“Omnia Numerorum Videntur Ratione Formata”. A ‘Computable World’ Theory in Early Medieval Philosophy

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Abstract. Digital philosophy is a speculative theory that places the bit at the foundation of reality and explains its evolution as a computational process. This theory reinterprets some previous philosophical intuitions, starting from the Pythagorean theory of numbers as the beginning of all things and as a criterion for the comprehension of reality. Significant antecedents of this computational philosophical approach can be found in the tradition of late antiquity and the early Middle Ages. One of the less investigated chapters of this 'pre-history' of digital philosophy can be found in the Ottonian Renaissance, when we can identify theorists of what has been called – in reference to modern authors as Leibniz – a 'computational paradigm'. The paper focuses on the works of Abbo of Fleury and Gerbert of Aurillac. Their theoretical basis is the famous verse of *Wis* 11, 21 (*Omnia creata sunt in numero mensura et pondere*).

Keywords. Digital Philosophy · Computational Paradigm · Metaphysics of Numbers · Medieval Philosophy · Ottonian Renaissance · Abbo of Fleury · Gerbert of Aurillac

1 Introduction

Digital philosophy is strictly speaking a new speculative theory, developed in recent decades by Edward Fredkin, Gregory Chaitin and Stephen Wolfram, who place the bit at the foundation of reality and explain the evolution of reality as a computational process. This theory actually reinterprets some previous philosophical intuitions, starting from the Pythagorean theory of numbers as the beginning of all things (on the metaphysical side) and as a criterion for the comprehension of reality (on the epistemological one).

Significant antecedents of this computational philosophical approach, however, can be found in the speculative tradition of late antiquity, as well as in the early Middle Ages. In particular, in Western thought there is a path that goes from Augustine of Hippo (354-430) to the School of Chartres (12th century), passing through Martianus Capella (4th-5th century), Severinus Boethius (480-524), Cassiodorus (485-580), the Venerable Bede (672/673-735) and Remigius of Auxerre

(841-908). The *De (institutione) arithmetica* of Boethius is a derivation of the *Introduction to Arithmetic* of Nicomachus of Gerasa, a Neo-Pythagorean roman philosopher and mathematician of the 2nd century d.C.

This book, which Boethius translated from Greek into Latin, was a very influential treatise on number theory and was considered a standard authority for many centuries, setting out the elementary theory and properties of numbers and containing the earliest-known Greek multiplication table. At the beginning of the *De arithmetica*, Boethius defines the *quadrivium* and in particular the study of the mathematics as a means to get closer to perfection and to perceive the infinite.

> *Omnia*, quaecumque a primaeva rerum natura constructa sunt, *numerorum videntur ratione formata*. Hoc enim fuit principale in animo conditoris exemplar.[1]

> *All created things*, which were built from the primeval nature of things, *are formed according to the rational structure of numbers*. For this was the principal copy in the mind of the builder.[2]

"All created things are formed according to the rational structure of numbers": Boethius therefore refers explicitly to the principle of Pythagorean thought, learned through the intermediation of Nicomachus.

One of the less investigated chapters of this 'pre-history' of digital philosophy is placed in the so-called Ottonian Renaissance, when we can identify some theorists of what has been called – in reference to modern authors as Leibniz – 'computational paradigm'. At the end of the 10th century, this paradigm becomes the theoretical background for the development of a concept that, although not in a systematic form, outlines the contours of an ordered vision of the world that updates the Pythagorean dream of perfection and numerical harmony.

2 Abbo of Fleury

One still largely misunderstood example of this mathematical approach from this historical and cultural context is the most speculative work of Abbo, abbot of Fleury Abbey in Saint Benoît-sur-Loire-from 988 to his death (1004). It is a commentary on an apparently negligible philosophical work, the *Calculus* of Victorius of Aquitaine, which was a calculation manual written primarily for exercises around 450 and consists mainly of multiplication tables. The only *intentio* of Victorius seems to be to ensure the correct calculation in every numerical problem of the disciplines of the *quadrivium*, the "artes quae numerorum ratione constant". However, the preface of the *Calculus* gives room for the commentator to theorize, precisely where Victorius defines arithmetic as the science of the unity that is the source of the multiplicity of the numbers:

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1 [4], I, 2, p. 14.

2 English translations of the Latin quotations are mine.

Unitas illa, unde omnis numerorum multitudo procedit, quae proprie ad arithmeticam disciplinam pertinet, quia vere simplex est et nulla partium congregatione subsistit, nullam utique recipit sectionem.[3]

It is that unity from which the entire multitude of numbers proceeds and which pertains particularly to the discipline of arithmetic because it is truly simple, does not subsist in a collection of parts, and in no way allows for division.

Abbo was inspired by this *sententia* and he wrote a long commentary, which goes far beyond the mere literal explanation typical of the glosses of that time. The intent of Abbo, who composed his work around 985, is to provide an 'introductory bridge' to arithmetic (*ysagoge arithmeticae*) in the form of an exposition (*sub expositionis tenore ad arithmeticam introductionis pontem construo*) that is a scholastic book, destined for the confreres who were less versed in the discipline.

In an effort to draw inspiration from the *Calculus* – a simple list of measures of various kinds and origin, essentially unreadable, for a universal 'measurement' of the cosmos – Abbo wants to emphasize how different systems of weight, capacity and size can converge into a global demonstration of the mathematical nature of all reality, despite the apparent diversity [8]. Abbo resorts to different sources that correspond to the three main spheres of his analysis: arithmetic, dialectic and cosmology. His favourite sources are the commentary of Macrobius on the *Somnium Scipionis* of Cicero, the commentary of Chalcidius on the Platonic *Timaeus*, the seventh book of *De nuptiis Philologiae et Mercurii* of Martianus Capella, the *Etymologiae* of Isidore of Seville, the *De arithmetica* and the *Consolatio philosophiae* of Boethius and his commentary on the *Categoriae* of Aristotle, the *De interpretatione* and the *Topica* of Cicero and the *De definitione* of Marius Victorinus.

The abbot of Fleury is therefore interested in deepening both the speculative the practical implications of mathematics and of the scientific literature available to him. Already in the work of Victorius some speculative arithmetic topics were joined to the traditional problems of *calculus*, but in the Commentary the practical purpose seems to be upstaged by the symbolic potential of the number, which for Abbo is not just a mere mathematical value, but a metaphysical principle as well.

In support of this reading, it should be remembered that as early as the beginning of the 9th century, it was no longer necessary to create a calendar or calculate the Easter day, because religious people could rely on the Easter tables written by the sixth century Egyptian Dionysius Exiguus and on the *De temporum ratione* of the Venerable Bede (composed about 703). The updates to the tables made by Abbo (the author of an important liturgical *computus*) and by the computists of his time do not constitute, therefore, developments of the technique, but are interesting variants from the figurative point of view.

The *Ephemerida* is the only item of Abbo's *computus* formally ascribed to him. It comprises three interconnected parts: an acrostic poem about astronomical topic, which is actually a *computus* table, a perpetual solar calendar to which the acrostic is the key and a lunar-letter sequence. The poem closely echoes the cosmological meters

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3 [20], praefatio, p. 3.

of *Consolatio philosophiae* [13].

Analyzing the so-called 'table and rules of Ramsey', Charles Burnett [6] showed that the purpose of the tables and the representations of Abbo were not to explain how to multiply the numbers but to show the way in which the numbers can occur and connect to each other. This is clear in particular from the arrangement of the elements in the table prepared by Abbo, who intentionally reverses the order, jeopardizing its practical usefulness but enhancing what is more interesting for him, i.e. the figurative representation of the derivation of all the numbers from the unity, collocated on the left and not – as usual – on the right side.[4]

According to Eva-Maria Engelen [9], the entire production of Abbo is suffused with the idea of showing the wonderful power of the number. Giulio d'Onofrio [8] defines the commentary on the *Calculus* as a profound treatise on the ability of mathematics to produce in the mind a high theological knowledge, through the perception and the conceptual representation of the harmony that God established in the creation. Abbo traces the typical path of the Christian philosophy rising from the visible things to the invisible things, in which lies the true meaning of the things perceptible by the senses, up to the inexplicable unity of the divine Trinity.

It is no coincidence that since the beginning of his work, the abbot of Fleury states that through the study of number, size and weight, it's possible to deepen the knowledge of the nature of the Creator: it's not just matter for calculation, it's a matter for contemplation. His definition of wisdom is inspired by the Boethian definition found in the *De arithmetica*: «The wisdom of God is the subtle contemplation and perfect knowledge of the things that are always in the same way, the comprehension of the whole truth. (*Est enim sapientia divinitatis subtilis contemplatio ac eorum quae semper eodem modo sunt perfecta cognitio, veritatisque integra comprehensio*)».[5]

The famous verse of *Wis* 11, 21 (*Omnia creata sunt in numero mensura et pondere*) is not for Abbo just a scriptural quotation: he gives to his work the subtitle "Number, size and weight" (*Tractatus de numero, pondere et mensura*), i.e. the triple means by which God has ordered the creation.[6] Stating that all the things were created and arranged by God not only according to number, the intelligible principle of the

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4 As for the definition of *abaci doctor* with whom Abbo qualifies himself, see [6], p. 138, "Ni dans «la table et les règles de Ramsey», ni dans le commentaire sur le *Calculus* de Victorius, Abbon ne donne d'instructions pour effectuer des opérations de calcul sur l'abaque, et il spécifie bien au lecteur, dans le commentaire sur le *Calculus*, qu'il n'est pas en train d'écrire un traité sur l'abaque (…) Il est vraisemblable qu'*abacus*, dans ce contexte, signifie «calcul» en général."

5 [1], p. 66. See [4], I, 1, p. 9.

6 See [17], p. XXVIII, "Abbo's Commentary is the most wide-ranging of his educational works, and displays a confidence that Creation is rational, numerical and knowable through any and all of the liberal arts. But Abbo's investigation is firmly set in a spiritual and contemplative dimension. (…) The main thrust of this tradition of exegesis was that an exploration of number, measure and weight will reveal the ordering principles of Creation and lead the soul closer to God."

Pythagorean and Neo-Platonic tradition, but also according to measure and weight, Abbo reiterates the Christian doctrinal tradition according to which it's possible, starting from the contemplation of these three quantitative elements, to go back to the knowledge of the principles, and even up to the Trinity.

The universe has a rational and intelligible structure, an essentially mathematical order. The abbot of Fleury discusses the relationship between the One and the many starting from a quotation of Chalcidius on the generation of the soul of the world (the *anima mundi*), which descends from the divine, simple and indivisible source of the plurality of the numbers.

The indivisible unity is the beginning and the end of all the divisible things. All reality is a development of this unity. The relationship between unity and multiplicity is indeed the theoretical core of the commentary, which appears to be directly based on the Pythagorean and Neo-Platonic realism of the *Arithmetica* and the *Consolatio*, for which – in the words of Barbara Obrist – the number *is* the being and the foundation of the cosmic model.

Abbo takes from the early medieval philosophical tradition the idea that the unity is the structure of the divine reason, the ideal model on the basis of which the world was created, the origin of multiplicity and of number itself: this tradition dates back to Boethius, who defines number as the mother of all things and unity as the privileged object of intellectual abstraction:

> Pronuntiandum est, nec ulla trepidatione dubitandum, quod quemadmodum per se constantis quantitatis unitas principium et elementum est, ita et ad aliquid relatae quantitatis aequalitas mater est.[7]
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> Quare constat primam esse unitatem cunctorum, qui sunt in naturali dispositione, numerorum et eam rite totius quamvis prolixae genitricem pluralitatis agnosci.[8]
>
> It must be declared and it must not be doubted with any trepidation that, just as the unity of the of the unchanging quantity in itself is the origin and first principle, so also the equality of a quantity related to something is the maternal source.
>
> Whereby it is agreed that first there is the unity of all the things that are in the natural disposition of numbers, and it is then rightly recognized as the mother of the entire plurality.

Compared to the Carolingian period, in which the study of the Bible is still the predominant science (the *De computo* of Rabanus Maurus – the so-called "praeceptor Germaniae" – follows the tradition of the *De temporum ratione* of the Venerable Bede and merely offers instructions in the elementary practice of calculating and *computus*), Abbo seems to assign a more speculative value to the scientific content.

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[7] [4], II, 1, p. 96.
[8] [4], I, 7, p. 20.

Unity, order and harmony can all be traced back to number, since the correct relationship between the parts is the one that respects the measure, which in turn is determined by the number.

3 Gerbert of Aurillac

Given that there are no lack of examples (above all, Constantine of Reims,) of the common program conducted by Abbo of Fleury and Gerbert of Aurillac – teacher and then bishop of Reims before being elected pope with the name of Sylvester II († 1003) – it is no wonder then that both shared a strong interest in calculation[9]. Like Abbo, Gerbert gave a decisive contribution to the discovery and the study of many arithmetical, geometrical and astronomical texts of the late antiquity.

His arithmetical works don't follow the previous tradition of *computus*, that was mainly related to the practical aspects of the discipline, because Gerbert learned the new elements of the Arabic science during his early trip to Catalonia, but also due to his skillful reworking of the traditional sources. As for the abbot of Fleury, in fact, his energetic impetus to the progress of the science has its speculative roots in the (onto)theological frame of Pythagorean-Platonic origin.

Richer of Saint-Remi, a pupil of Gerbert at Reims (one of the main centers for studying arithmetic) and his passionate biographer, writes that his teacher, a faithful follower of Boethius, divides philosophy into theology, mathematics and physics, and identifies the respective objects in *intellectibilia*, *intelligibilia* and naturalia.[10] This model will be retrieved by Boethius and some of his commentators, like Gilbert of Poitiers, the greatest metaphysician of the 12th century.

The entire production of Gerbert is animated by a dynamic conception of philosophical inquiry, open to exchanges with other disciplines, especially dialectic. Among his scientific works, we can highlight the *Regulae de numerorum abaci rationibus*, dedicated to the use of the *abacus* (the main innovation for the arithmetic studies), the *Scholium ad Boethii Arithmeticae Institutiones II, 10*, written at the invitation of the emperor Otto III, the *Epistola ad Adelboldum de causa diversitatis arearum trigoni aequilateri geometrice arithmeticeve expensi*, the *Libellus de numerorum divisione* and a *Fragmentum de norma rationis abaci* (both addressed to Constantine of Fleury in letter form), the *Liber Abaci*, and especially the treatise of Geometry (*Isagoge geometriae*), composed after 983 and designed as a completion of the Boethian writings on the *quadrivium.*

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9 Regarding the cooperation of Abbo and Gerbert in the creation of a new post-Carolingian Renaissance, based on the recovery and on the comparison with the arithmetical and logical works of Boethius, see the interesting reflections of Giulio d'Onofrio [7] in the edition of the *Excerpta isagogarum et categoriarum* (composed right under this joint program), especially pp. LXXXVII-CVI.

10 See [3], pp. 139–145: the traditional model of tripartite reality divided into sensible, mathematical and intelligible worlds is in Aristotle's *Metaphysics* E 1, where we also find a reference to common mathematics (so-called *mathesis universalis*) and an even more general science (*scientia generalis*).

In the prologue of the last work, the study of the abstract quantities and of the geometric figures is meant (through the explicit reference to *Wis* 11, 21) to be a tool to guide the human mind in the ascent from the multiplicity of the bodies to the harmony of the universe, the created «second number, measure and weight». The treatise begins with a programmatic statement:

> Utilitas vero disciplinae huius omnibus sapientiae amatoribus quam maxima est. Nam et ad animi ingeniique vires excitandas intuitumque exacuendam subtilissima, et ad plurima certa veraque ratione vestiganda, quae multis miranda et inopinabilia videntur, iocundissima, atque ad miram naturae vim, Creatoris omnia in numero et mensura et pondere disponentis potentiam et ineffabilem sapientiam contemplandam, admirandam et laudandam, subtilium speculationum plenissima est.[11]
>
> The advantage of this discipline is very great for the lovers of wisdom. In fact it is the most elevated discipline for stirring up the powers of the mind and genius, for sharpening one's vision and for investigating with the power of reason many definite and true things, which for many are wonderful, surprising and very pleasant. It is the fullest of the subtle speculative sciences for contemplating, admiring and praising the marvelous power of nature and the ineffable power and wisdom of the Creator who sets all things in order by number, measure and weight.

According to Gerbert, the study of mathematics should be pursued not only for practical purposes, but also in order to magnify the ordered beauty of reality. The author sees in geometric forms the the relationships among things that God has arranged by placing the parts into larger collections that in turn are traced to the unity, the conceptual center of gravity for Gerbert, as well for Abbo.

Gerbert derives from Boethius and Macrobius the emphasis on the metaphysical dimension of mathematics, the study of which is no longer, as in previous centuries, a functional clarification of the numbers that appear in the Scriptures, but is aimed at the identification of the universal plot of numerical relations that weaves the structure of creation.

He seems indeed to go a step further, if we focus on the difference between the definition of the geometry offered by Boethius (*disciplina magnitudinis immobilis, formarumque descriptio contemplativa*: discipline of motionless magnitude, and contemplative description of the forms) and that proposed by Gerbert (*Geometria est magnitudinum rationabiliter propositarum ratione vestigata probabilis dimensionis scientia*: Geometry is the science of probable measurement, traced out by the reasonable structure of rationally proposed magnitudes). His insistence on the rationality of the geometric science suggests that, according to the future Pope, this discipline constitutes a real instrument of knowledge of the world and of its mathematical plot.

Gerbert composed his *Geometria* after consulting in Bobbio the *excerpta* of the work of Euclid and the pseudo-Boethian *Geometria*, transmitted with the title of *Ars*

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[11] [13], pp. 50–51.

geometriae et arithmeticae, from which he derives the idea that the geometric structures reflect the divine wisdom. According to Gerbert, the geometric discipline is no longer a tool for the measurement of the land, as it was still to Cassiodorus and Isidore, but a way to reveal the mathematical structure of reality and the only divine source of the multiplicity.

In a letter (n. 187) to the emperor Otto III, who shared many of the studies of Gerbert, the future Pope reveals the deep reasons of his interest in arithmetic, which comes from the awareness that "in the numbers we can find the beginnings of all things", according to a principle expressed also in the *Libellus de numerorum divisione*, where we read that the knowledge of numbers is placed at the origin of all further knowledge:

> Nisi enim firmum teneretis ac fixum *vim numerorum vel in se omnium rerum continere primordia vel ex sese profundere*, non ad eorum plenam perfectamque noticiam tanto festinaretis studio.[12]
>
> If you don't keep firm and fixed *the power of numbers either to contain in themselves the elements of all things or to pour out the elements from themselves*, you will not rapidly progress with much zeal toward the full and perfect understanding of them.

According to Gerbert, the reduction of numbers to unity, that people could experience using the *abacus*, is not a meaningless mathematical exercise, but it actually reveals the process of creation: he is concerned with the wonderful suggestion of the science of numbers, that, penetrating the deep structure of the universe, reveals the higher laws of harmony. Incidentally, Gerbert of Aurillac's treatises on the abacus were the most influential in demonstrating the pedagogical and theoretical possibilities of the complex calculations this instrument made possible; he also innovated by replacing the cumbersome *calculi* with counters (*apices*) bearing symbols of the first nine digits.[13]

Gerbert is famous in mathematics especially for the abacus, and his reform of the calculation tool[14]. Its use necessarily implies the use of the Arabic numbering system, i.e. the positional decimal system. The abacus of Gerbert thus appears as the product of a synthesis of the traditional calculation tool of the Latin world and the innovative

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12 [24], p. 224.

13 [23] http://digital.library.mcgill.ca/ms-17/folio.php?p=56v.

14 See [16], p. 143–144, "The abacus was a practical and visual demonstration of the mathematical problem of creating numbers from unity, and Gerbert would have found it particularly attractive. (…) These manipulations on the abacus had great philosophic value for Gerbert, revealing in a graphic and tangible way the two essential qualities of all quantity. First, they demonstrated that the whole multitude of numbers existed as multiples by virtue of the relationships and connections between them, apparent in Gerbert's division of numbers into simple and composite types. Second, these manipulations showed that all quantity was growth from a single source, from unity." See [13], p. 7.

introduction of decimal numbers of Arabic origin.[15] Gerbert was, if not the creator, the greatest popularizer of the abacus in columns, which was to appear as a board divided vertically into 27 columns, gathered into groups of three, in which they were the apices (furniture squares of ivory or bone by means of which the numbers were composed and performed the operations).[16]

For Gerbert, therefore, the natural world is also a 'computable world' because it is governed by mathematical relations. Thus it is not surprising that, at the cathedral school of Reims, the study of arithmetic, *matheseos prima*, was a prerequisite to the study of the other disciplines of the *quadrivium*.

There may be other examples of teachers and scholars of the time, devoted to mathematical studies and interested in grasping their metaphysical aspect. Among these we can recall Adalboldo of Utrecht († 1026), Notker of Liege and especially Erigerius of Lobbes († 1007), author of mathematical writings (including the *Regulae de numerorum abaci rationibus*, a *Ratio numerorum abaci* and an Epistle to Hugonem of computistic argument), but also of a historical work, the *Gesta episcoporum Tungrensium, Trajectensium et Leodensium*, which opens with a chapter entitled *De numero, pondere et mensura*.

4 Concluding Remarks

This amazing epistemological value of medieval mathematics is the result of an approach to numbers, typical of these authors, in which the practical problems (the *computus*, the land surveying, the astrology) are based on theoretical issues of the first order. This speculative attitude is not unusual in the history of Western thought, but it is significant in that it will occur again in this era – frequently considered among the darkest ones – through the double reference, on the one hand to the scientific tradition that is rooted in 'numeric Pythagorean exemplarism', and, on the other hand, to the (Neo-)Platonic tradition. The first one developed with the contributions of the Venerable Bede, Martianus Capella, Rabanus Maurus and Odo of Cluny; the second one has its most immediate references in the concept of order developed by Augustine, in the universal harmony evoked by Boethius, in the Chalcidian theory of the *anima mundi*.[17]

So, at the dawn of the year one thousand, Abbo, Gerbert and Erigerius seem to teach us, scholars of the 21st century, the necessity of a virtuous dialogue between arithmetic and dialectic, between the disciplines of number and the *trivium*, between

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15 Guy Beaujouan [2] has established a link between the use of the abacus quotes and how they are represented in Arabic numerals in the oldest manuscript evidence. The abacus of Gerbert thus played a key role in the initial phase of implementation of the new numbering system.

16 [11], pp. 328–331.

17 It should be mentioned that the teaching of Abbo and Gerbert in arithmetic, contrary to what happened in the field of logic, had a modest sequel after the eleventh century and it remained confined to a limited geographical area.

‘mathematical thinking’ and philosophy, which seem to be the masters of digital humanities. Finally, since the knowledge of numbers is set by them at the beginning of any further knowledge, maybe it would not be inappropriate to apply to these philosophers – *mutatis mutandis* – the modern concept of *mathesis universalis*, which is the project of a general science – in this case, the *quadrivium* – that leads to a certain knowledge.

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