Algorithmic in the 12th Century: The Carmen de Algorismo by Alexander de Villa Dei

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Abstract. This paper aims to update the knowledge about one of the oldest medieval handbooks on calculation with Hindu-Arabic numerals in positional notation, the so-called Carmen de algorismo, also known as Algorismus metricus, and traditionally attributed to the French scholar Alexander de Villa Dei. This work had a remarkable spread during the Middle Ages in many European countries, alongside the Algorismus prosaicus by Johannes de Sacrobosco. In our study we will portray the overall picture of the spread of new techniques of calculation with Hindu-Arabic numerals in cultured circles and of the consequent literature, since it is different from the contemporary works called abacus books, devoted to merchant and practical calculations. Despite its importance, the work has not yet been thoroughly investigated both for its relative difficulty, because it is composed in verse by a refined author, and for the presence of a contemporary literature of the same content, starting precisely from the one by Sacrobosco.

Keywords: Algorithmic, algorismus, Villedieu, calculation, Sacrobosco, carmen.

1 What is an Algorismus?

The word algorismus is the Latinization of "al-Khwārizmī", the renown Persian scholar, born in Khwarazm (modern Uzbekistan) at the end of the 8th century CE. He worked in Baghdad, at the House of Wisdom, studying mathematics, geometry, astronomy, geography, history, and cartography.

Even though he had composed significant treatises about all these subjects [1], in medieval Europe his name was inextricably bound only to arithmetic, due to the numerous Latin translations of his lost work about calculation with Indian numerals. His name appeared at the beginning of the text, in the incipit, the so-called basmala (literally, in the name of God), and it was transliterated by European scholars during translations, probably in the 11th-12th centuries. Shortly, the actual meaning of the word was forgotten, and the term began referring to a written work, dedicated to the description of the 7 operations with Indian numerals.

In addition, when the link with al-Khwārizmī was completely lost, in the 16th century, scholars began making unfounded and fanciful hypotheses about the etymology of such a rummy word. After that time, the name of the Persian polymath fell into oblivion, and the algorismi suffered the same fate, due to the remarkable advances in European mathematics.

2 The Carmen de Algorismo

Starting from the 13^{th} century, many algorismi [2, 3] spread in Europe: in nearly all Western countries some outstanding mathematicians authored their own handbooks on the subject: Sacrobosco, Jordanus Nemorarius, Johannes de Lineriis, just to quote the most famous ones. Many of these works are however by unnamed authors: they were simply copied in miscellaneous manuscripts of scientific content, ranging from arithmetic to medicine, or to astronomy, as well as to strategy games, such as chess or checkers. Later, even vernacular works began disseminating these calculation techniques.

Due to their educational purpose, the authors tried to express the content in the clearest way, in a simple syntax, using a technical and consistent language, whose meaning is generally defined in the first paragraph, and giving as many examples as needed, for integers, decimals and for calculation in base 60. Sometimes, they also added a set of solved exercises.

One of these algorismi, however, beginning with the words “Haec algorismus ars praesens dicitur”, shows opposite features: the nearly 300 Latin hexameters appear very impenetrable, due both to the convoluted syntax[1], to the inconsistent use of technical terms, to the limitation to positive integers, and to the lack of examples, and of solved exercises. Moreover, the poem ends abruptly (a dextris digitum servando prius documentum), possibly indicating the loss of a final part.

2.1 The Title

One of the most intriguing characteristics of the work consists in its title. Scribes and/or librarians, while copying or cataloguing this poem, used different titles, highlighting different features.

Someone chose the word algorismus, possibly followed by the number set (in integris, integrorum), or by the reference to the poetic form (in metro, in versu, metricus, metrificatus, versificatus). Others underlined, first of all, its metric nature with the word carmen or versus (poem or verses), then they specified the content (de algorismo/algorithmo, de arithmetica, de arte algorismi); a small subset is in vernacular (Metrical arithmetic, The arithmetical poem, Poem on the algorismus). Another group emphasizes the educational purpose of the work (Libellus de algorismo, Regule algorismi, Treatise on arithmetic, Lectura algorismi metrici, Ars algorismi). One manuscript title alludes to the Indian origin of the numerals (Indorum ars numerandi).

As al-Khwārizmī’s work was possibly entitled “Kitāb al-Jam‘ wat-Tafrīq bi-Ḥisāb

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[1] In Latin poetry, the word order depends totally on the sequence of long and short syllabes required by the verse scheme, issuing a very irregular syntax.

al-Hind" (Book of Addition and Subtraction According to the Hindu Calculation), this last example looks as the most explicitly linked to the Arab tradition.

2.2 The Content

Before introducing the work content, a foreword is due: as no critical edition is currently available, we will use the transcriptions made by Halliwell [4], and by Steele [5]. Both authors transcribed manuscripts housed in the British library, dating back to the 15th century: Steele used the copy in the ms. Royal 8.C.iv, and, in different passages, he added some verses taken from the mss. Egerton 2622, and Royal 12.E.1. This is the reason why Halliwell's transcription length is 285 lines, while Steele's is longer (333).

In any case, the table of contents is the following: first of all, the author describes the "bis quinque Indorum figurae" (twice five numerals of the Indians) and their numerical meaning, and he explains how to write numbers in positional notation. Then the author lists the seven operations, and splits them into two groups, based upon their performing direction: the first group includes the operations made from the rightmost to the leftmost digit (addition, subtraction, doubling), while, in the second one, there are halving, multiplication, division, and root extraction, performed from left to right. After the root extraction, Steele adds also 6 verses about progressions, found in some manuscripts, while in some other mss. (such as BAV, Pal. Lat. 1393, or Erlangen, Universitätsbibliothek, 394) dating XIII or XIV c., after the explicit, 35 more verses about mental (intellectualis) calculation appear. This addendum begins with the words "Si digitus digitum multiplicat adspice per quot", and ends with "a maiore minus et summa videbitur eius"; in Bodleian Library, Digby 22, these lines are entitled Carmen de arte multiplicandi, and it is described in the catalogue as a different work, written by an anonymous author.

2.3 Commentaries

Due to the difficulty of the work topic, and to his teaching style not so user-friendly, many scholars added detailed glosses to familiarize readers with arithmetic. We have the names of two of these annotators: in the late 13th century, Thomas de Novo Mercato (Thomas of Newmarket), an arithmetician graduated MA in Cambridge, wrote a "Commentum in carmen de algorismo", and a less known Saxton authored a prose interpretation[2]. In 1360 a Rogerus de Saxton, bishop of Aberford, near York, is quoted in the wills by the bishop Richard Kellawe of Durham. In addition, we also have some copies, glossed by unknown scribes or scholars[3].

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[2] Witnessed in British Library, mss. Royal 8.C.IV, ff. 36b-38; 12.E.I, ff. 1-24; 12.F.XIX, f. 183; Sloane 513, ff. 26-43, Egerton 851, ff. 1-19v; and Add. 17716, ff. 123v-140.

[3] Oxford, Bodleian Library, ms. Digby 81, ff. 11-35; Prague, National Library, ms. XIII. H.3.h., ff. 75a-79a; Cambridge, University Library, ms. Ii.I.13, ff. 8-12; Gonville and Caius, ms. 76, ff. 87-93.

3 Calculation Techniques

After the brief introduction to numbers in positional notation, the author describes the seven operations: as it is already evident in the foreword, digits are written from the right, so, if a digit is set the first position (limes), it represents units; in the second, tens, and so on.

Another important remark is due: the author implicitly assumes that calculations are performed on a sand table, a technique that allows to delete and rewrite specific digits without transcribing at every passage all the numbers involved in the operation.

3.1 Addition, Subtraction, and Doubling

These three operations are performed from right to left, and possible carries are registered above the upper number, like nowadays. The result replaces the upper number (namely, the first addend, or the minuend) so that, once the operation has been completed, one can immediately check the correctness of the calculation by performing the opposite. Doubling however requires only one number, that is replaced by the result.

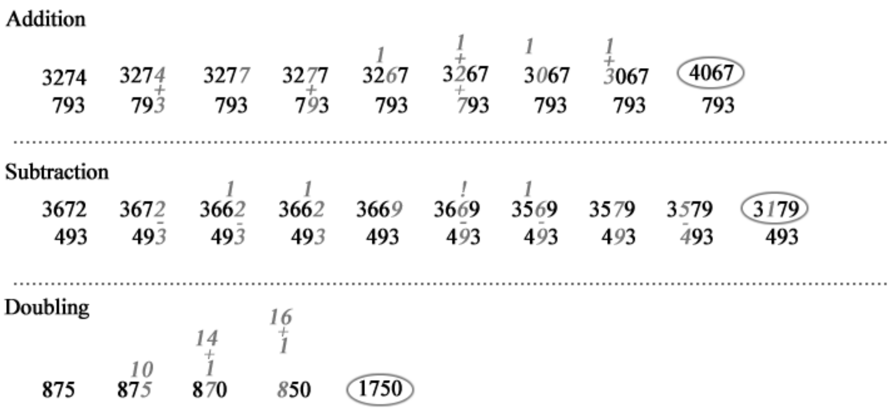

Fig. 1. Examples of Addition, Subtraction, and Doubling

3.2 Halving, Multiplication, and Division

These operations are performed from left to right, and possible carries are registered above the upper number, like nowadays. The result replaces the upper number (namely, the first factor, or the dividend) so that, once the operation has been completed, one can immediately check the correctness of the calculation by performing the opposite, and adding the possible remainder, as for division. Halving however requires only one number, that is replaced by the result. Possible partial carries or remainders are saved above.

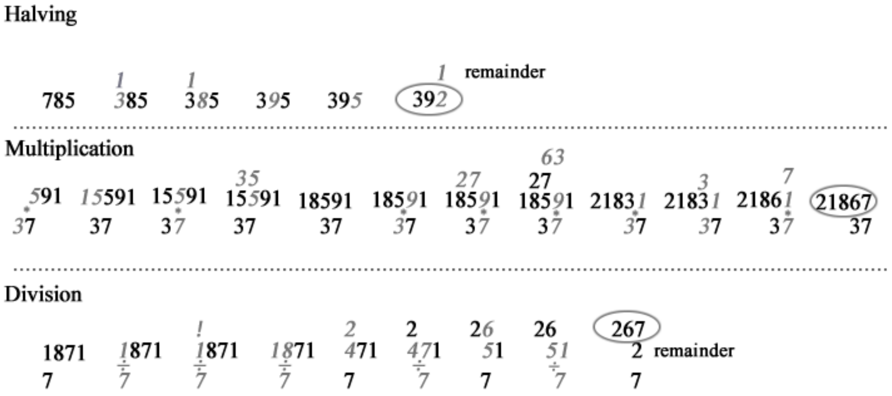

Fig. 2. Examples of Halving, Multiplication, and Division

3.3 Square and Cubic Root Extraction

These operations are also performed from left to right. Partial results are registered under the number. Information about how to write down single steps of calculation is quite generic and no numerical example is given in the text. Nevertheless, in the margin, for instance, the scribe of Cambridge, Trinity College, O.2.45, ff. 23-31, lists many perfect squares and cubes, related with their respective square and cubic roots, ranging from easier cases ($\sqrt{49}$ or $\sqrt{36}$) to more complicated ones, such as $\sqrt{196249}$. The passages of calculation are implicit. The algorithm described is similar to the modern one.

3.4 Progression

As previously noted, in some manuscripts, six verses[4] about progressions are added, though in different places.

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[4] Si sit continua progressio, terminus impar / per maius medium totalem multiplicato; / si par, per medium tunc multiplicato sequentem. / Sed, si continua non sit progressio, finis / impar, tunc maius medium se multiplicabit; / si par, per medium sibi multiplicato propinquum.

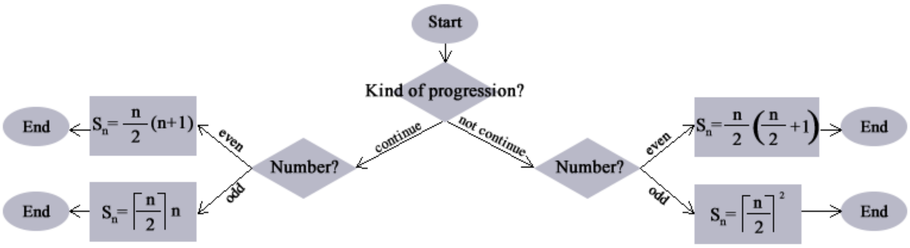

Fig. 3. The Progression Algorithm

3.5 Mental calculation

As previously referred, in some manuscripts, an addendum completes the work. Due to the absence of a canonical explicit of our text, possibly containing expressions like "hic explicit", "finis", etc., it is very challenging to deduce the original end, and even the relationship with these lines. Nonetheless, these verses witness the existence of further similar works in poetry, linked to another calculation tradition: the mental one. Such a tradition was strictly related with the Carmen: in its 15th-century old-English translation[5], the passage about the multiplicatio intellectualis of two different digits[6] is included, translated and accurately completed with a numerical example. It is obviously a calculation technique external to the Arab arithmetic dating back to al-Khwārizmī. At the end, a triangular multiplication table is given as a tool, useful to simplify calculation rules and to minimize the use of memory.

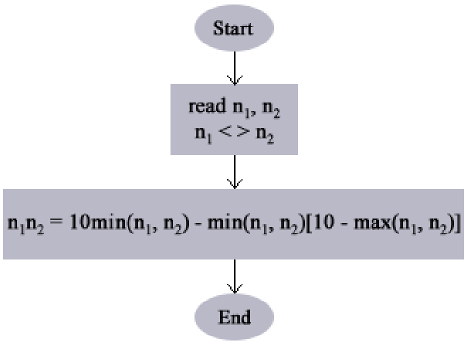

Fig. 4: multiplicatio intellectualis of two digits

4 The Spread of the Work Manuscripts

[5] Included in ms. British Library, Egerton 2622, ff. 136a-165°; ; the same rule is also quoted in Egerton 2261, ff. 225v-7v; and in München, Bayerische Staatsbibliothek, Clm 13021, f. 27.

[6] The case of $n_1 = n_2$ is not performed as multiplication, rather as doubling.

As far as we know at the moment, 161 manuscripts, in many cases accompanied by commentaries and marginal notes, contain at least some verses of the Carmen:

- with the following incipit “Haec Algorismus ars presens dicitur” and its orthographic variants “Hec Algorismus ars presens dicitur” (eTK 0080H; 0597E; 0597G), and “Hic Algorismus ars presens dicitur” (eTK 0614C);
- catalogued as Carmen de Algorismo, Algorismus metricus, and similar titles in the catalogue by Van Egmond, and in the data base Jordanus;
- attributed to Alexandre de Villedieu, even without incipit quotation;
- quoted in other catalogues, such as Assisi, Fondo Antico presso la Biblioteca del Sacro Convento, ms. 174, 113v-115c; Salamanca, Biblioteca Universitaria, ms. 2078, ff. 165r-169va.

Dating of 141 manuscripts has been possible; most of them (62) date back to the 14th century, 35 to the 13th century, 41 to the 15th, and only 3 to the 16th century: a sign of a decreasing interest about the work, and the topic.
From the geographical point of view, we can look at both their origin (in 33 cases, it is known), and at the countries of the libraries they are housed in.

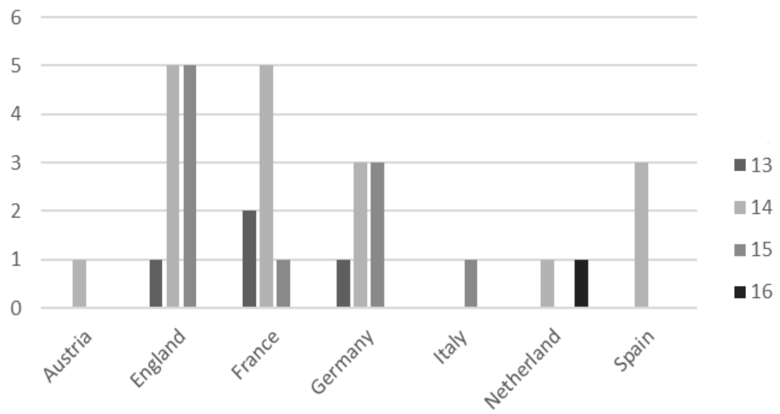

Fig. 5: number of manuscripts per country and century

Even though limited to the small available sample, England and France appear to be the most interested countries in the topic, starting from the century in which the work has been composed.

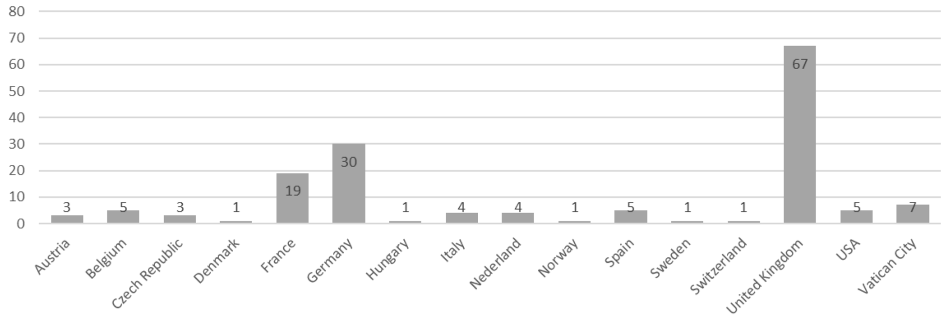

Fig. 6: number of manuscripts housed per country

Unsurprisingly, England appears the most interested country in the topic, followed by Germany and France. All other countries house only a little number of manuscripts. Three very ancient manuscripts are classified in the Kraus transaction catalogue (No. 4; 52; 155), but their present location is unknown.

5 The Source Question

The source of an Algorismus should be the already quoted work by al-Khwārizmī, though translated into Latin. As we have witnessed, the relationship with this source is not so immediate. The first issue to consider is the absence of an Arab copy of the whole work. The second one is that no complete translation is available, rather a very intricate tradition of partial translations, contaminated by many parallel traditions, as shown in Allard [3]. The third one is the use (or misuse) of numerals, which are referred to as 10 (bis quinque) in the Carmen, but are quoted as 9 in the Latin Arab tradition, and often written as 9 in the marginal glosses of our work. This is due to the inclusion (exclusion) of the zero in (from) the numerals set.

5.1 al-Khwārizmī

Even though the Kitāb al-Jam‘wat-Tafrīq bi-Ḥisāb al-Hind (Book of Addition and Subtraction According to the Hindu Calculation) is lost, four partial 12th-century Latin translations, deeply studied by Allard, survive:

- Dixit Algorizmi (DA);
- Liber Ysagogarum Alchorismi (LY), possibly authored by Adelard of Bath;
- Liber Alchorismi (LA) by John of Seville;
- Liber Pulueris (LP).

The works genres are obviously not comparable, but their contents approximately correspond; in Table 1, a synopsis is given, which however needs a further remark: in the Carmen, operations are performed only with natural numbers and in base 10, while, in the other works, non-negative rational numbers and base 60 are also taken into account. Cubic root calculation is present only in the Carmen, possibly due to a loss of the last part.

Table 1. Comparison of operations order in the arithmetic tradition

Operation	Carmen	DA	LY	LA/LP
Sum	1	1	2	1
Difference	2	2	3	2
Doubling	3	4	5	3
Halving	4	3	4	4
Multiplication	5	5	1	5
Division	6	6	6	6
Square Root	7	?	7	7
Cubic Root	8	?	?	?

5.2 Contamination of traditions

Despite the great similarity between the Carmen and LA/LP, under other respects, the works significantly differ. The limitations to the base 10 and to the numerical set N_0 suggest other links with independent techniques, such as Boëthian arithmetic, finger reckoning, and the abacus practise. Such contaminations have already been proposed by Allard in his study of al-Khwārizmī's al-Hind.

Another clue of the link to the Boëthian tradition consists in the use of a specific terminology, with a Greek nuance, due to its Pythagorean origin. Number one is sometimes referred to as monas or monos in the Carmen.

The relationship of the Carmen with finger reckoning is made evident by the use of specific terms, such as digiti (units, literally fingers), articuli (tens, namely finger joints), compositi (numbers composed by units and tens). This tradition is also connected with Boëthian arithmetic: Martianus Capella in his De nuptiis Philologiae et Mercurii, Bede the Venerable in his De computo vel loquela digitorum, refer to this practice.

During the Middle Ages, the necessity of a calculating tool was already big, due to the need of reckoning the dates of the movable feasts. Some important scholars had already written handbooks about the use of the abacus: Gerbert d'Aurillac and Adelard of Bath, just to quote the most famous authors. In most abacus trays, over time, Roman numerals were replaced by Hindu-Arabic ones in their Western form, which is also called ghubar (namely, dust): the same form is used in most of the Carmen manuscripts.

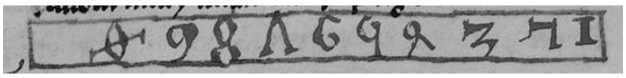

Fig. 7: Ghubar numerals including zero

Table 2. Representation of numerals in al-Khwārizmī's tradition.

Work	Definition	Representation
DA	Fecerunt (Yndi) IX literas, quarum figurae sunt he 987654321…	ghubar
LY	his VIIII figuris 987654321 tam integros quam minutias significantibus utuntur.	ghubar
LP/LA	Que figure et earum numerus et ordo est	987654321 and ghubar
Carmen	Talibus Indorum fruimur bis quinque figuris	ghubar including 0

Like in abacus trays and in later abacus treatises[7], the list of the numerals usually does not include zero, which is normally not considered as an actual digit, but a kind of placeholder, a symbol of absence. Therefore, in al-Khwārizmī's tradition, the numeral is introduced later and described in comparison with the letter O, or a small circle.

Table 3. Representation of zero in al-Khwārizmī's tradition

Source	definition	Representation
DA	(Yndi) posuerunt circulum paruulum in similitudine O litere	0
LY	Utuntur etiam ciffre	0 or τ
LP/LA	Circulus	-
CdeA	cifra vocatur; [Quae nil significat; dat significare sequenti.]	0 Ø

5.3 A New Arithmetic, far from Universities

As a matter of fact, in the 12th century, the curriculum studiorum in Paris included arithmetic, as a part of the quadrifaria mathesis (the four-fold learning), the so-called quadrivium, namely the four ways to reach what we would call scientific knowledge.
At the same time, a refined literary education could be achieved through the study of the disciplines of the trivium: grammar, rhetoric, and dialectics. One of the most famous handbooks of the 12th century was Alexander de Villa Dei's Doctrinale Puerorum, a Latin grammar in Leonine hexameters. Medieval students, both children and adults, needed to learn by heart a very wide range of topics, due to the high cost of manuscripts: the use of poetry could enhance their remembering, being an effective memory aid, thanks to rhythm, internal rhymes, and assonances. The Doctrinale quickly replaced Donatus' and Priscian's works in the official curriculum at the Sorbonne, but the Carmen fate would be very different.

Despite the many advantages of the new arithmetic, such as the reduced role of memory, the independence from a device (abacus), the contrast between speculative and practical arithmetic (logistics) survived and possibly increased [6]. The study of the

[7] In Italian vernacular handbooks they are referred to as the 9 digits on the abacus (9 fighure nell'abacho).

new arithmetic was not encouraged by the Sorbonne authorities, as even Roger Bacon refers: Studium Parisiense adhuc non habuit usum istarum quinque scientiarum (the university of Paris at the moment offers no classes of foreign languages, maths, perspective, experimental science, alchemy). Probably professors taught this subjects outside the university, being paid handsomely.

6 The Carmen Influence

6.1 Sacrobosco

Johannes de Sacrobosco is the alleged author of two 13th-century scientific works about arithmetic (Algorismus prosaicus), and astronomy (De sphaera). His biography is quite incomplete and uncertain, mainly as for his real name, his birthplace and also his education. He probably taught in Paris from 1221 until his death, in 1244 or 1256. His Algorismus became soon a popular handbook, copied in hundreds of manuscripts, due to its clarity and completeness, and later also printed. The Algorismus prosaicus and the Carmen explain the same topic: the differences between the two pertain mainly to the style, as the titles clearly show, and the length. As for the uncertain dating, a Sacrobosco's passage quoting some verses of the Carmen gives the opportunity to state which is the older work. In order to suggest a simple way to remember from which side (left or right) the various operations should be performed, Sacrobosco quotes the lines of the Carmen about operation verse; he introduces the quotation with only two words: "unde versus" (literally, whence come the verses …), without any reference to their author, as universally renown.

The two works were strictly connected by the likeness of their content, and, in a way, complementary: while Sacrobosco's Algorismus was more detailed and complete, and was best fitting the educational purpose of beginners, the Carmen text was shorter and therefore easier to be learnt by heart, at least in theory. This relationship is proved by the existence of another kind of Algorismus, called "in usum Cantabrigiensem" (Cambridge's style), which included both works in the same manuscript. In addition, some scribes[8] (or maybe some scholars) began "interleaving" the two works so that some lines of the Carmen appeared commented by the corresponding passage by Sacrobosco. The Carmen appeared difficult, especially if compared with the homonymous work in prose by Sacrobosco and this trick allowed students to study arithmetic with a double approach.

6.2 Vernacular Translations

As it turns out, the Carmen is very vital even in its Latin form, characterized by many variants, continuously updated, subject to numerous revisions and comments. The fact

[8] E.g. Universitätsbibliothek Erlangen-Nürnberg, ms. 436, 1r-10v

that it is a text aimed at teaching arithmetic, makes it an excellent candidate for vernacular translations, whose need was certainly felt by students who were not fluent in Latin, and maybe by merchants.

The first known French vernacular translations are witnessed by the ms. Paris, Bibl. Sainte-Geneviève, 2200 f. 150r, and by the ms. Paris, BNF, Français 2021 (Anc. 7929), ff. 154-155, both dating back to the 13th century.

These works, studied by several scholars [7-10], appear as the result of an accurate rework, useful for students who were not fluent in Latin, but in need of a basic education in arithmetic, as we can infer from the missing of the square and cubic root extraction operations. The contents are completely comparable with the Carmen.

An old-English version of the Carmen is “The Crafte of Nombrynge”, literally the Skills of Numbering, in the 15th-century ms. Egerton 2622, housed at the British Library, and edited by Steele. It is a translation and amplification of the Carmen glosses: the author, after quoting a passage of the Carmen in Latin, adds an old-English explanation, followed by numerical examples, or even by simple questions to be solved; then he moves to the following verses, until the lines devoted to mental multiplication. The introduction also includes the improbable etymology of the word algorismus, already present in Thomas of Newmarket’s glosses: the term is believed to derive either from the name of the author, an Indian king, named Algorym, or from the Greek algos, allegedly meaning craft, followed by the Latin rides, standing for number, according to the author. The same wrong origin appears in the glosses of the Carmen manuscripts, with some small variations: «algorismus ab inventore s(cilicet) ab algo quod est inductio et rismus quod est numerus» (algorismus comes from the author, i.e. algo, meaning introduction, and rismus, meaning number). The phonetical similarity between rismus and the actual Greek word for number (rhythmos) should also be taken into account, mainly due to the links with the Boëthian tradition.

The last vernacular translation here considered was made in a quite unexpected environment, as it is contained in the Norse Hauksbók, an epic anthology, which includes some mathematical works, such as Villedieu’s and Sacrobosco’s Algorismi and Fibonacci’s Liber Abaci [11]. It represents a kind of summa of the Medieval practical arithmetic in Latin, possibly translated for Norse merchants.

7 The Author

We left our remarks about the author as the last part of our work, due both to the fact that only a very small number of manuscripts contains a direct quotation of the author’s name and to the uncertainty of this attribution, as we will explain in the next paragraphs.

7.1 Life and works

Alexander de Villa Dei’s biography has been rewritten in details by Reichling [12], who studied both primary sources, such as his Latin grammar, the Doctrinale, and secondary, such as the historians who gave details about Alexander's life. In all the oldest manuscripts of the Doctrinale, the author is referred to as Alexander de Villa Dei, that is from Villedieu-les-Poêles, a small village in Bretagne, while other sources indicate Dol-de-Bretagne, or the episcopal city of Avranche as his birthplace. As for his date of birth, no trustworthy information is available; 1160-70 has been inferred by Reichling, starting from the edition of the Doctrinale, dating back to 1199.

A 15th-century manuscript[9] includes glosses about the author's education: he would have studied in Paris with two other fellows, the English Adolphus, and the French Ivo.

In order to enhance their memory, stressed by the huge number of Priscian's Latin grammar rules, they applied a technique possibly learned by the French poet Petrus Riga (c. 1140 - 1209), who had put the Bible into verses. They summarized each rule in a couplet of verses, far easier to remember; in order to earn some money, they applied this technique to many other grammar works. After some years, Adolphus went back to England and Ivo died, so Alexander remained the only owner of this educational material.

In 1199, when he was called by the bishop of Dol to teach Latin to his nephews, Alexander wrote the 2645 Latin hexameters of the Doctrinale, used for centuries as a grammar handbook. After the bishop's death, around 1205, information about Alexander becomes quite uncertain and controversial: it is therefore very difficult to determine whether Alexander was a Benedictine monk, as a 15th-century manuscript gloss refers, or, according to Renaissance sources (Trithemius and Wadding), a Franciscan friar [12]. The Franciscans arrived in France in the late 1210s, when Alexander would be an adult man, 40 or 50 years old. A modern Franciscan historian [13] suggests that the author could have entered the order in his late years, as witnessed by a 15th-century manuscript[10]. Glorieux [14] refers he returned from Dol to Avranche, where he was a canon at St. Andrew, and there he died on 29 July, possibly in 1240.

Contemporary sources unanimously attribute to Alexander the only Doctrinale, but, the Renaissance historian Trithemius adds some more texts, allegedly composed by the same author. This list includes De Computo ecclesiastico (about the calculation of movable feasts), De sphaera (about astronomy), De arte numerandi (the Carmen), and also letters, treatises, lectures the historian could not examine [15]. Many more works about grammar, and Bible will be added along centuries, as testified by the detailed Glorieux' review [14].

7.2 A Mathematician?

Despite the lack of direct sources, it is quite sure that Alexander was in some way part

[9] Paris, Bibliothèque de l'Arsenal, ms. 1038, f. 223b

[10] Perugia, Biblioteca Comunale, Lat. 112, f. 215, (1422): «Auctor huis libri [Doctrinale] fuit Alexander Parigiensis (sic) cognomine de Villa Dei; cum esset senex et non potuisset amplius legere, intravit ordinem minorem et ibi mortuus fuit.» The author of this book [the Doctrinale] was Alexander from Paris, named de Villa Dei; as he was old and couldn't teach any more, he entered the order and then died.

of the clergy. Whether he was a priest, a cleric, a canon, a Franciscan or Dominican friar, or a Benedictine monk, it is not so significant to our study. Nonetheless, this controversy inspired historians: while scholars were looking for evidences of Alexander's membership to a specific order, they found some worthwhile sources, that can be used to determine whether he actually was the author of the Carmen.

Hughes, for instance, quotes a passage of the Compendium, a history of the Franciscan order, written by the Franciscan Marianus de Florentia (1450-1523): "plurimi doctores florebant in Ordine ex quibus … Frater Alexander de Villa Dei, sacrarum litterarum professor" (Many scholars emerged in our order: among them brother Alexander de Villa Dei, professor of theology). These words, while celebrating the author as a great Franciscan scholar, do not refer to him as to a polymath, and no mention is made either to arithmetic, or to Latin grammar, possibly because Marianus wanted to emphasize the importance of his studies in theology, far more illustrious than those in liberal arts. Another possible explanation is that Marianus was not so well-informed about the whole work by Alexander. In any case, it is the second source in the Franciscan historical tradition that does not mention the Carmen.

A 13th-century source that appears absolutely expert about the topic (written works available in the 12th century) is Richard de Fournival (1201-1260), a philosopher and poet. In his Biblionomia (a list of 162 volumes, a sort of ideal library), in the shelf devoted to geometry and arithmetic, at the place n. 45, we find "Alkoharithim magistri Indorum liber de numerorum ratione" with the Apodixis by Jordanus Nemorarius and also "experimenta super algebra at abrakabala (sic)", "liber augmenti et diminutionis nidorum (sic) quam Abraham compilavit", "liber de invenienda radice", "alius Hermanni Secundi de opere numeri et operis materia" [16]. This quotation includes not only references to the Indo-Arab tradition translated in Latin, but also to contemporary works on the topic. In addition, we can exclude that Richard was not aware of Alexander's work, because, among the grammar books, we find the Doctrinale, correctly attributed to his author.

The attribution of the Carmen appears therefore quite controversial, as, while on the one hand some documents, though late and unfounded, testify Alexander's authorship, on the other hand, some different sources, which are contemporary and presumably very well-informed, remain silent about his work on arithmetic. Halliwell [4] agrees on the ascription, and quotes two 14th-century manuscripts to support this attribution: first of all, ms. Harley 3902, housed at the British Library, contains an introduction to the Compotus written by an anonymous author, where it is stated that the Compotus' author is the same as the Doctrinale's and the Algorismus metricus'; secondly, Halliwell had been informed by the French historian Chasles about the existence of a colophon with the same content, at the end of the copy of the Carmen included in ms. Lat. 7420A, housed at the Bibliothèque Nationale de France, and copied in Montpellier.

As last remark, we must add that a comparison between the Carmen and the Doctrinale issues more differences than similarities: both works are written in Latin hexameters, but the Carmen verses are not leonine (no internal rhymes); even taking into account that the content is not comparable, no recurring stylistic feature emerges; we have a prooemium in the Doctrinale, with the description of the work goal and of the audience, totally lacking in the Carmen; the use of the first person in the only Doctrinale; the relative stability of the text, opposed to the many versions of the Carmen.

8 Conclusion

The Carmen has unquestionably played a fundamental role in the spread of Arab arithmetic throughout Medieval Europe, due to his poetic form, and to the need for a brief summary of the calculation rules, useful for people who are already acquainted with the topic. A kind of complementarity with the very detailed Algorismus Prosaicus by Sacrobosco is proved by the fact that both works are often copied in the same codex, and sometimes "interleaved" one another. The Carmen is also a witness of a hybrid tradition, a kind of mathematical melting pot, where footprints of previous and contemporary cultures coexist, aiming at the same practical goal of calculation. The work itself appears as always in fieri: manuscripts containing exactly the same text are very rare, as different scholars felt free not only to comment the work, but also to update, to alter, or even to shorten it. Therefore, the Carmen can be considered as a kind of collective work, composed by many hands, possibly in various moments; science in verse was not uncommon in the Middle Ages, due to mnemonic reasons.

Alexander's authorship of the Carmen is questionable under many respects: the lack of contemporary and reliable sources, their distance in terms of time, the stylistic difference from the Doctrinale, the anonymous quotation by Sacrobosco, show that the ascription needs further studies to be really proved, as in other cases has been done [17].

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