision in the crisp form (binary response), we are intentionally highlighting that plenty of real-life decisions are perceived rather as weak or strong attitudes.

Premises describe customer's motivation towards certain products or services. In terms of needs theory (see [6]) premises are motivational stimuli, which elicit, control and sustain certain behaviors. These are all factors, which arise when an individual thinks of given decision in more general terms. It can be somehow called an initial or an *a priori* motivation. Authors treat all premises as a set of infinite amount of opinions or attitudes, from which while discussing a particular decision accounted and considered is only a relatively small and countable subset. Why small? Because people tend to simplify rather than complicate reality. Moreover, facing time constraints, people are aware that in order to efficiently manage one's time, even though there might be million possibilities, only a few are really worth discussing.

Second term present while discussing decision making processes are priorities. Authors intend to use this term in the context of a second set of beliefs (or in other words as a second set of motivational stimuli). Priorities allow us to take into account reassessed attitude towards one particular choice. In this paper, term premises concerns more general case, while priories describe certain precise choices. Analogically, priorities can be expressed scaled, for example as a number from the interval $[0;1]$. Priorities are recognized and evaluated later than premises, and their purpose is to provide a perspective of how one particular choice satisfies one's needs. In this context, they may be perceived as an *aposteriori* motivation, arising when subject has faced particular product or service. Of course, a set of priorities evaluations might be drastically different than premises.

3 Triangular norms

In this section we recall basic notions of fuzzy sets and generalization of fuzzy connectives maximum and minimum to triangular norms. We will be expressing fuzzy sets in the form of membership functions. Namely, a fuzzy set A defined in the universe X is a mapping $\mu : X \to [0,1]$ or $\mu_{A,X} : X \to [0,1]$ if the names of the set and the universe should be explicitly stated.

The Zadeh's model of fuzzy sets can be described as a system similar to set theory $(F(X), \cup, \cap, \overline{})$, where $F(X)$ denotes fuzzy sets over the universe X and $\cup$, $\cap$, $\overline{}$ denote union, intersection and complement. This system is clearly interpreted as $([0,1]^X, \max, \min, 1-)$, where $[0,1]^X$ denotes all mappings from the universe X into the unit interval $[0,1]$, i.e. the space of membership functions, and max, min and $1-$ applied to membership functions implement union, intersection and complement. We do not pay attention to the interpretation of fuzzy sets in terms of a lattice L^X.

In this paper study of fuzzy sets system was enriched with triangular norms used in place of the max and min operators. Note: max and min are triangular norms as well.

Triangular norms, i.e. t-norms and t-conorms, are mappings from the unit square $[0,1] \times [0,1]$ onto the unit interval $[0,1]$ satisfying axioms of associativity, commutativity, monotonicity and boundary conditions (cf. [7, 9] for details), i.e.:

Definition 1. *t-norms and t-conorms are mappings $p : [0,1] \times [0,1] \to [0,1]$, where p stands for both t-norm and t-conorm, satisfying the following axioms:*

1.	$p(a, p(b,c)) = p(p(a,b), c)$	*associativity*
2.	$p(a,b) = p(b,a)$	*commutativity*
3.	$p(a,b) \leq p(c,d)$ *if* $a \leq c$ *and* $b \leq d$	*monotonicity*
4.	$t(1,a) = a$ *for* $a \in [0,1]$	*boundary condition for t-norm*
	$s(0,a) = a$ *for* $a \in [0,1]$	*boundary condition for t-conorm*

t-norms and t-conorms are dual operations in the sense that for any given t-norm t, we have a dual t-conorm s defined by the De Morgan formula

$$s(a,b) = 1 - t(1-a, 1-b)$$

and vice-versa, for any given t-conorm s, we have a dual t-norm t defined by the De Morgan formula

$$t(a,b) = 1 - s(1-a, 1-b)$$

Duality of triangular norms causes duality of their properties. Note that the min/max is a pair of dual t-norm and t-conorm. The selected known t-norms and their dual t-conorms are given in Table 1. Note that Hamacher product is

Table 1. Selected triangular norms, dual t-norms and t-conorms are placed in one row

t-norm		t-conorm	
minimum	$min(x,y)$	maximum	$\max(x,y)$
product	$x \cdot y$	probabilistic sum	$x + y - x \cdot y$
Lukasiewicz	$\max(0, x+y-1)$	bounded sum	$\min(a+b, 1)$
nilpotent minimum	$\begin{cases} \min(x,y) & \text{if } x+y > 1 \\ 0 & \text{otherwise} \end{cases}$	nilpotent maximum	$\begin{cases} \max(x,y) & \text{if } x+y < 1 \\ 1 & \text{otherwise} \end{cases}$
drastic	$\begin{cases} y & \text{if } x = 1 \\ x & \text{if } y = 1 \\ 0 & \text{otherwise} \end{cases}$	drastic	$\begin{cases} y & \text{if } x = 0 \\ x & \text{if } y = 0 \\ 1 & \text{otherwise} \end{cases}$
Hamacher product	$\begin{cases} 0 & \text{if } x = 0 = y \\ \dfrac{x \cdot y}{x + y - x \cdot y} & \text{otherwise} \end{cases}$	Einstein sum	$\dfrac{x+y}{1 + x \cdot y}$

the chosen representative of the parametric class of Hamacher norms. t-norms and t-conorms are bounded by minimum t-norm and maximum t-conorm, i.e. for any t-norm t, any t-conorm s and any $x, y \in [0,1]$:

$$t(x, y) \leq \min(x, y) \leq \max(x, y) \leq s(x, y) \quad (1)$$

We will be discussing consumers' decision making process modeled with triangular norms. In this study consumers' decision making is interpreted in the following way:

- first, a consumer attempts purchasing a product or a service considering general needs for it. This solicitude results in a series of *necessity factors* corresponding to needs. The *necessity factors* are expressed as values of a membership functions and we call them *premises*
- then, (s)he confronts the general needs with properties of a concrete item or offer, which fits the general need to some extent. The confrontation produces *fitting factors*, again expressed as values of a membership function. We call the fitting factors *priorities*,
- we assume that the property of a given item/service cannot increase corresponding need, it rather may soften the need. We say that priorities *moderate* premisses and implement the moderation with applying a t-norm. Note that according to formula 1 the result of moderation cannot exceed weaker of the premise and the priority,
- finally we aggregate moderated premisses and priorities with a t-conorm. We assume here that needs vote for purchasing, therefore they either strengthen each other, or at least not weaken themselves. Note that the formula 1 shows that the aggregation with any t-conorm will produce a result not weaker than the stronger moderated premise/priority.

In this study we consider only positive premisses. A discussion on negative premisses, i.e. premisses which vote against purchasing, is not in the scope of this paper. We only wish to note that initial discussion on this subject was undertaken in [5]. That attempt, utilizing balanced fuzzy sets [4] in decision making, combines both types of premisses: positive and negative. In frames of another strive positive and negative premisses are aggregating separately and then final decision is made on these aggregation. So called intuitionistic fuzzy sets [1, 2] or vague fuzzy sets [3] can be used as vehicles for such aggregation.

4 Case Study

To be able to compare described in section 3 models and observe how their properties are reflected while modeling decision making processes, we introduce a case study. In this paper we will continue a discussion on a decision making process regarding purchase of a car (by analogy to: [5]). In order to be able to compare results, we will discuss the same set of eight pairs of premises and priorities for five different customers.

We will be analyzing following attributes encouraging purchase of a car:

- if you have to take care of babies, it is easier to transport them in a car than by public transport,
- shopping with a car is very convenient,
- in a city, where decision maker lives, car allows you to go faster than by bus or by tram,

- having a nice car manifests consumer's good taste and his wealth,
- with a car you can easily make weekend trips to nearby places,
- car allows you to travel at any time you'd like, you are not dependant on any timetables,
- if the weather is bad, driving a car is better than waiting on the bus stop,
- car allows to transport plenty of luggage without overworking.

First, we describe customers and their vectors of premises and priorities. Next, in subsection 4.2 we discuss methodology of applying triangular norms for computing the decision.

4.1 Consumer's vectors description

The analysis revolves around five customers (named A, B, C, D and E). All of them are discussing the same set of attributes regarding a purchase of a car. Selection of vectors of premises and priorities describing customer's preferences was performed by authors. We chose these particular values in order to capture different real-life situations.

Customer A was assigned with following vector of premises:

$$A_{premises} = [\, 0.00,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20 \,] \tag{2}$$

Customer A evaluated all premises as weak ones. First premise is considered as not important at all (is equal to 0). He is not convinced that having a car is necessary. Similarly, vector of priorities for customer A expresses his lack of strong positive opinions regarding one particular car.

$$A_{priorities} = [\, 0.00,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20 \,] \tag{3}$$

A is consequently convinced, that there is no strong motivation for him to buy a car. We expect that consumer his final decision regarding the purchase of a car will be weak.

Customer B was assigned with following vector of premises:

$$B_{premises} = [\, 1.00,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20 \,] \tag{4}$$

B has evaluated first premise as extremely important. He has small babies and having a car would be very helpful to transport them. The rest of premises were evaluated as rather not influencing. Below shown is vector $B_{priorities}$, which gathers customer B's priories evaluation towards one particular car.

$$B_{priorities} = [\, 1.00,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20 \,] \tag{5}$$

Analogically, first priority was evaluated as extremely important - he believes that this particular car is suitable and would play its role as a help while transporting his family. The rest of priorities still have weak impact on his behavior.

Customer C is described by a following vector of premises:

$$C_{premises} = [\, 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80 \,] \tag{6}$$

All premises were recognized as very important. Analogically, C's vector of priorities $C_{priorities}$ contains high values.

$$C_{priorities} = [\, 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80 \,] \tag{7}$$

C is strongly convinced that buying car is necessary. Moreover, this one particular car he is analyzing meets his expectations. We expect that C's decision should be strong positive.

Customer D has following values attached to premises regarding purchase of a car in general:

$$D_{premises} = [\, 0.80,\ 0.30,\ 0.70,\ 0.00,\ 0.10,\ 0.60,\ 0.90,\ 0.30 \,] \tag{8}$$

His preferences are varied. There are several premises recognized as important and several recognized as very weak. There is even one attribute (premise number 4: car as a way to manifest one's wealth and social status), which in D's opinion initially does not matter at all. Vector $D_{priorities}$ describes D's priorities towards particular car.

$$D_{priorities} = [\, 0.10,\ 0.80,\ 0.20,\ 0.90,\ 0.60,\ 0.10,\ 0.00,\ 0.80 \,] \tag{9}$$

This customer's strengths of all priorities are drastically different than strengths of corresponding premises. When D has faced this one particular decision, he drastically reevaluated his opinions. We will observe how different moderating operators would cope with these vectors. Situation captured in $D_{premises}$ and $D_{priorities}$ corresponds to a case, when facing one particular choice we recognize several new important features of given product. In such case, old premises lose their impart on the final decision. This can be caused for example by very persuasive marketing communications, when customer starts to analyze new priorities, which concern one particular car. Successful marketing campaign makes customer D believe that these new projected features of advertised car are even more important than original premises.

Customer E was assigned with following vector of premises:

$$E_{premises} = [\, 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80,\ 0.80 \,] \tag{10}$$

Customer E has reviewed his opinions regarding purchase of a car in general and decided that all premises are very strong. But when one particular car was analyzed, he reevaluated the importance of named priorities and it turned out that all attributes are rather insignificant. Vector $E_{priorities}$ presents reevaluated priorities.

$$E_{priorities} = [\, 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20,\ 0.20 \,] \tag{11}$$

Situation described with vectors $E_{premises}$ and $E_{priorities}$ happens when at first we are strongly convinced that some product is a necessity. Next, when we face a particular decision, we are surprised to notice that what has been so appealing has lost its charm and we are rather disappointed.

4.2 Results of aggregation

In following subsections we discuss results obtained for A, B, C, D and E after applying operators described in section 3. We discuss three different methodologies of moderation and aggregation:

1. various t-norms for premises and priorities moderation and max t-conorm for final decision calculation,
2. min t-norm for premises and priorities moderation and various t-conorms for final decision aggregation,
3. various t-norms for premises and priorities moderation and their dual t-conorms for final decision calculation.

Chosen strategy of applying triangular norms was dictated by literature review and our intuition. Authors picked popular triangular norms and combined them in order to obtain different results.

Moderation with different t-norms and aggregating with max t-conorm In this subsection we will discuss decisions made for customers A, B, C, D and E basing on maximum t-conorm and vectors of premises and priorities moderated with different t-norms. At first, we would like to show exemplar outputs of moderation. Please note that all calculations were performed according to formulas given in Table 1.

Table 12 contains exemplar results of premises and priorities moderation using minimum t-norm for consumers A and C.

A:	0.0	0.2	0.2	0.2	0.2	0.2	0.2	0.2
C:	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8

(12)

In the same fashion calculated were moderated values for consumers B, D and E. Analogically, we moderated premises with priorities using other t-norms, namely: product, Lukasiewicz t-norm, Nilpotent minimum, Drastic t-norm and Hamacher product. Due to space limitations these calculations are not discussed to greater extent. Please note, that none of results obtained via moderation using t-norms different than minimum would give us values greater than ones received with the minimum (property explicitly stated in formula 1).

Next, we would like to discuss final decisions obtained for consumers A, B, C, D and E after applied maximum t-conorm. Table 13 contains values of decisions aggregated with max for vectors moderated using various t-norms. The top row of table 13, contains information about applied t-norms and t-conorms.

	minimum maximum	product maximum	Lukasiew. maximum	Nilpotent maximum	Drastic maximum	Hamacher maximum
A:	0.20	0.04	0.00	0.00	0.00	0.29
B:	0.90	0.90	0.90	0.90	0.90	0.90
C:	0.80	0.64	0.60	0.80	0.00	0.67
D:	0.30	0.24	0.10	0.30	0.00	0.28
E:	0.20	0.16	0.00	0.00	0.00	0.19

(13)

Results of minimum t-norm and maximum t-conorm (first column) show that only B and C are strongly convinced about the purchase of one particular car. All other customers express weak opinions. Consumer B represents a situation, when gathered is one strong positive information and several weak premises and priorities. Decision computed for consumer C is also strong positive one, what is no surprise, as all evaluations both of premises and priorities are high.

In this first case, consumers A, D and E are not convinced about the purchase of given car. Even though some of them have recognized several strong priorities (or premises) the process of moderation disregarded them and the decision was computed as a weak one.

Second column contains values received by maximum t-conorm aggregation on vector of moderated premises and priorities. Moderation was performed using product t-norm. In the second case, only B and C made strong positive decisions. Through properties of multiplication of numbers not greater than 1 the output of moderation contains mostly low numbers. Therefore, max t-conorm as an aggregation operator is more than necessary so that we can somehow balance low results received through multiplication. Applied product t-norm and maximum t-conorm for E gave us result equal to 0.16. Previously discussed combination of operators also returned equally weak choice. The weakest value of aggregated decision was computed for A. Product t-norm strongly reflected the fact that all premises and priorities for A were weak ones.

Third column of table 13 contains decisions obtained with a combination of Lukasiewicz t-norm and maximum t-conorm. Analogically to the previous case, consumers B and C are the only ones, who are convinced about the purchase of a car. Consumers A and E's decisions are equal to 0 - they are absolutely indifferent towards the decision regarding a particular car. This is first combination of t-norms and t-conorms, which returns such a strict output.

Fourth column in table 13 presents values obtained after applying Nilpotent minimum t-norm and max t-conorm for A, B, C, D, and E. Results of aggregation obtained with Nilpotent minimum t-norm and maximum t-conorm are close to results obtained using Lukasiewicz and maximum norms. B and C are strongly convinced about the purchase of the car, while A, D and E would not buy the car. In this case output for C is more satisfying (is higher) than in the previous case - enhanced was the fact that C's both vectors have all strong positive values.

Fifth column contains the results obtained after applying Drastic t-norm and maximum t-conorm. As we expected, all uncertain cases were eliminated. Positive decision was computed only for consumer B.

Sixth column in table 13 presents the output of moderation and aggregation using Hamacher product t-norm and maximum t-conorm for all customers. Noticeable is that B and C are strongly convinced about the purchase of the car. At the same time A, D and E's decisions are weak positive.

Moderating with min t-norm and aggregating with various t-conorms Second part of our experiment was to observe the differences between decisions aggregated using different t-conorms. In this approach in each case premises and priorities were moderated using minimum t-norm. Table 14 compares final decisions obtained using t-conorms as aggregation operators. The top row of table 13, contains information about applied t-norms and t-conorms.

	minimum maximum	minimum prob. sum	minimum bound. sum	minimum Nilpotent	minimum Drastic	minimum Einstein
A:	0.20	0.79	1.00	0.20	1.00	0.89
B:	0.90	0.98	1.00	1.00	1.00	0.99
C:	0.80	1.00	1.00	1.00	1.00	1.00
D:	0.30	0.71	1.00	0.30	1.00	0.81
E:	0.20	0.83	1.00	0.20	1.00	0.92

(14)

Results obtained using min t-norm for moderation and various t-conorms for output calculation are unsatisfactory. Apart from the first and fourth case, when we applied min-max dual operators and minimum/Nilpotent maximum norms, all other results compute strong positive decisions for each customer. Especially undesirable effects obtained were for bounded sum and Drastic t-conorms.

We find second methodology rather unsuitable. Nevertheless, basing on discussed results, we observed how different t-conorms aggregate sequences of the same values. For example, let's discuss aggregation of an 8-element sequence of 0.2s. We observe that two t-conorms, namely maximum and Nilpotent maximum maintain the same result and it is 0.2. All other discussed t-conorms gradually saturate and tend to 1. Norm, which is returning 1 instantly is Drastic t-conorm. Norm, which is tending most slowly to 1 is bounded sum. The higher are the values of aggregated sequence, the faster the result tends to 1. For a sequence of 0.8s, only after three operations the result is saturated. Of course, by definition of triangular norm, the result of aggregation is always bounded by 1.

In consequence, gradual saturation of the result means that certain triangular norms, including bounded sum, probabilistic sum and Einstein sum incorporate following property: the more positive motivational stimuli one recognizes the closer the decision gets to 1. Second corollary is following: one strong positive argument (moderated premise with priority) can be replaced by several weak arguments. Discovering these properties is very important from the point of view of consumer decision making modeling. Presented properties of aggregation operators are highly desirable - economists prove that people tend to simplify cognitive processes. What does it mean in the context of decision making? Saturation of named t-conorms allows us to model a situation, when customer, even though there might be an infinite number of premises, is able to efficiently make a decision basing on relatively small set of arguments. Presented t-conorms allow to represent nontrivial aspects of the decision making processes, including behavioral biases. We believe that applying them in the neoclassical theory of consumer's choice might be increase its accuracy and would allow us to describe real-life situations more accurately.

Moderating with different t-norms and aggregating with their dual t-conorms Last, but not least, we would like to discuss the decisions regarding purchase of a car computed basing on various dual t-norms and t-conorms. Table

15 shows decisions made for consumers A, B, C, D and E. The top row informs us, which t-norm and t-conorm was applied in order to compute particular decision.

	minimum maximum	product prob. sum	Lukasiewicz bounded sum	Nilpotent Nilpotent	Drastic Drastic	Hamacher Einstein
A:	0.20	0.23	0.00	0.00	0.00	0.58
B:	0.90	0.92	0.90	0.90	0.90	0.98
C:	0.80	1.00	1.00	1.00	0.00	1.00
D:	0.30	0.60	0.20	0.20	0.00	0.78
E:	0.20	0.75	0.00	0.00	0.00	0.91

(15)

Obtained results prove again that the choice, of which operators apply for computing the decision has major impact on the output. The differences between particular results are substantial. It is vividly seen in the case of customer C, whose decisions vary from 0 to 1, depending on the applied norm. Variety of properties incorporated in different aggregating operators would allow us to use ones, which would describe decision making phenomena the best.

Decisions obtained for D and E are analogical to ones received with the first methodology. We find outputs computed using operators product/probabilistic sum and Hamacher product/Einstein sum for D and E as too high. These two vectors were constructed to reflect consumers who are undecided (D) or who are disappointed with one particular car described by vector of priorities (E).

Customers B and C were assigned with strong positive decisions in all cases, except from one (Drastic t-norm and t-conorm for consumer C). Comparing the third methodology with the first and the second one, we see that C's decisions were computed before as weaker. For further research left is the topic, which methodology is more suitable.

Finally, we'd like to discuss decisions computed using dual t-norms and t-conorms for consumer A. Third approach computed rather optimistic results. In comparison, for A's decisions, first methodology in all cases computed values non greater than using the third approach. Third approach for the following three dual operators: Lukasiewicz t-norm/bounded sum, Nilpotent minimum/Nilpotent maximum, Drastic t-norm and Darstic t-conorm computed A's decisions as zeros. We find this results as too conservative, since these norms disregard all preferences, which even though are weak, but still they exist. As we mentioned in [5], Kahneman and Tversky proved that people tend to overweight small probabilities and underweight moderate and high probabilities, c.f. [10]. Therefore justified would be the choice of conservative operators for the case of consumer C and rather optimistic operators for A.

5 Conclusions

In the article we discuss how different triangular norms can be applied to model decision making processes based on positive premises only. We support our paper with the case study of five customers who decide about one particular car basing on the same set of attributes. These five cases represent different real-life situations of people, who show varied attitudes towards both a decision regarding purchase of a car in general and different opinions regarding one particular car, about which the decision is made. We apply three methodological approaches of how to calculate the decision using t-norms and t-conorms. First one uses various t-norms for premises and priorities moderation and maximum t-conorm to compute the output. Second approach applies minimum t-norm for vector's moderation and various t-conorms for decision calculation. Third approach uses dual t-norms and t-conorms. First and third methodologies bring us satisfactory results. Second one we find rather not suitable. We noticed that several t-norm/t-conorm combinations would allow us to model more conservative (weak) decisions, while some other are optimistically enhancing the result. An example of a set of operators strengthening the decision is Hamacher product and Einstein sum. Examples of t-norms, which applied to vectors of premises and priorities compute rather weak decisions, for almost each t-conorm, are Lukasiewicz, Drastic and Nilpotent minimum t-norms. Important conclusions were introduced while discussing aggregation of sequences using bounded sum, probabilistic sum and Einstein sum norms. Saturation of decision computed using named operators ideally reflects human's tendency for simplification. Presented triangular norms allow to represent nontrivial aspects of the theory of consumer's choice. Possibility of incorporating behavioral biases into consumer decision making models would allow us to develop a methodology, which would describe real-life phenomena more precisely and more extensively.

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