Calculation and Analysis of Double-Axis Elliptical-Parabolic Compond Flexure Hinge

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[1]Calculation and analysis of double-axis elliptical-parabolic Compond flexure hinge

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Abstract. A double-axis flexure hinges combined with elliptical and parabolic curve is presented in this paper. The design equations for its compliance computation have been deduced by application of Castigliano's displacement theorem after the anlysis of the stucture，which were analyzed by the use of the software MATLAB, and comfirm the validity of the model with finite element analysis software ANSYS.

Keywords: flexure hinge；mixed flexure hinge；compliance；Castigliano' theorem

Introduction

Flexible hinge eliminates its idling running and mechanical friction by using small angle elastic deformation and self-recovery characteristics in the transmission process , and can get highter resolution of displacement. It is been widely used in various occations, such as small angular displacement, high-precision rotation of occasions, especially in precision measurement, positioning and other fields because of its small-volume, gapless, no mechanical friction,no lubrication, in-gage and high sensitivity of the transmission structure[1-3].

People have been made a series of studies on the single-axis oval, chamfered, parabolic and hyperbolic flexure hinge[4-5] since 1965 Paros and Weisbord derived circular flexure hinge flexibility formul. However,they have done little reaserch about

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the compound flexure hinge in addition to Chengui Min's [6] on the right circular and oval compound flexure hinge. Elliptical flexible hinge has a lager range of rotation but lower accuracy,and Parabolic with small range of rotation but high precise compared to the Circular incision hinge; for both accuracy and range of motion exercise, combining the elliptic and parabolic type flexure hinge will take the advantages of both flexible hinges, so this paper will present elliptic - parabolic compound flexible hinge.

1 Elliptie - parabolic compound flexible hinge

Elliptic and parabolic confound flexible hinge is composed of two half elliptical and parabolic form, as shown in figure1, oval-side as a fixed end and the Parabolic side as the rotation-side, and make the following assumptions:

（1）Elliptic and parabolic incision have the same thickness,and both are t.

（2）Elliptic and parabolic are tangented at the intersection, or smooth over.

（3）Two incision length in the X direction are m。

（4）Ellipse major axis m is greater than the minor axis n, when m=n,that is circurlar and parabolic compound flexible hinge.

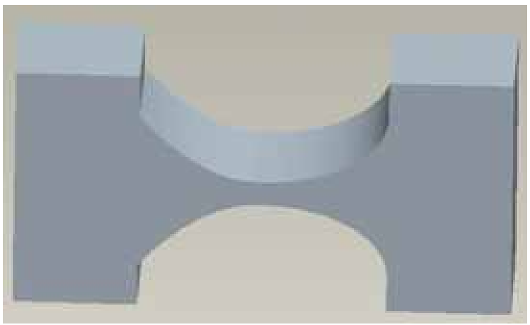

Fig. 1. elliptic - parabolic hinge pro/e model

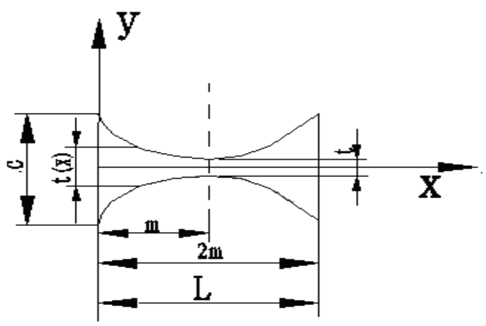

Fig. 2. the structure parameters of compound hinge

2 Flexibility Formulas

Assumptions based on cantilever small deformation,bending deformation is generated by the force and moment. The impact of axial load has considered, while the impact of shear and torsion were not. The structure parameters of axial elliptic - parabolic compound flexure hinge were shown in Figure 2,and the force analysis as shown by Figure 3, the flexibility formulas of compound hinge were deduced based on mechanical card's second theorem:

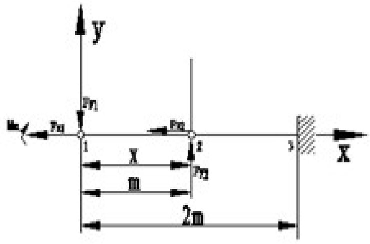

Fig. 3. the force analysis of compound hinge

$$\begin{Bmatrix} \theta_{z1} \\ y_1 \\ x_1 \end{Bmatrix} = \begin{Bmatrix} C_{\theta-M} & C_{12} & 0 \\ C_{12} & C_Y & 0 \\ 0 & 0 & C_X \end{Bmatrix} \begin{Bmatrix} M_{z1} \\ F_{y1} \\ F_{x1} \end{Bmatrix} \quad (1)$$

According to reciprocal theorem known $\theta_{z1} = C_{11}M_{z1} + C_{12}F_{y1}$,it can be obtained from the left of formula (1) with the use of cartesian displacement：

$$\begin{cases} \theta_{z1} = \dfrac{\partial u}{\partial M_{z1}} \\ y_1 = \dfrac{\partial u}{\partial F_{y1}} \\ x_1 = \dfrac{\partial u}{\partial F_{x1}} \end{cases} \quad (2)$$

By the material mechanics, without considering the shear and torsion, the deformation energe of the compound flexible hinge is：

$$U = \frac{1}{2}\left[\int_l \frac{F_{X1}^{\ 2}}{EA(x)} dx + \int_l \frac{M_Z^{\ 2}}{EI_z(x)} dx \right] \quad (3)$$

and $F_x = F_{x1}$, $Mz = M_{z1} + F_{y1}x$, $A(x) = bt(x)$, $I(x) = \left(bt(x)^3\right)/12$

The coordinate system as shown in figure 2, variable thickness equations of dual-axis flexure hinge as follow：

$$t(x) = \begin{cases} t + 2\left[n - \dfrac{n}{m}\sqrt{2mx - x^2}\right] & 0 \le x \le m \\ t + 2\left[2p(x-m)^2\right], m < x \le l \end{cases} \quad (4)$$

To the type of substitution (1) (2) (3) can obtain the flexibility formulas：

$$C_X = \frac{1}{EB}\left\{ m * \frac{-\pi\sqrt{-t(4n+t)} - (8n+4t)Arc\tan\left(\dfrac{2n+t}{\sqrt{-t(4n+t)}}\right)}{4n\sqrt{-t(4n+t)}} + \frac{Arc\tan\left(\dfrac{2m\sqrt{p}}{\sqrt{t}}\right)}{2\sqrt{pt}} \right\} \quad (5)$$

$$C_Y = \frac{12}{EB}\left\{ m^3 \frac{16c^3\left(-80c^4 + 24c^3t + 8c^2(3+2\pi)t^2 + 4c(1+2\pi)t^3 + \pi t^4\right)}{16c^3t^2\left(4c+t\right)^2} + m^3 \frac{(2n+t)^3(6n^2 - 4nt - t^2)}{4n^3\left(-t(4n+t)\right)^{\frac{5}{2}}} Arc\tan\left(\frac{\sqrt{-4nt - t^2}}{t}\right) \right.$$

$$\left. + \frac{m(112p^2m^4 + 56pm^2t - t^2)}{32p^2t^2(4pm^2 + t)^2} + \frac{(12pm^2 + t)Arc\tan\left(\dfrac{2m\sqrt{p}}{\sqrt{t}}\right)}{64p^{\frac{3}{2}}t^{\frac{5}{2}}} \right\} \quad (6)$$

$$C_{\theta-M}=\frac{12}{EB}\left\{m\frac{6n^2+4nt+t^2}{t^2(2n+t)(4n+t)^2}-m\frac{6n(2n+t)}{\left(-4nt-t^2\right)^{\frac{5}{2}}}Arc\tan\left(\frac{2n+t}{\sqrt{-4nt-t^2}}\right)+\frac{m\left(12pm^2+5t\right)}{8t^2\left(4pm^2+t\right)^2}+\frac{3Arc\tan(\frac{2m\sqrt{p}}{\sqrt{t}})}{16\sqrt{p}t^{\frac{5}{2}}}\right\} \tag{7}$$

From the formulas of elliptical-parabolic compound hinges which have deduced,it can easily see that the flexibility is inversely proportional to the thickness B and modulus of elasticity E.

2.1 The verification of Closed-loop compliance formula

Design two dual-axis elliptical – parabolic compound hinges, its parameters as follows: thickness B is 5mm, length of oval long axle 5mm, the minor axises 3mm and 5mm, the thinnest thickness in the middle is 1mm, the total width 10mm. FEM experimental parameters are:

modulus of elasticity：E = 110 $\times 10^9$ Nm^{-2}，Poisson ratio： μ = 0. 3.

parabol parameters：p=1/16 Fx1 = Fy1 = 1 N Mz1 = 1 N.m.

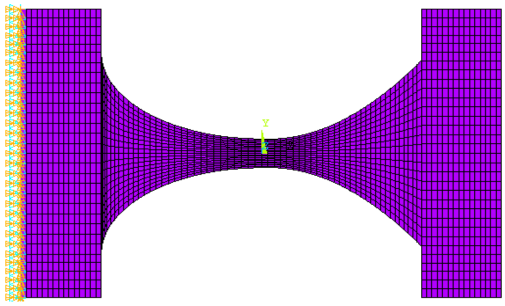

Fig. 4. the ansys geometric model of compound

Models with different parameters in the finite element software to verify compliance formulas, choose the shell63 finite element, making finite element analysis to compound flexure hinge(As shown in figure 4), analytical method(AM) and finite element results were shown in Table 1. Known from the data in the table, finite element method(FEM) and the exact formula is less than 10% deviation.

Table 1, analytical and finite element calculation about flexibility

	t/m	m/m	n/m	AM	FEM
$C_{11}(N^{-1}m^{-1})$	0.001	0.005	0.003	1.44	1.37
	0.001	0.005	0.005	1.08	1.14
$C_{22}(N^{-1}m^{-1}\times 10^{-6})$	0.001	0.005	0.003	6.87	7.24
	0.001	0.005	0.005	6.31	6.52
$C_{33}(N^{-1}m^{-1}\times 10^{-8})$	0.001	0.005	0.003	1.43	1.37
	0.001	0.005	0.005	1.35	1.44

2.2 Performance Analysis

From the formulases (5) to (7),it is easy to know that the formula of flexibility is inversely proportional to modulus E and thickness B; the relationship between the CX, CY, Cθ-M and the short axis length n、 incision thickness t were shown in figure 5-7.

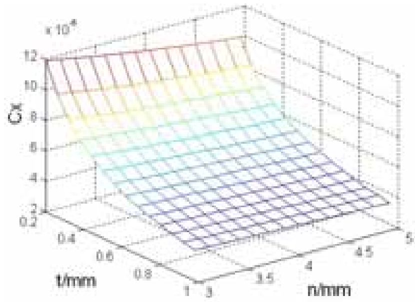

Fig. 5.the relationship between Cx and n、t

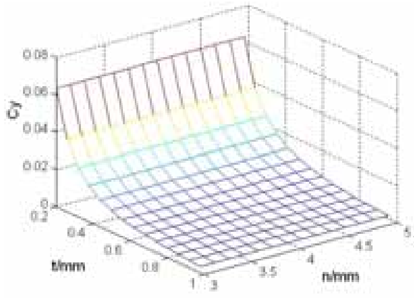

Fig. 6. the relationship between Cy and n、t

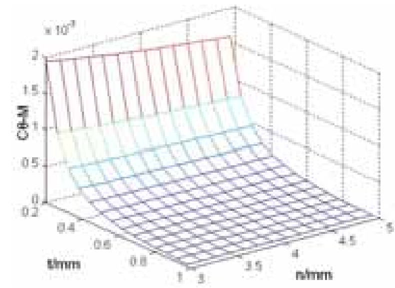

Fig. 7. the relationship between Cθ-M and n、t

（1）We can see from figure 5, the relationships between the flexibility of x-axis direction Cx and t, n are in the overall smooth, keep the t unchanged, with the increase of n, Cx will decrease linearly; if the n unchanged, Cx decrease with t increases and changes in relatively fast as t in the range of 0.2-0.4, Cx changes in relatively fast ,in conclusion the stiffness of X axis is large relatively.

（2）The relationship between flexibility Cy and t、n as shown in Figure 6，Cy will reach maximum when t equals to 0.2,and Cy will change sharply as t in the range of 0.2-0.4 and decrease smoothly when t is greater than 0.4, Cy has no significant change with n.

（3）From the figure 7,we can see that the C θ -M obtains maximum when t equals to 0.2 and decreases as n increase,especiallly when t in the range of 0.2-04, it decreases sharply； when t is greater than 0.6,the C θ -M decreases slowly and close to zero.

From the anlysis above all,this conpound hinge has large rigidity in the direction of x –axis and good flexibility in y axis direction and rotate around Z axis,so it is very suitable as rotation flexibility hinge.

3 Conclusions

This paper presents a new type of compound hinge oval - parabolic compound flexible hinge. And analized its structure, its flexibility formulas were deduced based on card's second law and vertified by the finite element software,find the errors are within 8%, which demonstrates the correctness of the flexibility formulas. Then analyzed the relationship between the flexibility and the thickness B and the short axis lengh n with matlab software,and got some conclusions, it could be a theoretical references in future industry and agriculture fields.

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