A Type of Arithmetic Labels about Circulating Ring

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Absteact

The graph composed with several rings is a kind of important and interesting graph, many scholars studied on the gracefulness of this kind of the graph, The reference [1] is given the gracefulness of m *kinds* C_4 *with one common point. In this paper, we researched the arithmetic labels of four kinds graph:* $C_{8,1,n}$ 、$C_{8,2,n}$ 、$C_{8,3,n}$ 、$C_{8,4,n}$ *, and we proved they are all* $(d,2d)$ *--arithmetic graph.*

Key Word: Arithmetic graph；labeling of graph；$C_{8,i,n}$

1. Introduction

The graph in this paper discussed are undirected、no multiple edges and simple graph, the unorganized state of definitions and terminology and the symbols in this graph referred to reference[2][3].

There are two kinds of labels of the graph: one is the reduced label, is to say that in order to get the label of one edge you should reduce the endpoints of the edge; the other is additive label, for the same you should acttive the endpoints of one edge to get the label of the edge. For example, the well-known of "Gracefulness" is reduced. the "Compatible labels" is additive. In 1990, B.D.Achaya and S.M.Hegde import the concept of "Arithmetic lables"(referred to reference[2]), which is a more extensive additive label, it have applied value on solution to question of the joint ventures in rights and obligations.

Definition 1.1 For $G=(V,E)$, if there is a mapping f (called the vertices v of the label) from $V(G)$ to the set of nonnegative integer N_0, meet:

(1) $f(u)\neq f(v)$， which $u\neq v$， and u，$v\in V(G)$；

(2) $\{f(u)+f(v)|uv\in E(G)\}=\{k,k+d,\cdots,k+(q-1)d\}$.

Then we call graph G is a (k,d) --arithmetic graph.

2. Main results and certification

Theorem 2.1 $C_{8,1,n}$ is a $(d,2d)$ --arithmetic graph.

Proof As the graph shown on fig.1.

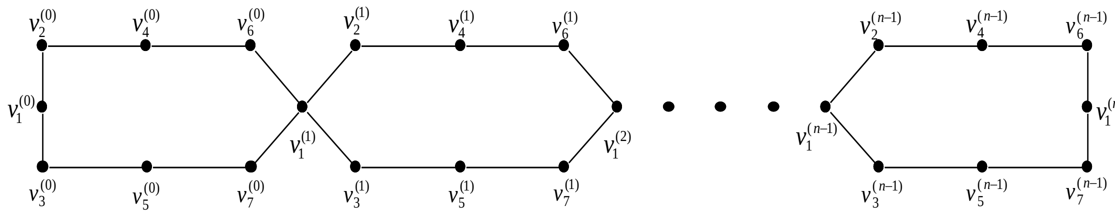

Fig.1.The Graf $C_{8,1,n}$

Label all vertices as follows:

$f(v_1^{(i)})=8id(i=0,1,2,\cdots,n)$, $f(v_2^{(i)})=8id+d(i=0,1,2,\cdots,n-1)$,

$f(v_3^{(i)})=8id+3d(i=0,1,2,\cdots,n-1)$, $f(v_4^{(i)})=8id+6d(i=0,1,2,\cdots,n-1)$,

$f(v_5^{(i)})=8id+2d(i=0,1,2,\cdots,n-1)$, $f(v_6^{(i)})=8id+5d(i=0,1,2,\cdots,n-1)$,

$f(v_7^{(i)})=8id+7d(i=0,1,2,\cdots,n-1)$.

Now we proof that the mapping f is arithmetic labeling of $C_{8,1,n}$.

We can see the mapping f meet $f(u)\neq f(v)$ which $u\neq v$ and $u,v\in V(C_{8,1,n})$.

Next we prove $\{f(u)+f(v)|uv\in E(C_{8,1,n})\}$ is an arithmetic progression in the way of mathematical induction.

When $n=1$

Then $f(v_1^{(0)})=0$, $f(v_2^{(0)})=d$, $f(v_3^{(0)})=3d$, $f(v_4^{(0)})=6d$, $f(v_5^{(0)})=2d$, $f(v_6^{(0)})=5d$, $f(v_7^{(0)})=7d$, $f(v_1^{(1)})=8d$.

Therefore $\{f(u)+f(v)|uv\in E(C_{8,1,1})\}$

$$=\{f(v_1^{(0)})+f(v_2^{(0)}),\ f(v_1^{(0)})+f(v_3^{(0)}),\ f(v_3^{(0)})+f(v_5^{(0)}),\ f(v_2^{(0)})+f(v_4^{(0)}),\ f(v_5^{(0)})+f(v_7^{(0)}),\ f(v_4^{(0)})+f(v_6^{(0)}),\ f(v_6^{(0)})+f(v_1^{(1)}),\ f(v_7^{(0)})+f(v_1^{(1)})\}$$

$$=\{d,3d,5d,7d,9d,11d,13d,15d\}$$

is an arithmetic progression, and the common difference is $2d$.

Suppose when $n=k-1$, we know

$$\{f(u)+f(v)|uv\in E(C_{8,1,k-1})\}=\{d,d+1\times 2d,d+2\times 2d,\cdots,d+(8k-9)\times 2d\}$$

is an arithmetic progression, and the common difference is $2d$.

Then when $n=k$

$$\{f(u)+f(v)|uv\in E(C_{8,1,k})\}$$

$$=\{f(u)+f(v)|uv\in E(C_{8,1,k-1})\}\cup\{f(v_1^{(k-1)})+f(v_2^{(k-1)}),\ f(v_1^{(k-1)})+f(v_3^{(k-1)}),\ f(v_3^{(k-1)})+f(v_5^{(k-1)}),\ f(v_2^{(k-1)})+f(v_4^{(k-1)}),\ f(v_5^{(k-1)})+f(v_7^{(k-1)}),\ f(v_4^{(k-1)})+f(v_6^{(k-1)}),\ f(v_6^{(k-1)})+f(v_1^{(k)}),\ f(v_7^{(k-1)})+f(v_1^{(k)})\}$$

$$=\{d,d+1\times 2d,d+2\times 2d,\cdots,d+(8k-9)\times 2d\}\cup\{d+(8k-8)\times 2d,\ d+(8k-7)\times 2d,\ d+(8k-6)\times 2d,\ d+(8k-5)\times 2d,\ d+(8k-4)\times 2d,\ d+(8k-3)\times 2d,\ d+(8k-2)\times 2d,\ d+(8k-1)\times 2d\}$$

$$=\{d,d+1\times 2d,d+2\times 2d,\cdots,d+(8k-1)\times 2d\}$$

is an arithmetic progression, and the common difference is $2d$.

In sum for the arbitrary $n\in N_0$, the mapping $f:V(C_{8,1,n})\to N_0$ meet:

(1) $f(u)\neq f(v)$ when $u\neq v$ and $u,v\in V(C_{8,1,n})$;

(2) $\{f(u)+f(v)|uv\in E(C_{8,1,n})\}=\{d,d+1\times 2d,d+2\times 2d,\cdots,d+(8n-1)\times 2d\}$.

Therefore $C_{8,1,n}$ is a $(d,2d)$--arithmetic graph.

Theorem 2.2 $C_{8,2,n}$ is a $(d,2d)$--arithmetic graph.

Proof As the graph shown on fig.2.

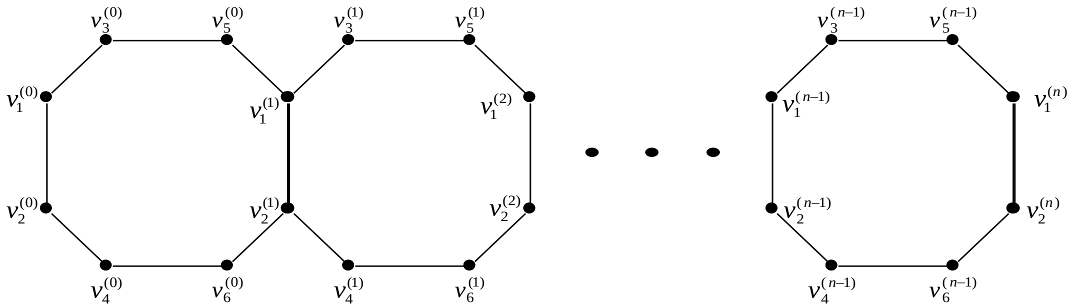

Fig.2.The Graf $C_{8,2,n}$

Label all vertices as follows:

$f(v_1^{(i)}) = 7id\,(i = 0,1,2,\cdots,n)$, $f(v_4^{(i)}) = 7id + 6d\,(i = 0,1,2,\cdots,n-1)$,

$f(v_2^{(i)}) = 7id + d\,(i = 0,1,2,\cdots,n)$, $f(v_5^{(i)}) = 7id + 2d\,(i = 0,1,2,\cdots,n-1)$,

$f(v_3^{(i)}) = 7id + 3d\,(i = 0,1,2,\cdots,n-1)$, $f(v_6^{(i)}) = 7id + 5d\,(i = 0,1,2,\cdots,n-1)$.

Now we proof that the mapping f is arithmetic labeling of $C_{8,2,n}$

We can see the mapping f meet $f(u) \neq f(v)$ which $u \neq v$ and $u,v \in V(C_{8,2,n})$.

Next we prove $\{f(u) + f(v) | uv \in E(C_{8,2,n})\}$ is an arithmetic progression in the way of mathematical induction.

When $n = 1$

Then $f(v_1^{(0)}) = 0$, $f(v_2^{(0)}) = d$, $f(v_3^{(0)}) = 3d$, $f(v_4^{(0)}) = 6d$, $f(v_5^{(0)}) = 2d$, $f(v_6^{(0)}) = 5d$, $f(v_1^{(1)}) = 7d$, $f(v_2^{(1)}) = 8d$.

Therefore $\{f(u) + f(v) | uv \in E(C_{8,2,1})\}$

$= \{f(v_1^{(0)}) + f(v_2^{(0)}),\ f(v_1^{(0)}) + f(v_3^{(0)}),\ f(v_3^{(0)}) + f(v_5^{(0)}),$
$f(v_2^{(0)}) + f(v_4^{(0)}),\ f(v_5^{(0)}) + f(v_1^{(1)}),\ f(v_4^{(0)}) + f(v_6^{(0)}),\ f(v_6^{(0)}) + f(v_2^{(1)}),\ f(v_1^{(1)}) + f(v_2^{(1)})\}$

$= \{d, 3d, 5d, 7d, 9d, 11d, 13d, 15d\}$

is an arithmetic progression, and the common difference is $2d$.

Suppose when $n = k-1$, we know

$\{f(u) + f(v) | uv \in E(C_{8,2,k-1})\} = \{d, d + 1\times 2d, d + 2\times 2d, \cdots, d + (7k-7)\times 2d\}$

is an arithmetic progression, and the common difference is $2d$.

Then when $n = k$

$\{f(u) + f(v) | uv \in E(C_{8,2,k})\}$

$= \{f(u) + f(v) | uv \in E(C_{8,2,k-1})\} \cup \{f(v_1^{(k-1)}) + f(v_3^{(k-1)}),\ f(v_3^{(k-1)}) + f(v_5^{(k-1)}),$
$f(v_2^{(k-1)}) + f(v_4^{(k-1)}),\ f(v_5^{(k-1)}) + f(v_1^{(k)}),\ f(v_4^{(k-1)}) + f(v_6^{(k-1)}),\ f(v_6^{(k-1)}) + f(v_2^{(k)}),\ f(v_1^{(k)}) + f(v_2^{(k)})\}$

$= \{d, d + 1\times 2d, d + 2\times 2d, \cdots d + (7k-7)\times 2d\} \cup \{d + (7k-6)\times 2d,\ d + (7k-5)\times 2d,$
$d + (7k-4)\times 2d,\ d + (7k-3)\times 2d,\ d + (7k-2)\times 2d,\ d + (7k-1)\times 2d,\ d + 7k\times 2d\}$

$= \{d, d + 1\times 2d, d + 2\times 2d, \cdots, d + 7k\times 2d\}$

is an arithmetic progression, and the common difference is $2d$.

In sum for the arbitrary $n \in N_0$, the mapping $f : V(C_{8,2,n}) \to N_0$ meet:

(1) $f(u) \neq f(v)$ when $u \neq v$ and $u,v \in V(C_{8,2,n})$;

(2) $\{f(u)+f(v) | uv \in E(C_{8,2,n})\} = \{d, d+1\times 2d, d+2\times 2d, \cdots, d+7n\times 2d\}$.

Therefore $C_{8,2,n}$ is a $(d,2d)$ --arithmetic graph.

Theorem 2.3 $C_{8,3,n}$ is a $(d,2d)$ --arithmetic graph .

Proof As the graph shown on fig.3.

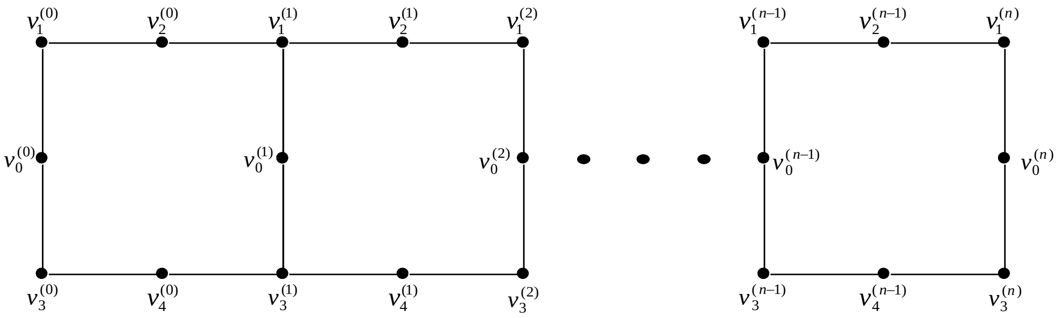

Fig.3.The Graf $C_{8,3,n}$

Label all vertices as follows:

$f(v_0^{(i)}) = 8id (i = 0,1,2,\cdots,n)$, $f(v_1^{(i)}) = 4id + d (i = 0,1,2,\cdots,n)$,

$f(v_3^{(i)}) = 4id + 3d (i = 0,1,2,\cdots,n)$, $f(v_2^{(i)}) = 8id + 6d (i = 0,1,2,\cdots,n-1)$,

$f(v_4^{(i)}) = 8id + 2d (i = 0,1,2,\cdots,n-1)$.

Proof in imitation of Theorem 2.1.

For the arbitrary $n \in N_0$, the mapping $f : V(C_{8,3,n}) \to N_0$ meet:

(1) $f(u) \neq f(v)$ when $u \neq v$ and $u,v \in V(C_{8,3,n})$;

(2) $\{f(u)+f(v) | uv \in E(C_{8,3,n})\} = \{d, d+1\times 2d, d+2\times 2d, \cdots, d+(6n+1)\times 2d\}$.

By (1), (2) we know the mapping f is arithmetic labeling of $C_{8,3,n}$. Therefore $C_{8,3,n}$ is a $(d,2d)$ --arithmetic graph.

Theorem 2.4 $C_{8,4,n}$ is a $(d,2d)$ --arithmetic graph.

Proof As the graph shown on fig.4.

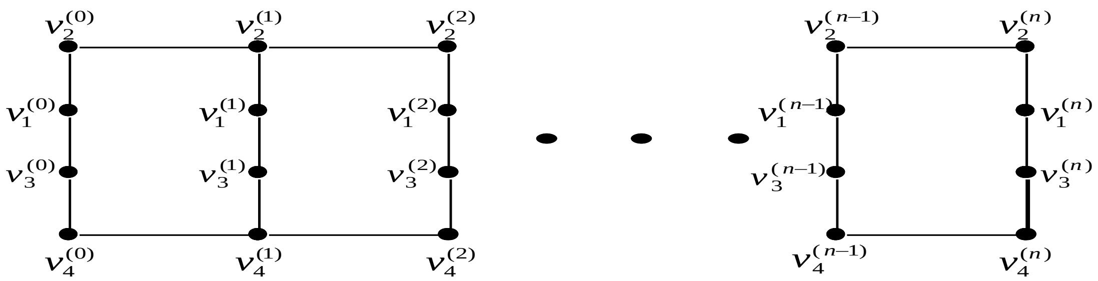

Fig.4.The Graf $C_{8,4,n}$

Label all vertices as follows:

$f(v_1^{(i)}) = 5id (i = 0,1,2,\cdots,n)$, $f(v_2^{(i)}) = 5id + d (i = 0,1,2,\cdots,n)$,

$f(v_3^{(i)}) = 5id + 3d(i = 0,1,2,\cdots,n)$, $\quad$ $f(v_4^{(i)}) = 5id + 2d(i = 0,1,2,\cdots,n)$.

Proof in imitation of Theorem 2.2.

For the arbitrary $n \in N_0$, the mapping $f : V(C_{8,4,n}) \to N_0$ meet:

(1) $f(u) \neq f(v)$ when $u \neq v$ and $u, v \in V(C_{8,4,n})$;

(2) $\{f(u) + f(v) | uv \in E(C_{8,4,n})\} = = \{d, d + 1 \times 2d, d + 2 \times 2d, \cdots, d + (5n+2) \times 2d\}$.

By (1), (2) we know the mapping f is arithmetic labeling of $C_{8,4,n}$. Therefore $C_{8,4,n}$ is a $(d, 2d)$ --arithmetic graph.

3. Labels of Some Special Graph

In order to explan the correctness of the aforementioned labels, we give the arithmetic labeling of $C_{8,1,3}$ $C_{8,2,3}$ $C_{8,3,3}$ and $C_{8,4,3}$.

(1) The arithmetic labeling of $C_{8,1,3}$ on fig.5.

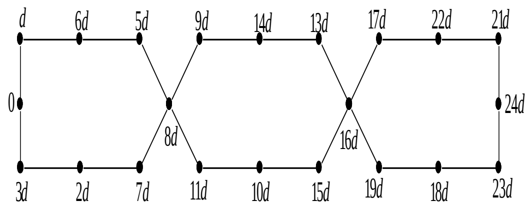

Fig.5. The Graf $C_{8,1,3}$

(2) The arithmetic labeling of $C_{8,2,3}$ on fig.6.

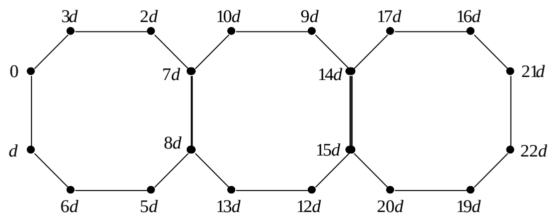

Fig.6. The Graf $C_{8,2,3}$

(3) The arithmetic labeling of $C_{8,3,3}$ on fig.7.

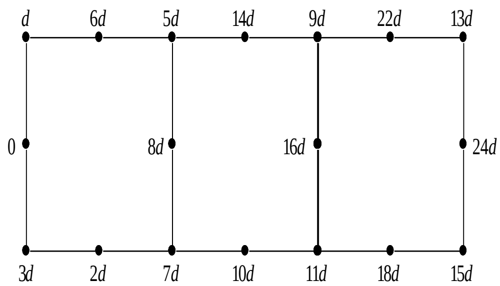

Fig.7. The Graf $C_{8,3,3}$

(4) The arithmetic labeling of $C_{8,4,3}$ on fig.8.

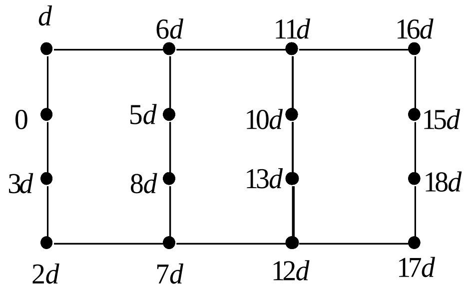

Fig.8. The Graf $C_{8,4,3}$

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