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Abstract: The monitoring of mechanical systems aims at detecting and diagnosing damages, in general by using output-only vibration measurements under ambient excitation. In this paper, a method is proposed for the localization of stiffness changes in a structure. Based on mechanical grounds, damage is located in elements of a structure with zero stress when a load is applied that is in the null space of the transfer matrix difference between the nominal reference and the damaged state. This load vector is estimated from system identification in both reference and damaged states, and the stress is computed based on a finite element (FE) model of the structure in the reference state. In this work, we address two sources of errors in this computation that lead to stress that is only approximately zero in the damaged elements, which are (1) estimation errors due to noise and finite data, and (2) modal truncation errors due to a limited number of identified modes in comparison to the number of modes present in the FE model that characterizes the structural behavior. To address (1), we propose a statistical evaluation of the stress estimates for a decision on the damaged elements, by propagating the covariance from system identification results to the covariance of the stress. To address (2), several stress estimates are obtained for different mode sets and Laplace variables in the evaluation of the transfer matrices, and jointly evaluated in a hypothesis test. Damage localization results are presented in a simulation study and on experimental data from a damaged beam in the lab.

Keywords: Damage localization, statistical tests, mechanical system, vibrations.

1. INTRODUCTION

The detection and localization of damages based on measured vibration data are fundamental tasks for structural health monitoring (SHM) to allow an automated damage diagnosis [Farrar and Worden, 2007]. Damages can be modeled as changes in the stiffness of the underlying mechanical system. They induce changes in the dynamic properties of the system, which can be monitored through vibration measurements. A particular difficulty for SHM is caused by the absence of known system inputs, since the structural excitation is usually only ambient, leading to an output-only monitoring problem.

To link changes in the dynamic properties to the physical changes in the structure, a finite element (FE) model of the structure can be used. Many methods for vibration-based *damage localization* infer on the stiffness parameters of a FE model. A nominal model from the reference state of the structure is updated to reproduces the dynamic response from the data of the damaged state. Comparing the updated stiffness matrices with the original ones provides damage location and extent [Brownjohn et al., 2001]. While model updating-based approaches are in principle applicable to arbitrary structures, they are often too poorly conditioned to be successful in practice. The parameter size of FE models of real structures is usually much larger than the number of identified parameters from measurements, leading to an ill-posed problem [Friswell, 2007]. Alternative methods confront measurement data to a FE model to analyze changes in the structure in a more indirect way, without updating. Empirical approaches [Fan and Qiao, 2011] and approaches with a more profound theoretical background have been developed, e.g. using sensitivity-based hypothesis tests for detecting the changed stiffness parameters [Döhler et al., 2016] or analyzing structural flexibility changes for damage localization with the stochastic dynamic damage locating vector (SDDLV) approach [Bernal, 2010].

In the SDDLV approach, a vector in the null space of the difference between the transfer matrices of the healthy and damaged systems is obtained from the modal estimates. It has been shown that when applying this load vector to the FE model of the healthy structure, then the resulting stress field is zero at the damaged element. Propagating

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the covariance of the modal estimates [Reynders et al., 2008, Döhler and Mevel, 2013] to the stress estimates in a sensitivity-based approach, hypothesis tests for decision making on damaged and undamaged elements were developed in [Döhler et al., 2013, Marin et al., 2015].

In this work, the SDDLV framework is extended with the joint evaluation of stress estimates from transfer matrix estimates based on different identified mode sets. This is necessary to take all the identified modes from output-only measurements into account, since the output-only nature of the data puts a constraint on the number of modes that can be used for the transfer matrix estimate. Closely linked to this problem is the choice of the Laplace variable s in the computation of the transfer matrix, which has an impact on the modal truncation error in the estimate. With the stress estimation at different values of s and at different mode sets to include all available modal information, and its joint statistical evaluation in a hypothesis test for a decision on the damaged elements, it is expected that modal truncation errors are mitigated. This will be supported by experimental results.

This paper is organized as follows. In Section 2, the system models and parameters are defined, and the problem of damage localization with the SDDLV method is stated. In Section 3, the new statistical extension of the method is developed. Finally, an application for damage localization on a beam structure is reported in Section 4.

2. PROBLEM OUTLINE

The behavior of mechanical structures subject to unknown ambient excitation can be described by the differential equation

$$\mathcal{M}\ddot{\mathcal{X}}(t) + \mathcal{C}\dot{\mathcal{X}}(t) + \mathcal{K}\mathcal{X}(t) = f(t) \tag{1}$$

where t denotes continuous time; $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are mass, damping, and stiffness matrices, respectively; the state vector $\mathcal{X}(t) \in \mathbb{R}^m$ is the displacement vector of the m degrees of freedom of the structure; and $f(t)$ is the external unmeasured force (random disturbance).

Observed at r sensor positions by displacement, velocity or acceleration sensors, system (1) can also be described by a continuous-time state space system model [Juang, 1994]

$$\begin{cases} \dot{z}(t) = Az(t) + Be(t) \\ y(t) = Cz(t) + De(t) \end{cases} \tag{2}$$

where the state vector $z = \begin{bmatrix}\mathcal{X}^T & \dot{\mathcal{X}}^T\end{bmatrix}^T \in \mathbb{R}^n$ with $n = 2m$, the output vector $y \in \mathbb{R}^r$, the system matrices

$$A = \begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}, \tag{3}$$

$$C = \begin{bmatrix} L_d - L_a\mathcal{M}^{-1}\mathcal{K} & L_v - L_a\mathcal{M}^{-1}\mathcal{C} \end{bmatrix} \in \mathbb{R}^{r\times n}, \tag{4}$$

with selection matrices $L_d, L_v, L_a \in \{0,1\}^{r\times m}$ indicating the positions of displacement, velocity or acceleration sensors, respectively. Since $f(t)$ is unmeasured, it can be substituted with a fictive force $e(t) \in \mathbb{R}^r$ acting only in the measured coordinates and that regenerates the measured output. A mode of the system is denoted as the pair (λ_j, φ_j) of eigenvalue and observed eigenvector with $A\phi_j = \lambda_j\phi_j$, $\varphi_j = C\phi_j$.

The transfer function $G(s) \in \mathbb{C}^{r\times r}$ of the system can then be derived as [Bernal, 2010]

$$G(s) = R(s)D + D,$$

where

$$R(s) = C(sI - A)^{-1} \begin{bmatrix} CA \\ C \end{bmatrix}^{\dagger} \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{5}$$

under the condition that the system order satisfies $n \leq 2r$. Note that $R(s)$ can be computed from output-only system identification, e.g. using subspace methods [Marin et al., 2015], while matrices B and D cannot be identified for the computation of $G(s)$.

The difference between the transfer matrices in both damaged (variables with tilde) and healthy states is $\delta G(s) = \tilde{G}(s) - G(s)$. Damage is considered as changes in the structural stiffness properties of system (1), while mass remains unchanged. Then, the matrices $\delta G(s)$ and $\delta R(s)^T = \tilde{R}(s)^T - R(s)^T$ have the same null space since $\tilde{D} = D$ [Bernal, 2010]. The desired load vector $v(s) \in \mathbb{C}^r$ is obtained from the null space of $\delta R(s)^T$ using a singular value decomposition (SVD), thus it can be estimated entirely on measurements in healthy and damaged states. Finally, the load $v(s)$ is applied to the FE model of the healthy structure to compute stress at all structural elements, stacked in vector $S(s)$, which yields a linear relationship

$$S(s) = \mathcal{L}_{\text{model}}(s)v(s)$$

defined through the FE model of the healthy state. In theory, the stress components in vector $S(s)$ that correspond to damaged elements are zero [Bernal, 2010]. Since subspace identification yields asymptotically Gaussian estimates, the estimated stress is also asymptotically Gaussian distributed. To decide if the estimated stress components are zero, its covariance has been estimated and the appropriate hypothesis test has been proposed in [Döhler et al., 2013, Marin et al., 2015].

The problem considered in this work is the estimation of $S(s)$ for damage localization when $n \leq 2r$ in (5) is not satisfied. In this case, only a limited number of the identified modes can be used, inducing further modal truncation errors besides the "classical" modal truncation errors that result from model reduction. Note that the effective model order of system (2) that can be estimated from measurements is usually much lower than the order of a numerical FE model (1) or of the actual structure (being usually infinite).

3. STATISTICAL EVALUATION WITH SDDLV METHOD

Denote $\Theta = \{(\lambda_i, \varphi_i) : i = 1, \ldots, n\}$ the set of all identified modes. To take all these modes into account, we propose to use subsets θ_j, $j = 1, \ldots, n_m$, containing each n_j modes with $n_j \leq 2r$, and $\Theta = \bigcup_{j=1}^{n_m} \theta_j$. Then, for each mode set θ_j a "modally truncated" transfer matrix difference is evaluated for Laplace variable s_j in the vicinity of the respective modes to limit modal truncation errors, and the stress $S^j(s_j)$ is computed at all elements. The stress for a particular element t in this vector is denoted as $S_t^j(s_j)$.

Then, several stress estimates for each structural element are available that are in theory zero at the damaged elements. For each structural element t, let the respective stress values from the different estimates be stacked in vector S_t with

$$S_t = \begin{bmatrix} S_t^1(s_1) \\ \vdots \\ S_t^{n_m}(s_{n_m}) \end{bmatrix}. \tag{6}$$

For a decision about damage in element t, $S_t = 0$ is tested against $S_t \neq 0$ in a hypothesis test. Since S_t is asymptotically Gaussian distributed, the test statistic amounts to

$$s_t = S_t^T \Sigma_t^{-1} S_t, \tag{7}$$

where Σ_t is the covariance of S_t and detailed in the following. The test statistic s_t is asymptotically χ^2 distributed, central if element t is damaged and with a non-centrality parameter for non-damaged elements. Thus, s_t can be compared to a threshold for a decision.

The estimation of the covariance Σ_t is based on a sensitivity analysis, propagating the sample covariance estimate of the output covariances in the subspace identification to the covariance of the stress estimates. To allow a joint covariance evaluation, the sensitivity of each component in vector S_t needs be related to a common random variable. Each stress estimate $S_t^j(s_j)$ depends on different modes of the system, which are estimated in the same subspace identification using the output covariance Hankel matrices $\mathcal{H}$ and $\tilde{\mathcal{H}}$ in the healthy and damaged states, respectively. Thus, the sensitivity of each stress estimate can be developed in dependence of these common Hankel matrices, yielding

$$\Delta S_t^j(s_j) = \tilde{\mathcal{J}}_t^j \mathrm{vec}(\Delta\tilde{\mathcal{H}}) - \mathcal{J}_t^j \mathrm{vec}(\Delta\mathcal{H})$$

for first-order perturbations $\Delta(\cdot)$. The technical details of this development are partly based on [Marin et al., 2015] and are not detailed here for brevity. Then, stacking the different stress estimates in (6) yields

$$\Delta S_t = \tilde{\mathcal{J}}_t \mathrm{vec}(\Delta\tilde{\mathcal{H}}) - \mathcal{J}_t \mathrm{vec}(\Delta\mathcal{H}),$$

where

$$\mathcal{J}_t = \begin{bmatrix} \mathcal{J}_t^1 \\ \vdots \\ \mathcal{J}_t^{n_s} \end{bmatrix}, \tilde{\mathcal{J}}_t = \begin{bmatrix} \tilde{\mathcal{J}}_t^1 \\ \vdots \\ \tilde{\mathcal{J}}_t^{n_s} \end{bmatrix}.$$

Finally, the desired covariance can be estimated as

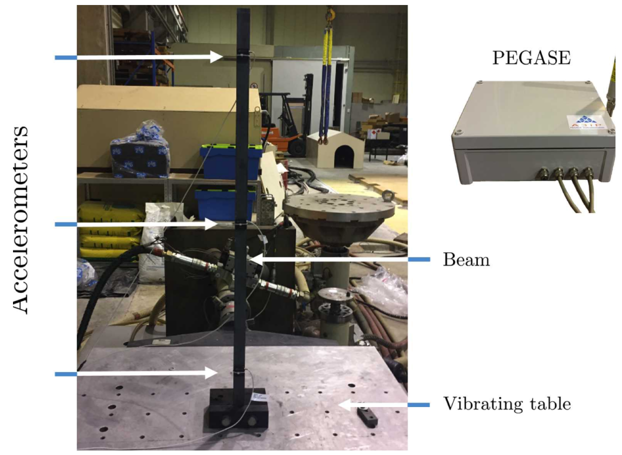
$$\Sigma_t = \tilde{\mathcal{J}}_t \Sigma_{\tilde{\mathcal{H}}} \tilde{\mathcal{J}}_t^T + \mathcal{J}_t \Sigma_{\mathcal{H}} \mathcal{J}_t^T,$$

where $\Sigma_{\mathcal{H}} = \mathrm{cov}(\mathrm{vec}(\mathcal{H}))$ can be easily obtained as the sample covariance estimate of the Hankel matrix of output covariances [Döhler and Mevel, 2013].

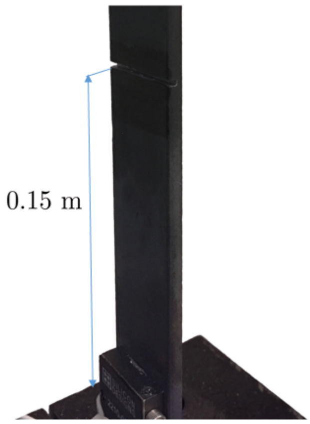
4. APPLICATION

4.1 Experimental setup

The proposed algorithm has been applied on vibrational data of a cantilever beam as shown in Figure 1. The beam length is 1 m and is embedded on 5 cm at its lower end. The excitation for the tests was provided by a hydraulic shaker. A broadband random acceleration signal was induced via a vibrating table at the bottom of the structure. Response of the beam was measured in the transverse direction with three accelerometer sensors (0.1, 0.5 and 0.9 m of the vibrating table, i.e. 0.05, 0.45 and 0.85 m of the embedding), at a sampling frequency of 2000 Hz and 20,000 samples. Data are recorded and stored with a PEGASE platform, which is a smart wireless sensor



(a) Experimental setup for the beam in healthy state



(b) Damaged beam.

Fig. 1. Experimental setup

systems performing real-time monitoring [Le Cam et al., 2016]. With the practical difficulties to access structures, to collect data and then perform off-line and remote computation, this platform offer an advantage compared to classical measurement systems.

A damage was performed by introducing a cut along the length of the beam, at 0.2 m to the vibrating table (i.e. 0.15 m to the embedding), as depicted in Figure 1(b). Experimental data for the damaged state were recorded similarly as for the healthy structure.

4.2 FE model

The underlying healthy finite element model is required for the method and assumed to be known. The structure of

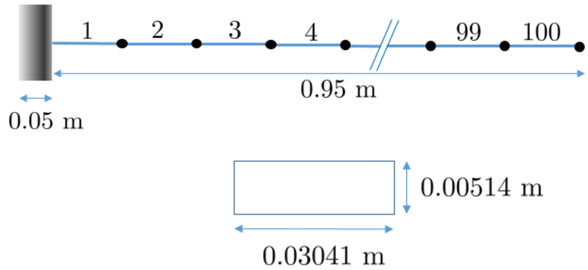


Fig. 2. FE model of the beam.

Table 1. Eigenvalues λ_j of beam.

Mode j	Healthy state	Damaged state	Mode set
1	$-0.55 + 28.4\mathrm{i}$	$-1.10 + 26.1\mathrm{i}$	θ_1
2	$-0.42 + 178.9\mathrm{i}$	$-0.46 + 178.3\mathrm{i}$	
3	$-0.15 + 506.2\mathrm{i}$	$-0.15 + 505.3\mathrm{i}$	θ_2
4	$-1.32 + 984.5\mathrm{i}$	$-13.54 + 975.5\mathrm{i}$	
5	$-0.71 + 1630.9\mathrm{i}$	$-0.48 + 1605.5\mathrm{i}$	θ_3
6	$-2.35 + 2416.0\mathrm{i}$	$-1.33 + 2387.5\mathrm{i}$	

0.95 m length is equally divided into 100 three-dimensional beam elements (6 degrees of freedom per node) as shown in Figure 2. The density and elastic modulus of the beam are 7800 kg/m^3 and $2.1 \cdot 10^{11}$ Pa, respectively, and the area of cross-section is 0.03041×0.00514 m^2. The damage position corresponds to elements 16-17 of the model.

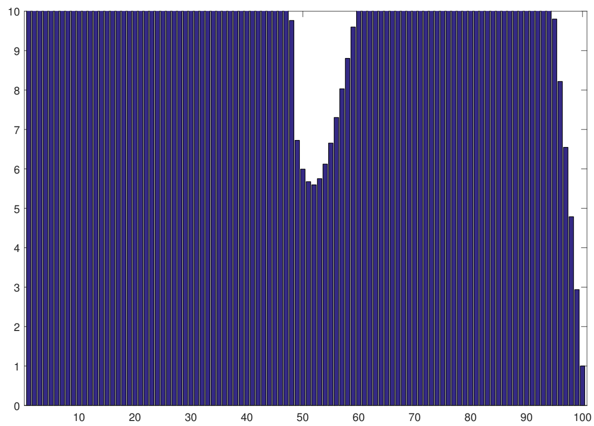
4.3 System identification

After downsampling and decimation of the data by factor 2, six well-estimated bending modes were obtained in the healthy and damaged states from the measurement data using subspace identification, together with their covariance. The identified frequencies are shown in Table 1 for each mode. Then the set of identified modes is split into three mode sets θ_1, θ_2 and θ_3 containing two modes each, respectively.

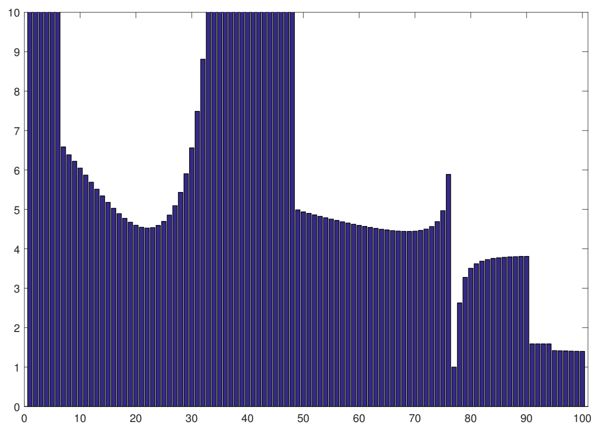
4.4 Damage localization results

The localization results are computed at all elements from the experimental datasets in both healthy and damaged states. The computation of the stress and its covariance for the statistical evaluation in (7) is carried out for three different mode sets, each with s-values in the vicinity of the respective mode sets. First, one s-value is chosen for each set with $s_1^1 = -1 + 190\mathrm{i}$ for mode set θ_1, $s_2^1 = -1 + 500\mathrm{i}$ for θ_2 and $s_3^1 = -1 + 1700\mathrm{i}$ for θ_3. In a second step, an additional s-value is chosen for joint evaluation for each mode set as $s_1^2 = -1 + 200\mathrm{i}$ for θ_1, $s_2^2 = -1 + 950\mathrm{i}$ for θ_2 and $s_3^2 = -1 + 2500\mathrm{i}$ for θ_3. To compare the ratios of the test statistics between the healthy and damaged elements, the computed values are normalized in the figures such that the smallest value is 1.

In Figures 3(a) and 3(b), the stress $S_t^1(s_1^1)$ is obtained for all 100 elements t for the single mode sets θ_1 and the respective test statistic is shown. It can be seen that neither the estimated stress nor its statistical evaluation can correctly indicate the damage at elements 16-17, which is possibly due to the modal truncation error. However, when using the joint statistical evaluation based on multiple mode sets, it is seen that damage can be localized correctly at elements 16-17 in Figures 4(a)-4(b).

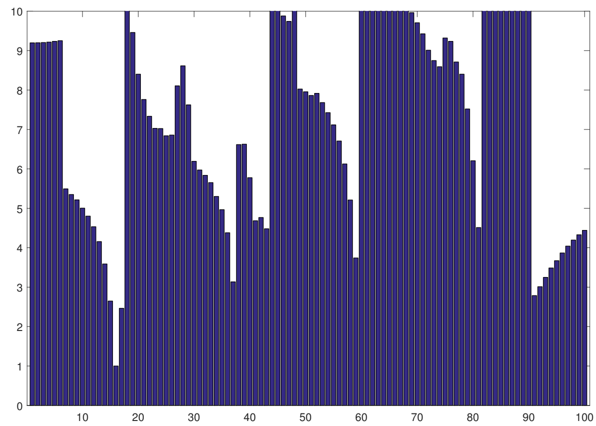


(a) Estimated stress S_t from data

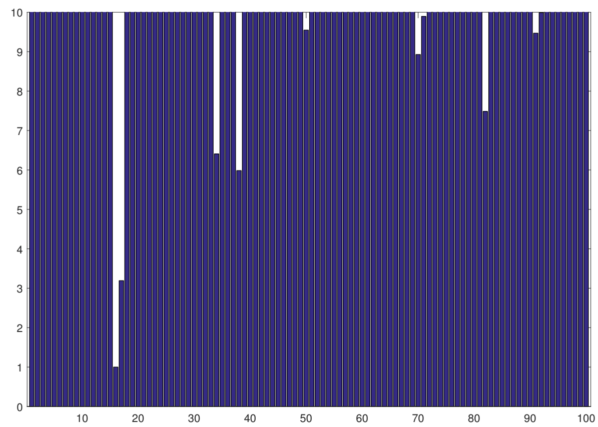


(b) Test statistic s_t

Fig. 3. Localization results for all elements using single mode set θ_1 at $s_1^1 = -1 + 190\mathrm{i}$ (experimental data).



(a) 1 s-value



(b) 2 s-values

Fig. 4. Test statistic s_t based on multiple mode sets θ_1, θ_2 and θ_3 (experimental data).

The use of one more s-value in Figure 4(b) increases the ratio to the undamaged elements compared to Figure 4(a), leading to a clearer localization.

4.5 Comparison to simulated data

Based on the above FE model, vibration data at the three sensor positions has been simulated from white noise excitation in both reference and damaged states, where the Young and shear modulus were reduced by 50% in elements 16 and 17 in the damaged case. White measurement noise with 5% magnitude of the simulated outputs was added. The simulated data is of length 10,000 at sampling frequency 1000 Hz after decimation, as in the previous section. From the data, the first six modes are identified and split into three mode sets as above.

The statistical evaluation using only the first mode set (as in Figure 3(b)) is shown in Figure 5, where the damage cannot be localized. Results using all mode sets with 2 s-values for each mode set (as in Figure 4(b)) are shown in Figure 6, where the lowest values of the test statistic are correctly found at the damaged elements.

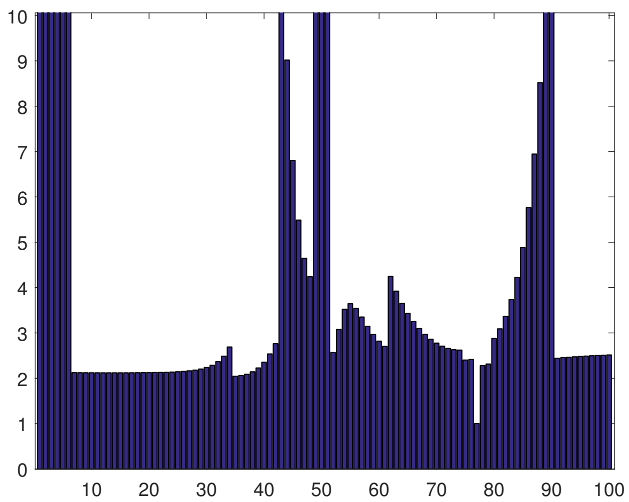


Fig. 5. Test statistic s_t using single mode set θ_1 at $s_1^1 = -1 + 190i$ (simulated data).

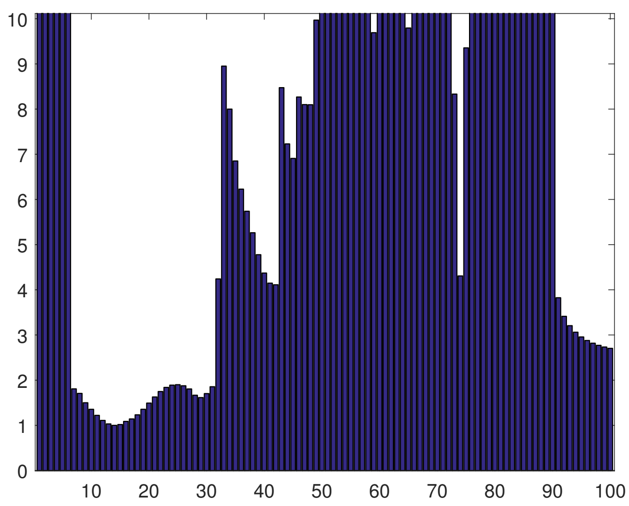


Fig. 6. Test statistic s_t based on multiple mode sets θ_1, θ_2 and θ_3, with 2 s-values each (simulated data).

5. CONCLUSIONS

In this work, damage localization with the SDDLV approach has been extended with a statistical approach considering multiple identified mode sets. The underlying stress computation in this approach using multiple mode sets increases the information content about the damaged or non-damaged elements of the structure, compared to evaluation from a limited number of modes due to a previous constraint of the approach on the number of modes. The method has been successfully applied to vibration measurements of a damaged cantilever beam, where damage was correctly localized with a small number of sensors, while the previous approach with a limited number of modes failed.

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