Fault tolerant remote parity detection and entanglement stabilization

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Loss-tolerant parity measurement for distant quantum bits

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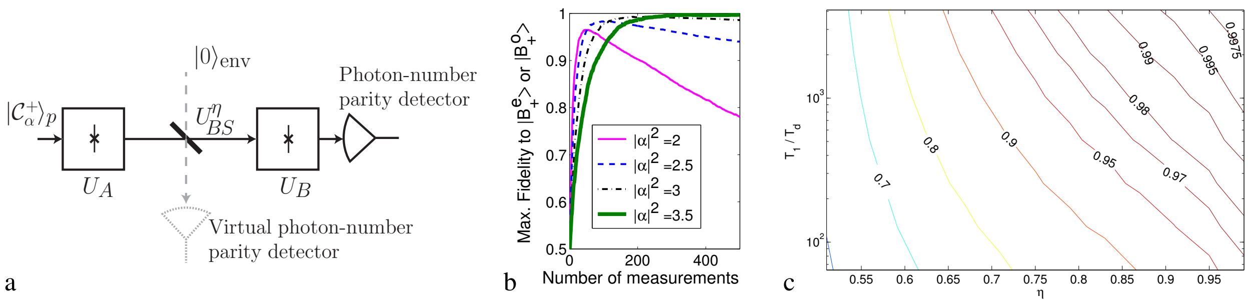
Abstract: Thanks to cat-state probes, we propose a scheme to measure the parity of two distant qubits, while ensuring that losses on the quantum channel between them does not destroy coherences within the parity subspaces.

The correlation of distant systems through entanglement is a hallmark of quantum physics and plays a fundamental role in envisioned quantum technology. E.g. towards future quantum computers, quantum teleportation could transport information between few-qubits processing units and well-protected memory units [1]. A major challenge towards enabling these applications is that generating entangled states between distant systems must rely, itself, on a quantum channel. Microwave experiments have demonstrated how to deterministically entangle separate quantum subsystems via parity measurements [2], yet with a fidelity directly limited by the quality of the quantum channel. Heralding and distillation allows to generate high-fidelity entanglement, with channel losses affecting preparation success only [3–5]. The present proposal aims at *deterministically stabilizing* entanglement, with minimal resources, thanks to a channel-loss-tolerant joint parity measurement.

An ideal *Eigenstate-Preserving Quantum Non-Demolition* (EP-QND) measurement of the joint parity of two qubits would have outputs $+,-$, with associated projectors $Q_+ = |00\rangle\langle 00| + |11\rangle\langle 11|$, $Q_- = |01\rangle\langle 01| + |10\rangle\langle 10|$. This can be effectively achieved in a standard way with a single probe qubit that starts in state $|0\rangle$, consecutively undergoes a CNOT conditioned on each individual target qubit, and finally is measured in the canonical basis $|0\rangle, |1\rangle$. The issue is that when the probe undergoes channel losses between its interactions with the two target qubits, entanglement is in general irremediably lost. However, as already noticed in a distillation context [4], things can get much better if the channel loss can be restricted to a operator, e.g. bit-flip. In this case indeed, the QND character is preserved and only the contrast of the parity measurement is reduced.

The key of our proposal is therefore and implementation where the physical noise channel on the probe, i.e. the photon loss operator, reduces to a stochastic bit-flip effect. This is obtained by encoding the logical $|0\rangle, |1\rangle$ states of the probe into so-called "cat states" of an electromagnetic field pulse: $\left|\mathscr{C}_\beta^\pm\right\rangle = (|\beta\rangle \pm |-\beta\rangle)\,/\,\mathscr{N}_\beta^\pm$, $\mathscr{N}_\beta^\pm = \sqrt{2 \pm 2\, e^{-2|\beta|^2}}$, where $|\beta\rangle$ denotes a coherent state (quasi-classical state) of phasor amplitude $\beta \in \mathbb{C}$. The normalization constants $\mathscr{N}_\beta^\pm$ rapidly approach $\sqrt{2}$ as β increases. In our scheme, shown on Figure a, the probe field is initially prepared in $|\mathscr{C}_\alpha^+\rangle_p$ and interacts with two qubit-cavity target systems in a cascaded manner. These are performed by unitary operations U_A and U_B which perform the CNOT switch from $\left|\mathscr{C}_\beta^\pm\right\rangle$ to $\left|\mathscr{C}_\beta^\mp\right\rangle$, at the respective amplitude β of the field reaching the respective qubit ($\beta = \alpha$ for U_A and $\beta = \sqrt{\eta}\alpha$ for U_B). Indeed between the two target setups the probe is exposed to losses, modeled as mixing with the vacuum state $|0\rangle_{\text{env}}$ of an ancillary mode through a beam-splitter unitary operator U_{BS}^η with transmittance $\sqrt{\eta}$ and reflectance $\sqrt{1-\eta}$. Finally a measurement projects the probe's state onto $\left|\mathscr{C}_{\sqrt{\eta}\alpha}^+\right\rangle$ (+) or $\left|\mathscr{C}_{\sqrt{\eta}\alpha}^-\right\rangle$ (-) e.g. through parity discrimination.

The evolution implied by one measurement iteration on the target qubits' joint state, can be described by two superoperators $\mathbb{K}_+$ and $\mathbb{K}_-$, depending on the detection outcome (+) or (-). In absence of channel loss, $\eta = 0$, we have $\mathbb{K}_\pm(\rho) = Q_\pm \rho Q_\pm$. Finite channel loss $\eta > 0$ has a primary and a secondary effect. The primary effect is to reduce the contrast, i.e. $\mathbb{K}_+(\rho) = (1-\xi) Q_+ \rho Q_+ + \xi Q_- \rho Q_-$ with $0 < \xi < 0.5$. This leaves invariant a state of definite parity, e.g. $\mathbb{K}_-(|\psi\rangle\langle\psi|) = \mathbb{K}_+(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi|$ for any $|\psi\rangle = c_1|00\rangle + c_2|11\rangle$. However, such $|\psi\rangle$ will give both (+) and (-) detections, just with predominance for (+), while a perfect parity observation would never give (-) detections. The resulting convergence from an initial state $(|e\rangle + |g\rangle)(|e\rangle + |g\rangle)/2$ towards a Bell state of definite parity $\left|B_+^e\right\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ or $\left|B_+^o\right\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ is visible as the fast initial evolution on Figure b, for various probe field intensities $|\alpha|^2$ and transmission efficiency $\eta = .75$. The convergence rate can be computed as approximately $r_{\text{parity}} = \frac{1}{2}\log\left(\frac{1-e^{-4|\alpha|^2}}{1-e^{-4(1-\eta)|\alpha|^2}}\right)$. The secondary effect of channel loss is a slight violation of the QND character, i.e. a loss of coherence inside the same parity subspace $\mathtt{span}(|00\rangle, |11\rangle)$. This loss of decoherence occurs at a rate approximately $r_{\text{dephasing}} = \frac{1}{2}\log\left(\frac{1-e^{-4|\alpha|^2}}{1-e^{-4\eta|\alpha|^2}}\right)$. On Figure b, this leads to the slow decay of fidelity. The latter is only due to the fact that $\left|\mathscr{C}_\beta^+\right\rangle$ and $\left|\mathscr{C}_\beta^-\right\rangle$ have slightly different energies, such that the rate of photon loss contains slight information allowing to discriminate in which of the two states we were after interaction with the first target system. Accordingly, this loss becomes negligible for a choice of $\alpha \gg 1$. As a tradeoff, larger α imply slower convergence via r_{parity}, which can be detrimental when the target qubits' finite lifetime is taken into account.



Thanks to the near EP-QND character of our parity measurement, the long-term fidelity decay can be countered in order to stabilize a particular Bell state with a simple feedback protocol. To stabilize $|00\rangle + |11\rangle$, (i) apply a π-pulse around the X-axis on the first qubit whenever the measurements estimate a probability higher than $1/2$ to be in the odd parity subspace, (ii) after that, apply a $\pi/2$-pulse on both qubits around the Y-axis irrespectively of the detection result. The measurement back-action favors convergence towards the dominant parity, the π pulse correcting the parity whenever this is not even. This pushes the state towards the span of $|00\rangle \pm |11\rangle$ without favoring the target $+$. The two $\pi/2$-pulses then leave $|00\rangle + |11\rangle$ untouched and send the undesired $|00\rangle - |11\rangle$ onto $|01\rangle + |10\rangle$, such that the next parity measurement stochastically moves the corresponding population as well towards the target. Figure c shows a simulation of the expected steady-state fidelity achievable by selecting the optimal value of α, as a function of channel loss rate η and expected bit-flips per measurement iteration T_d/T_1. A 99% entanglement fidelity appears within reach of state-of-the-art experiments.

Indeed, all the required operations for this proposal have been individually implemented within the framework of quantum superconducting circuits. The strong dispersive coupling of a transmon qubit to a high-Q cavity mode, provides universal controllability of the state of the quantum harmonic oscillator modeling the cavity mode [6, 7], with a coupling strength, and hence the number of gate operations that limits a single measurement duration, more than three orders of magnitude larger than both the qubit and the cavity decay rates. Recent experiments realizing a variable coupling between cavity modes and a transmission line [8], provide the catch-gate-release capability on the propagating microwave field. The final measurement can be performed in any convenient basis after tweaking U_B.

The authors thank the Agence Nationale de la Recherche for financial support under grant ANR-14-CE26-0018. For more details, see our paper [9].

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