Grid Generation for Fusion Applications

Hervé Guillard, Jalal Lakhlili, Adrien Loseille, Alexis Loyer, Ahmed Ratnani

Grid Generation for Fusion Applications

H. Guillard[1], J. Lakhlili[2], A. Loseille[3], A. Loyer[1], A.Ratnani[2]

[1] Inria Sophia Antipolis Méditerranée and Côte d'Azur University, LJAD, CNRS , France
[2] Max-Planck-Institut für Plasmaphysik, Garching, Germany
[3] Inria Saclay Île-de-France, France

Magnetized plasma

- An extremely anisotropic medium :
  - Transport coefficients in the parallel (to the magnetic field) and perpendicular directions differ by several order of magnitude : $k_{//}/k_{\perp} \sim 10^9$
  - Dynamics is completely different in the parallel and perpendicular direction : incompressible in the perp direction, compressible (and supersonic in the SOL) in the parallel direction : plasma slides along the magnetic field lines
- Plasma - wall interaction (Bohm's boundary condition) : Plasma enter into the wall at supersonic velocity :

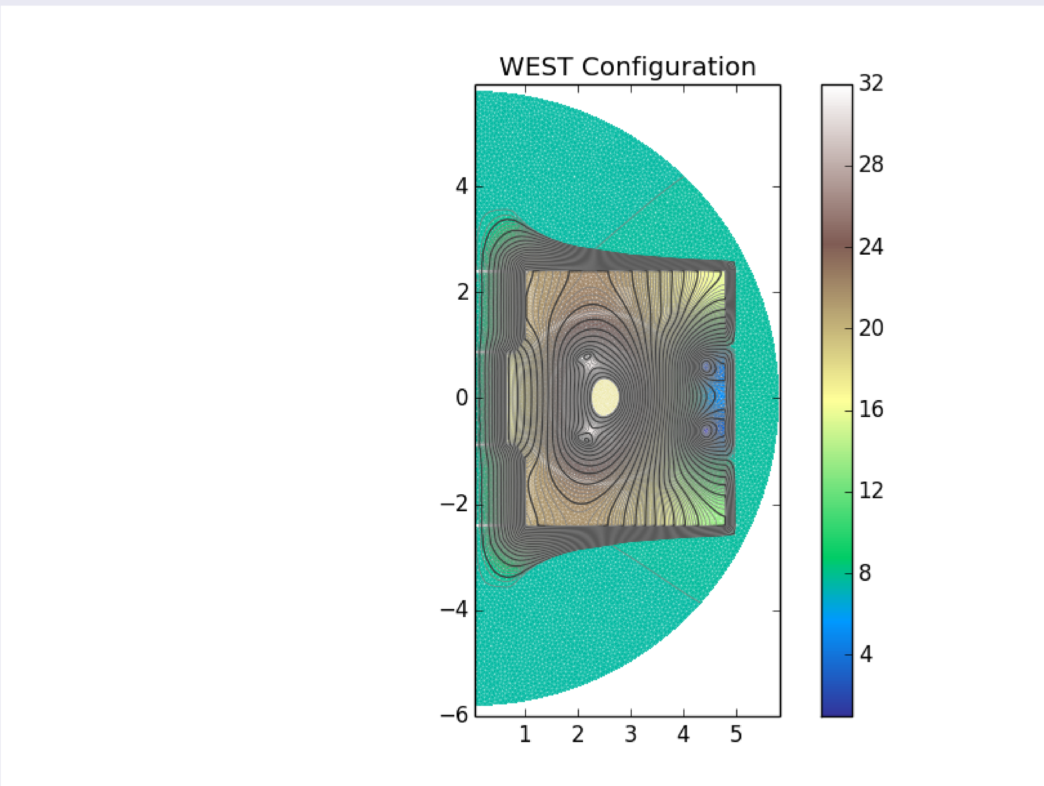
In magnetized plasma, plasma flows along the magnetic field lines

Meshes adapted to magnetic flux surfaces

Grad-Shafranov solvers

For realistic geometries, magnetic field has to be computed by numerical Grad-Shafranov solvers

$$-\nabla\left(\frac{1}{\mu r}\nabla\psi\right) = \begin{cases} \mathbf{J}(x,\psi) & \text{plasma} \\ \mathbf{J}(\text{voltage}, \partial_t\psi) & \text{coils} \\ 0 & \text{elsewhere} \end{cases}$$



Block-structured grid

1. Block-structured meshes :
   - Identify the sub-domains (blocks–patches) Ω_n
   - Construct the mappings between logical grid $[0,1]x[0,1]$ and physical Ω_n
2. How many blocks and definition of the patches ?
   This is a **segmentation problem :**
   Given a domain Ω : Find a number n and n patches Ω_n such that :
   $$\Omega = \cup\Omega_n \text{ with } \mathring{\Omega}_n \cap \mathring{\Omega}_m = \emptyset$$
   and there exists a one to one mapping between Ω_n and the the unit square :
   $$\exists\phi_n : [0,1]x[0,1] \leftrightarrow \Omega_n$$
3. for flux surface aligned grid, the segmentation problem can be solved by **Morse theory**

Segmentation Problem and Morse Function

f is a Morse function if all its critical points are regular.

Critical points
Let C^r be the space of $r \geq 2$ differentiable scalar field defined on Ω. $\mathbf{p} \in \Omega$ is a critical point of f if $\nabla f(\mathbf{p}) = \mathbf{0}$.

Regular Critical points
A critical point is regular if the Hessian of f at $\mathbf{p}$ is invertible.

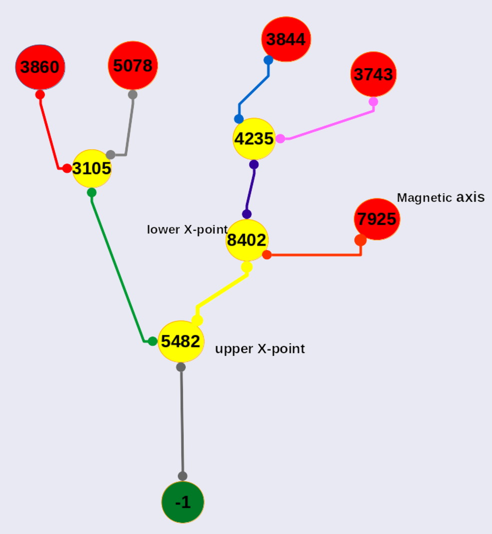
In 2-D
the only possible critical points of a Morse function are :
Maxima index = 2 Minima index = 0
Saddles index = 1

Topology of a Morse function : The topological set of the iso-contours of f consists of connected components that are either

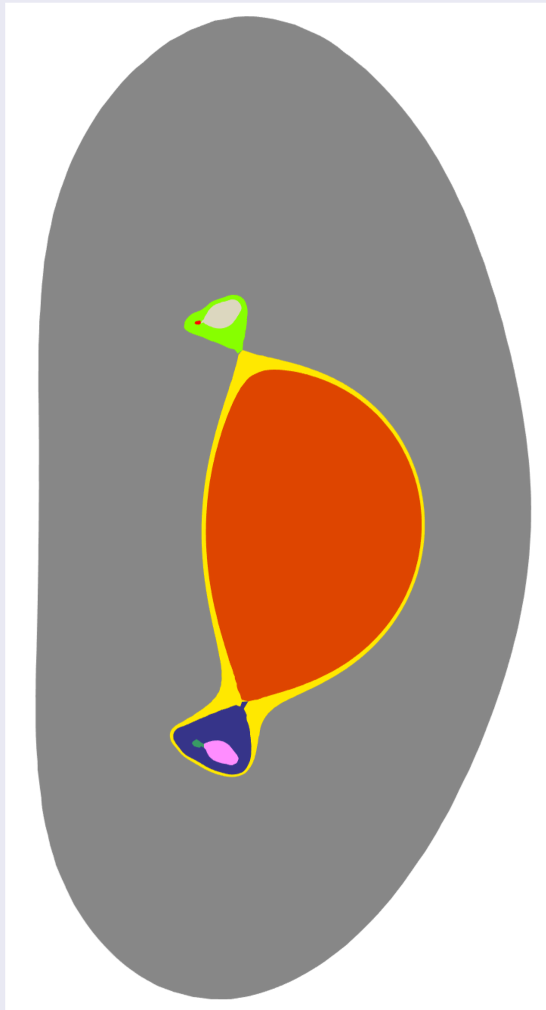
- Circle cells which are homeomorphic to open disks
- Circle bands which are homeomorphic to open annulus

The Reeb graph

- Compute the connected components by computing the Reeb Graph



- Obtain the associated domain segmentation

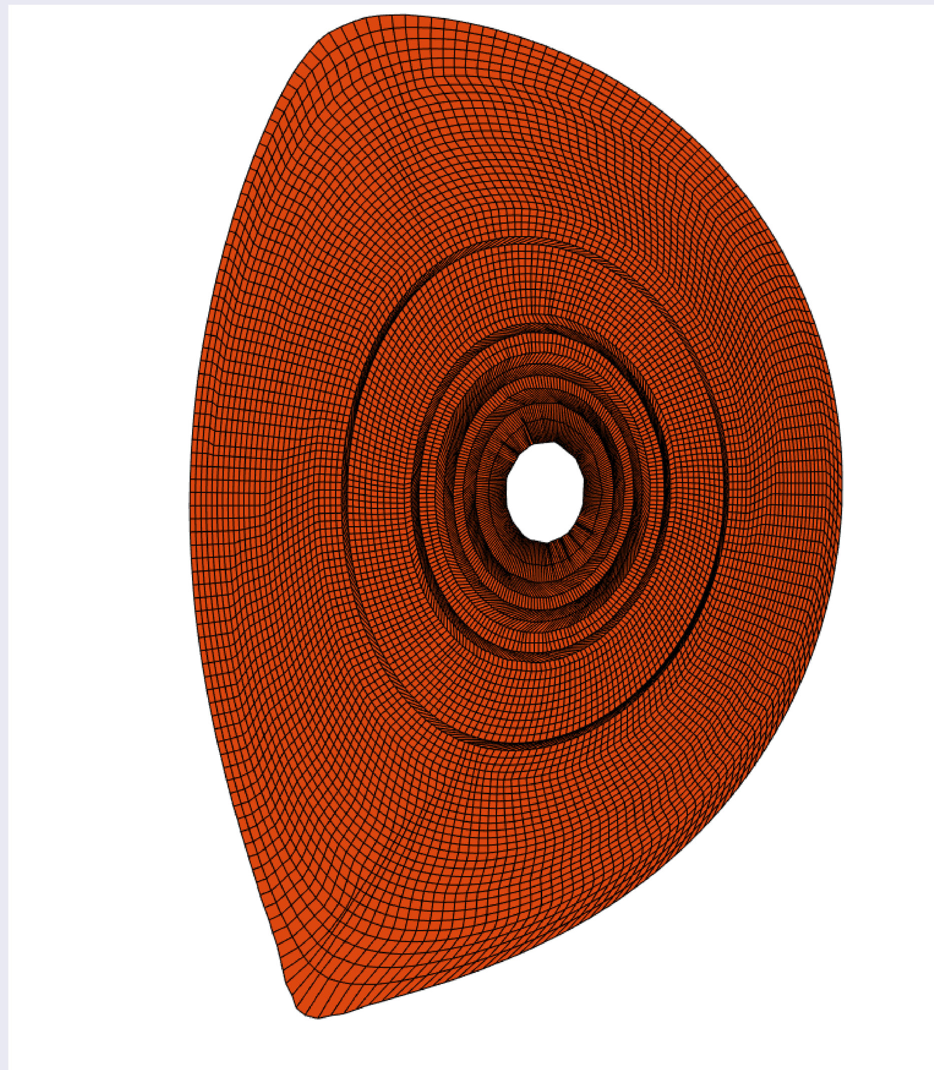


each subdomain correspond to an edge of the Reeb graph and in each subdomain, the function is monotone

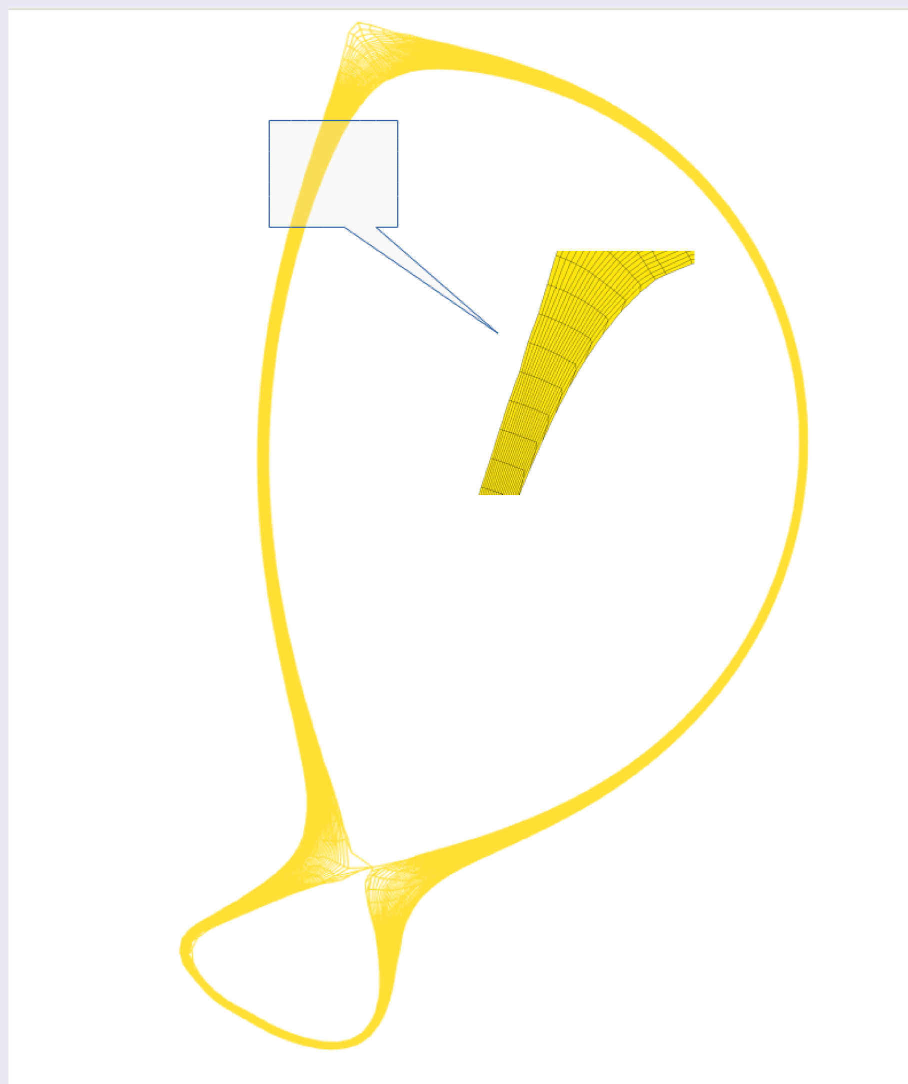
Construction of the mapping of Ω_n

Orthogonal mesh constructed by intersecting
–The isolines
–Streamline integration of $\frac{dx}{ds} = \nabla\psi$

- Leaf of the Reeb Graph : homeomorphic to a disk
  Example : Reeb Graph edge : [7925-8402]



- Internal edge of the Reeb graph : homeomorphic to annulus
  Example : Reeb Graph edge : [7925-8402]



Future works

- More accurate and smooth GS solvers
- Add additional mapping techniques : Elliptic solvers + equidistribution
- Topological simplification of the Reeb-Graph
- Construction of $\mathcal{C}^1$ mappings
- Take into account the vacuum chamber boundary