Identification, estimation and control for linear systems using measurements of higher order derivatives

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Identification, estimation and control for linear uncertain systems using measurements of higher order derivatives

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In this paper, the problem of output control for linear uncertain systems with external perturbations is studied. Firstly, it is assumed that the output available for measurement is only the higher order derivative of the state variable, instead of the state variable itself (for example, the acceleration for a second order plant), and the measurement is also corrupted by noise. Then via series of integration, an identification algorithm is proposed to identify all unknown parameters of the model and all unknown initial conditions of the state vector. Finally, two control algorithms are developed, adaptive and robust, both provide boundedness of trajectories of the system. The efficiency of the obtained solutions is demonstrated by numerical simulation.

1 Introduction

The problem of output control for uncertain linear systems with external perturbations, is one of the most important problems in the control theory. A model-based controller design requires an identification process of the uncertain system, for which full state variables need to be either measured or estimated. Usually observers for such systems are designed under assumption that, only the output variables are available for measurement, but not their derivatives [1]. However, there are cases where the measurement of high-order derivatives of the output state can be more convenient than that of output state itself, for example, it is much more easier for an individual to attach an accelerometer to the end-effector of a robot manipulator, than to mount a position decoder to the motor rotor inside the robot body. This paper addresses the mentioned problem under assumption that, only the higher order derivative of the state is measurable.

In the present work, we are mainly motivated by model identification, state estimation and control in a robot manipulator application, when sensors are installed inside the robot motors at the basements of joints, but due to flexibility of joints, the end-point position of the joint does not always coincide with the value given by motor position decoder under rigid geometry of the robot. In order to improve the control performance in such a case, an accelerometer can be installed at the end effector (at the end of a joint), to obtain additional information about acceleration at that point. Next, the problem arises as how to use this information for the model parameters identification, state estimation and control. In this paper, a general case of linear uncertain system with high-order derivative measurement is considered. A method using recursive integrals to identify system parameters and to estimate real-time state is developed, based on the results, an adaptive controller and a robust controller are proposed, and simulation results are given.

The outline of this paper is as follows. After preliminaries in Section 2, the problem statement is given in Section 3. The identification, estimation and control algorithms are presented in Section 4 . Numerical simulations for a simple second-order example are described in Section 5.

2 Preliminaries

The real numbers are denoted by $\mathbb{R}$, $\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$, the euclidean norm for a vector $x \in \mathbb{R}^n$ is denoted as $|x|$, and for a measurable and locally bounded input $u : \mathbb{R}_+ \to \mathbb{R}$, the symbol $||u||_{[t_0,t_1]}$ denotes its L_∞ norm:

$$||u||_{[t_0,t_1]} = ess \sup_{t \in [t_0,t_1]} |u(t)|,$$

if $t_1 = +\infty$ then we will simply write $||u||_\infty$. We will denote as $\mathcal{L}_\infty$ the set of all inputs u with the property $||u||_\infty < \infty$. The symbols I_n, $E_{n\times m}$ and E_p denote the identity matrix with dimension $n \times n$, the matrix with all elements equal to 1 with dimensions $n \times m$ and $p \times 1$, respectively. For a matrix $A \in \mathbb{R}^{n\times n}$, the vector of its eigenvalues is denoted as $\lambda(A)$, $\lambda_{\min}(A) = \min(\lambda(A))$, and $||A||_2 = \sqrt{\max_{i=\overline{1,n}} \lambda_i(A^T A)}$ (the induced L_2 matrix norm). The conventional results and definitions on L_2/L_∞ stability for linear systems can be found in [2].

3 Problem statement

Let us consider a SISO linear uncertain system of the following form:

$$y^{(n)}(t) = \sum_{i=0}^{n-1} a_i y^{(i)}(t) + b_0[u(t) + \varpi(t)], \quad (1)$$

where $y(t) \in \mathbb{R}$ is the system "position", $y^{(i)}(t)$ for $i = 1,\ldots,n$ are derivatives of $y(t) = y^{(0)}(t)$, the vector $x = [y, \dot{y}, \ldots, y^{(n-1)}]$ represents the state of (1); $u(t) \in \mathbb{R}$ is the system control input and $\varpi \in \mathcal{L}_\infty$ is the input disturbance. It is assumed that the coefficients a_i, $i = 0,\ldots,n-1$ and b_0 are unknown constants, $b_0 \neq 0$.

In this work we will assume that, only the corrupted signal

$$\psi(t) = y^{(n)}(t) + v(t) \quad (2)$$

is available for measurement, where $v \in \mathcal{L}_\infty$ is the measurement noise.

Assumption 1. *The case $y^{(n)}(t) = \sum_{i=0}^{n-1} a_i y^{(i)}(t) + \sum_{j=0}^{m} b_j u^{(j)}(t) + \varpi(t)$ for some $1 \le m \le n$ can be also treated by the proposed below approach. However, for brevity of presentation only the case of* (1) *is considered.*

It is required to stabilize the system (1), (2) at the origin, and ensure boundedness of all trajectories, in the presence of bounded disturbances and noises.

4 Main results

To proceed, we need the following assumptions.

Assumption 2. *For all $i = 1,\ldots n$, the signals*

$$v_i(t,t_0) = \int_{t_0}^{t} v_{i-1}(s,t_0)ds$$

with $v_0(t,t_0) = v(t)$ are essentially bounded for all $t \ge t_0 \ge 0$.

In this work, we will use the convention $v_i(t) = v_i(t,0)$.

Assumption 3. *There is a known constant $V_\infty > 0$ such that* $\max\{||\varpi||_\infty, ||v_0||_\infty, \ldots, ||v_n||_\infty\} \le V_\infty$.

Note that the integration of a high frequency noise $v(t)$ (usual for inertial sensors) leads to amplitude decreasing for $v_i(t)$ with $i \ge 1$, i.e., integration acts as a filter in this case.

4.1 Identification

Define variables for $t_0 \ge 0$ as bellow:

$$\psi_0(t,t_0) = \psi(t),$$
$$\psi_i(t,t_0) = \int_{t_0}^{t} \psi_{i-1}(s,t_0)ds \quad \forall i = 1,\ldots n,$$

then by recursive integration, we obtain for all $i = 1,\ldots n$:

$$y^{(i)}(t) = \psi_{n-i}(t,t_0) + \sum_{j=1}^{n-i} y^{(n-j)}(t_0)\frac{(t-t_0)^{n-i-j}}{(n-i-j)!} - v_{n-i}(t,t_0). \quad (3)$$

Substituting the obtained expressions for the derivatives $y^{(i)}(t)$ in (1), we get:

$$\psi_0(t,t_0) = \sum_{i=0}^{n-1} a_i \left(\psi_{n-i}(t,t_0) + \sum_{j=1}^{n-i} y_{t_0}^{(n-j)} \frac{(t-t_0)^{n-i-j}}{(n-i-j)!}\right) + b_0[u(t) + d(t)],$$

where $y_{t_0}^{(i)} = y^{(i)}(t_0)$. By Assumption 2,

$$d(t) = \varpi(t) + b_0^{-1}[v_0(t,t_0) - \sum_{i=0}^{n-1} a_i v_{n-i}(t,t_0)]$$

is a new essentially bounded disturbance, and according to Assumption 3, we have $||d||_\infty \le (1 + |b_0^{-1}|[1 + \sum_{i=0}^{n-1}|a_i|])V_\infty$.

Performing a direct expansion, we can observe that

$$\sum_{i=0}^{n-1} a_i \sum_{j=1}^{n-i} y_{t_0}^{(n-j)} \frac{(t-t_0)^{n-i-j}}{(n-i-j)!} = \sum_{i=0}^{n-1} \frac{(t-t_0)^i}{i!} \sum_{j=1}^{n-i} a_{n-i-j} y_{t_0}^{(n-j)},$$

then

$$\begin{aligned}\psi_0(t,t_0) &= \textstyle\sum_{i=0}^{n-1} a_i \psi_{n-i}(t,t_0) + \sum_{i=0}^{n-1} \frac{(t-t_0)^i}{i!} \sum_{j=1}^{n-i} a_{n-i-j} y_{t_0}^{(n-j)} \\ &\quad + b_0[u(t) + d(t)] \\ &= \omega(t,t_0)\theta + b_0 d(t),\end{aligned} \quad (4)$$

where the regressor vector

$$\omega(t,t_0) = [\psi_n(t,t_0), \ldots, \psi_1(t,t_0), \frac{(t-t_0)^{n-1}}{(n-1)!}, \frac{(t-t_0)^{n-2}}{(n-2)!}, \ldots, t-t_0, 1, u(t)]$$

is composed of known signals (integrals of the measurable output $\psi_0(t,t_0)$, functions of time t and $u(t)$), and the vector

$$\begin{aligned}\theta = [&a_0, \ldots, a_{n-1}, \\ &a_0 y_{t_0}^{(n-1)}, a_1 y_{t_0}^{(n-1)} + a_0 y_{t_0}^{(n-2)}, \ldots, \\ &\sum_{j=1}^{n-1} a_{n-j-1} y_{t_0}^{(n-j)}, \sum_{j=1}^{n} a_{n-j} y_{t_0}^{(n-j)}, b_0]^T\end{aligned}$$

contains all unknown parameters of the regression model (4),and all unknown initial conditions of the state y.

There exist many methods to solve the equation (4), with respect to θ minimizing the noise influence [3]. One of the simplest consists in multiplication of both sides in (4) by $\omega^T(t,t_0)$,

$$\omega^T(t,t_0)\psi_0(t,t_0) = \omega^T(t,t_0)\omega(t,t_0)\theta + \omega^T(t,t_0)b_0 d(t),$$

and integration till un instant when the matrix $M(t,t_0) = \int_{t_0}^{t} \omega^T(s,t_0)\omega(s,t_0)ds$ becomes nonsingular, then

$$\hat{\theta}(t,t_0) = M^{-1}(t,t_0)\int_{t_0}^{t} \omega^T(s,t_0)\psi_0(s,t_0)ds \quad (5)$$

is an estimate of θ with the estimation error:

$$|\theta - \hat{\theta}(t,t_0)| \leq |b_0|\lambda_{\min}^{-1}(M(t,t_0))||\omega^T(\cdot,t_0)||_{[t_0,t]}||d||_\infty.$$

The non-singularity of $M(t,t_0)$ is related to the property of persistence of excitation in (4) [4], which we have to impose.

Assumption 4. *For any $t \geq 0$, there exist $T > 0$ and $\mu > 0$ such that, $\lambda_{\min}(M(t+T,t)) \geq \mu$.*

Note that the norm $||\omega(\cdot,t_0)||_{[t_0,t]}$ is growing with time, thus a minimal imposition of T is desirable by a selection of $u(t)$.

Proposition 1. *Let Assumptions 2, 3, 4 be satisfied, and there exist $\mathbb{T} > 0$ such that $||x||_{[0,\mathbb{T}]} + ||u||_{[0,\mathbb{T}]} < +\infty$, then there exists $\Theta > 0$ such that in (5),*

$$|\theta - \hat{\theta}(kT,(k-1)T)| \leq \Theta$$

for all $1 \leq k \leq \frac{\mathbb{T}}{T}$.

Proof. By construction and imposed assumptions $|\theta - \hat{\theta}(kT,(k-1)T)| \leq |b_0|\mu^{-1}||\omega(\cdot,(k-1)T)||_{[(k-1)T,kT]}||d||_\infty$. By assumptions 2 and 3, $||d||_\infty < +\infty$, and μ is a non-zero real by Assumption 4. It is necessary to evaluate $||\omega(\cdot,(k-1)T)||_{[(k-1)T,kT]}$, but all components dependent explicitly on time are bounded on the interval $[(k-1)T,kT]$, u is bounded by conditions of the proposition, and ψ_i are bounded by construction and due to boundedness of x. Therefore, $||\omega(\cdot,(k-1)T)||_{[(k-1)T,kT]} < +\infty$ while $||x||_{[(k-1)T,kT]} + ||u||_{[(k-1)T,kT]} < +\infty$, that was necessary to prove.

Consequently, according to Proposition 1, on any finite time interval ((1) is a linear system, thus no finite-time escape is possible and for any finite $\mathbb{T} > 0$ the property $||x||_{[0,\mathbb{T}]} + ||u||_{[0,\mathbb{T}]} < +\infty$ is satisfied), the identification algorithm (5) provides a solution with a bounded error. The crucial step in (5) is the resetting of all integrators after the period of time T, in order to avoid integration drift and unboundedness of the regressor ω.

Assume that, for $t_0 \geq 0$ in the conditions of Proposition 1 (Assumptions 2, 3, 4 are satisfied), the estimate $\hat{\theta}(t_0+T,t_0)$ is obtained and $|\theta - \hat{\theta}(t_0+T,t_0)| \leq \Theta$ for some $\Theta > 0$, then from the definition of θ, it is easy to check that, the estimate $\hat{\eta}(t_0+T,t_0)$ of the constant vector

$$\eta = [a_0,\ldots,a_{n-1},y_{t_0}^{(0)},\ldots,y_{t_0}^{(n-1)},b_0]$$

can be calculated with the property

$$|\eta - \hat{\eta}(t_0+T,t_0)| \leq \Theta',$$

for some $\Theta' > 0$ related with Θ. Indeed, $a_0,\ldots,a_{n-1}$ and b_0 are the first and the last elements of θ, respectively, next $y_{t_0}^{(n-1)}$ can be found from the value of $n+1$ element of θ, and next recursively for all $y_{t_0}^{(i)}$. Denote

$$\hat{\eta} = [\hat{a}_0,\ldots,\hat{a}_{n-1},\hat{y}_{t_0}^{(0)},\ldots,\hat{y}_{t_0}^{(n-1)},\hat{b}_0],$$

then for all $i = 0,\ldots,n-1$, we have the state estimate

$$\hat{y}^{(i)}(t) = \psi_{n-i}(t,t_0) + \sum_{j=1}^{n-i}\hat{y}_{t_0}^{(n-j)}\frac{(t-t_0)^{n-i-j}}{(n-i-j)!}. \quad (6)$$

Defining the state estimation error $e_i(t) = y^{(i)}(t) - \hat{y}^{(i)}(t)$, we obtain that

$$e_i(t) = \sum_{j=1}^{n-i}(y_{t_0}^{(n-j)} - \hat{y}_{t_0}^{(n-j)})\frac{(t-t_0)^{n-i-j}}{(n-i-j)!} - v_{n-i}(t,t_0). \quad (7)$$

Denote $\hat{x} = [\hat{y}^{(0)},\hat{y}^{(1)},\ldots,\hat{y}^{(n-1)}]$ and $e = [e_0,\ldots,e_{n-1}]$. Finally, the estimate of $\varpi(t)$ from (1) can be derived as bellow:

$$\hat{\varpi}(t) = \hat{b}_0^{-1}[\psi_0(t,t_0) - \sum_{i=0}^{n-1}\hat{a}_i\hat{y}^{(i)}(t)] - u(t).$$

4.2 State space representation for controller design

To simplify forthcoming analysis, we suppose that the control gain is given (or the value of $\hat{b}_0$ after identification has been obtained correctly).

Assumption 5. *The non-zero value b_0 is known.*

In such a case, the discrepancy $|b_0 - \hat{b}_0|$ can be used for the constant Θ' evaluation.

In the state space form, the system (1) can be written as follows:

$$\begin{cases} \dot{x} = Ax + B(u+\varpi), \\ Y = \hat{y}^{(n-1)} = y^{(n-1)} + \delta = Cx + \delta, \end{cases} \quad (8)$$

where $x = [y,\ y^{(1)},\ \ldots,\ y^{(n-1)}]^T$ is the state vector, $Y \in \mathbb{R}$ is the estimate of $y^{(n-1)}$, served as a new measured output with new measurement noise δ, and

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & & \\ a_0 & a_1 \ \ldots & a_{n-1} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix},$$
$$C = [0,\ldots,0,1].$$

4.3 Adaptive control

There exists an unknown vector $k = [k_1,\ k_2,\ \ldots,\ k_n]^T \in \mathbb{R}^n$, such that $D = A - Bk^T$ is a given matrix in the canonical controllability form with desired eigenvalues, then (8) becomes:

$$\dot{x} = Dx + B(u + k^T x + \varpi). \tag{9}$$

By the definition of k and x, and from (3), we have

$$\begin{aligned} k^T x = & \sum_{i=0}^{n-1} k_{i+1}\psi_{n-i}(t,t_0) - \sum_{i=0}^{n-1} k_{i+1}v_{n-i}(t,t_0) \\ & + \sum_{i=0}^{n-1} k_{i+1}[\sum_{j=1}^{n-i} y^{(n-j)}(t_0)\frac{(t-t_0)^{n-i-j}}{(n-i-j)!}]. \end{aligned}$$

Introduce $m = i + j$, then we have

$$\begin{aligned} k^T x + \sum_{i=0}^{n-1} k_{i+1}v_{n-i}(t,t_0) = \sum_{i=0}^{n-1} k_{i+1}\psi_{n-i}(t,t_0) + \\ \sum_{m=1}^{n} \frac{(t-t_0)^{n-m}}{(n-m)!}\left(\sum_{j=1}^{m} k_{m-j+1}y^{(n-j)}(t_0)\right). \end{aligned}$$

Denote $\bar{\omega}^T(t,t_0)$ as

$$\begin{aligned} \bar{\omega}^T(t,t_0) = & [\psi_n(t,t_0), \psi_{n-1}(t,t_0), \ldots, \psi_1(t,t_0), \\ & \frac{(t-t_0)^{n-1}}{(n-1)!}, \frac{(t-t_0)^{n-2}}{(n-2)!}, \ldots, 1], \end{aligned}$$

and $\bar{\theta}$ as

$$\begin{aligned} \bar{\theta} = & [k_1, k_2, \ldots, k_n, k_1 y^{n-1}(t_0), \sum_{j=1}^{2} k_{3-j}y^{(n-j)}(t_0), \ldots, \\ & \sum_{j=1}^{n} k_{n-j+1}y^{(n-j)}(t_0)]^T, \end{aligned}$$

where $\bar{\omega}^T(t,t_0)$ is the new regressor vector, and $\bar{\theta} \in \mathbb{R}^{2n}$ is the vector of unknown constant parameters, then we have

$$k^T x + \sum_{i=0}^{n-1} k_{i+1}v_{n-i}(t,t_0) = \bar{\omega}^T(t,t_0)\bar{\theta}. \tag{10}$$

Substitute (10) into (9), and define a new perturbation signal $\phi(t) = \varpi(t) - \sum_{i=0}^{n-1} k_{i+1}v_{n-i}(t,t_0)$, we obtain

$$\dot{x} = Dx + B(u + \bar{\omega}^T(t,t_0)\bar{\theta} + \phi(t)). \tag{11}$$

Choose the control law in the following form:

$$u(t) = -\bar{\omega}^T(t,t_0)\hat{\bar{\theta}}(t), \tag{12}$$

where $\hat{\bar{\theta}} \in \mathbb{R}^{2n}$ is the estimate of $\bar{\theta}$ to be calculated, then the system (8) with the control (12) takes the form:

$$\dot{x}(t) = Dx(t) + B\left[\bar{\omega}^T(t,t_0)[\bar{\theta} - \hat{\bar{\theta}}(t)] + \phi(t)\right].$$

There are many ways to derive $\hat{\bar{\theta}}$ using direct or indirect adaptive control theory [5]. For example, by designing an adaptive observer [6, 7] (the only difficulty is to select a solution providing a better robustness with respect to v_i and ϖ) in the following form:

$$\begin{aligned} \dot{z}(t) &= Dz(t) - B\bar{\omega}^T(t,t_0)\hat{\bar{\theta}}(t), \\ \dot{\Omega}(t) &= D\Omega(t) - B\bar{\omega}^T(t,t_0), \\ \dot{\hat{\bar{\theta}}}(t) &= -\gamma\Omega^T(t)C^T[Y(t) - Cz(t) + C\Omega(t)\hat{\bar{\theta}}(t)], \end{aligned} \tag{13}$$

where $\gamma > 0$ is a tuning parameter, $z \in \mathbb{R}^n$ and $\Omega \in \mathbb{R}^{n\times 2n}$ are two auxiliary variables. For the system (8), by adopting the control input (12) with the adaptive observer (13), we obtain for an error $\varepsilon = x - z + \Omega\bar{\theta}$ that

$$\dot{\varepsilon}(t) = D\varepsilon(t) + B\phi(t),$$

which explains the structure of the used adaptation law:

$$\dot{\hat{\bar{\theta}}}(t) = -\gamma\Omega^T(t)C^T[C\varepsilon(t) + \delta(t) + C\Omega(t)\{\hat{\bar{\theta}}(t) - \bar{\theta}\}].$$

Since D is a Hurwitz matrix and $\phi \in \mathcal{L}_\infty$ by Assumption 3, then we have $\varepsilon \in \mathcal{L}_\infty$ with the norm asymptotically proportional to $||\phi||_\infty$, and the discrepancy $\bar{\theta} - \hat{\bar{\theta}}(t)$ possesses the same property [8], provided that the variable $\Omega^T(t)C^T$ is persistently excited.

Assumption 6. *For any $t \geq 0$, there exist $T' > 0$ and $\nu > 0$, such that*

$$\int_t^{t+T'} \Omega^T(s)C^T C\Omega(s)ds \geq \nu I_{2n}.$$

Generally, Assumptions 4 and 6 are related, but we prefer to state them separately, one for identification and the other one for control phase, respectively.

Theorem 1. *Let Assumptions 2, 3, 5 and 6 be satisfied. Then for the system* (1), (2)*, with the control law* (12) *and the adaptive observer* (13)*, for any $t_0 \geq 0$ and $\mathbb{T} > 0$, the variables x, z, Ω, $\hat{\bar{\theta}}$ stay bounded on the interval* $[t_0, t_0 + \mathbb{T}]$.

Proof. Note that on any finite time interval $[t_0, t_0 + \mathbb{T}]$, the regressor $\bar{\omega}$ is bounded (consequently, Ω has the same property since D is Hurwitz), and using Lemma 1 of [8] and Assumption 6, we obtain boundedness of the discrepancy $\bar{\theta} - \hat{\bar{\theta}}(t)$. Boundedness of the variables x and z follows the Hurwitz property of D and the boundedness of all external signals in the right-hand side of the differential equations describing dynamics of these variables.

Note that in the considered case, it is hard to state an asymptotic result, or consider the system on unbounded interval $[0,+\infty]$, since the regressor $\bar{\omega}$ depends on the powers of t for $n \geq 2$ and it is asymptotically unbounded.

4.4 Robust control

Based on the results given by the identification process, a simple static feedback can be applied:

$$u = -\kappa^T \hat{x}, \tag{14}$$

where the vector of coefficients $\kappa \in \mathbb{R}^n$ is selected in a way to ensure that the matrix $H = A - B\kappa^T$ is Hurwitz. To choose κ, if the identification results are of high precision, then an initial κ can be given using pole placement method with respect to $\hat{A}$ and $\hat{B}$, and can be then adjusted based on the control performance.

Theorem 2. *Let Assumptions 2 and 3 be satisfied. Then in the system* (1), (2), *with the control* (14), *for any* $t_0 \geq 0$ *and* $\mathbb{T} > 0$, *the variable* x *stays bounded on the interval* $[t_0, t_0 + \mathbb{T}]$.

Proof. Substituting the control (14) in (8) (an equivalent state space representation of (1), (2)), we obtain:

$$\begin{aligned}\dot{x} &= Ax + B(-\kappa^T \hat{x} + \varpi) \\ &= Hx + B(\kappa^T e + \varpi).\end{aligned}$$

On any finite time interval, the error e is bounded, the same property has the signal ϖ, while the matrix H is Hurwitz, thus the state variable is bounded.

5 Simulations

5.1 Model specification

We select $n = 2$, $t_0 = 0$, and the model is specified as:

$$\begin{gathered} a_0 = 2,\ a_1 = -4,\ b_0 = 2, \\ v(t) = 0.0025\sin(25t),\ \varpi(t) = 0.002\sin(t), \\ x(0) = [0.5\ 5]^T. \end{gathered}$$

In state space representation, we have

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 2 \end{bmatrix},\ C = [0,\ 1],$$

then the system is unstable.

5.2 Identification

During the identification phase, to guarantee an excitation of the system from the input and to not drive too much the system by the input, a bounded periodic signal with components of different frequencies is take as

$$u(t) = -\cos(8t) + \sin(\frac{1}{4}t) - \cos(5t).$$

The step time is set to be 1ms, after 1000 samples (1 second), based on (5), we obtain the estimates as:

$$\begin{gathered} \hat{a}_0 = 2.015,\ \hat{a}_1 = -3.997,\ \hat{b}_0 = 2.000, \\ \hat{y_0}^{(0)} = 0.468,\ \hat{y_0}^{(1)} = 4.991. \end{gathered}$$

The identified parameters and initial conditions are very close to the real values. According to (7), the state estimation error can be expressed as

$$\begin{cases} e(t) = y(t) - \hat{y}(t) = 0.009\,t + 0.032 - v_2(t), \\ \dot{e}(t) = \dot{y}(t) - \dot{\hat{y}}(t) = 0.009 - v_1(t). \end{cases} \tag{15}$$

From (15), the error on the velocity estimation is bounded, but the error on the position estimation increases with time. In fact, the second-order system is a particular case, since y can also be estimated by:

$$\hat{y}' = (\hat{\ddot{y}} - \hat{a}_1\hat{\dot{y}} - \hat{b}_0 u)/\hat{a}_0, \tag{16}$$

where $\hat{\dot{y}}$ is the measurement, in this way, the position estimation error will be bounded.

5.3 Adaptive controller

For the proposed adaptive control scheme, taking

$$D = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix},\ \gamma = 0.4$$

as parameters of the adaptive control algorithm (12), (13), this gives $k = (B^TB)^{-1}B^T(A-D) = [k_1, k_2]^T = [3, 0]^T$, and $\bar{\theta} = [k_1,\ k_2,\ k_1 y^{(1)}(t_0),\ k_2 y^{(1)}(t_0) + k_1 y^{(0)}(t_0)]^T = [3, 0, 15, 1.5]^T$. In practice, $\bar{\theta}$ is unknown due to the unknown model parameters and initial conditions. To estimate $\bar{\theta}$, firstly, a time-independent estimate can be driven from the identification results; then by taking this static estimate as initial value, a dynamical estimation can be performed with the observer (13).

Using the identification results in Section 5.2, the time-independent estimate of k, denoted as $\hat{k}_c$, is given as

$$\hat{k}_c = [\hat{k}_{c1},\ \hat{k}_{c2}] = (\hat{B}^T\hat{B})^{-1}\hat{B}^T(\hat{A} - D) = [3.0074, 0.0013]^T, \tag{17}$$

where

$$\hat{A} = \begin{bmatrix} 0 & 1 \\ \hat{a}_0 & \hat{a}_1 \end{bmatrix},\ \hat{B} = \begin{bmatrix} 0 \\ \hat{b}_0 \end{bmatrix}.$$

For the proposed adaptive control process, the time-dependent estimate of $\bar{\theta}$, denoted as $\hat{\bar{\theta}} = [\hat{\bar{\theta}}_1, \hat{\bar{\theta}}_2, \hat{\bar{\theta}}_3, \hat{\bar{\theta}}_4]^T$, can be initialized as in (18), using the identified initial conditions and $\hat{k}_c$. Then the evaluation of $\hat{\bar{\theta}}$ with time follows the observer (13).

$$\begin{aligned}\hat{\bar{\theta}}(t_0) &= [\hat{k}_{c1},\ \hat{k}_{c2},\ \hat{k}_{c1}\hat{y}_0^{(1)},\ \hat{k}_{c2}\hat{y}_0^{(1)} + \hat{k}_{c1}\hat{y}_0^{(0)}]^T \\ &= [3.0074, 0.0013, 15.0101, 1.4142]^T.\end{aligned} \quad (18)$$

Because of the good result of the identification of the model parameters and the initial conditions, $\hat{\bar{\theta}}(t_0)$ is rather close to $\bar{\theta}$. For the simulation, the first 1 second is devoted to the identification process, the time-trace of the update of the elements of $\hat{\bar{\theta}}$ during the following 7 seconds is shown in Figure 1. The control performance with respect to position and velocity tracking of application of the adaptive controller is illustrated in Figure 2. We observed that the error $e(t)$ increased with time while $\dot{e}(t)$ stays always quite close to zero. This can be explained according to (15) that, the position error increases with time while the velocity error is bounded.

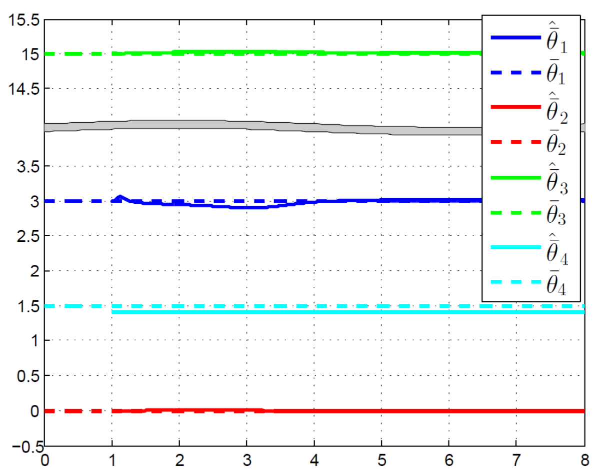

Fig. 1. Time-trace of $\hat{\bar{\theta}}$

5.4 Robust controller

We take the same desired matrix, i.e. $H = D$, and then $\kappa = \hat{k}$ as the gains of the robust control (14), while $\hat{k}$ is determined by (17), the results of the robust controller are shown in Figure 3. Compared to the performance of the adaptive controller, the robust control converges more quickly than the adaptive one, and it is easier to implement.

5.5 Periodical identification

As stated in Theorem 1 and Theorem 2, both the adaptive and the robust controllers guarantee the boundedness of the state variable on a bounded time interval, to attenuate the time-increasing state errors (such as the position in Figure 2 and in Figure 3), the identification process can be performed periodically.

Lack of persistent excitation, the input signals from $t = 7s$ to $t = 8s$ cannot be used for the proposed identification method. So, the authors repeated the sinusoidal input signal in Section 5.2, from $t = 8s$ and lasts for $1s$. This may give undesired behavior of the system, but the time-increasing errors can be attenuate. Table 1 gives the comparison of the position estimation before and after the second identification.

Table 1. Position estimation with and without periodical identification

	adaptive control	robust control
$y(t = 8)$	0.0721	0.1266
$\hat{y}(t = 8)$	-0.0205	0.0336
$\hat{y}(t = 8)$ re-identified at $t = 9s$	0.0795	0.1350

6 Conclusion

The problem of model identification and output control has been studied in this work for linear time-invariant SISO system with completely unknown parameters, external disturbances, while the output is represented by derivative $y^{(n)}$ corrupted by a bounded noise. It is shown that by introducing n recursive integrals and using a simple identification algorithm, some estimates on vector of unknown parameters and states can be obtained. Next, these information can be used in control, adaptive or robust, to provide boundedness of the state vector of the system. Due to integration drift all results are obtained on final intervals of time. Efficacy of the proposed identification and control algorithms is demonstrated in simulations.

The authors have tried to implement the identification method for a robot application case: using accelerometer instead of link-side angular position encoder to identify the joint model parameters (linear acceleration can be transformed to angular acceleration). However, in practice, the output of an accelerator on an axis is always corrupted by the projection of the gravitational acceleration on the axis. To remove this undesired "noise", the angular position of the link need to be known a priori, which cannot be done without additional sensors. The application of the identification and control method in this paper is expected in the future work.

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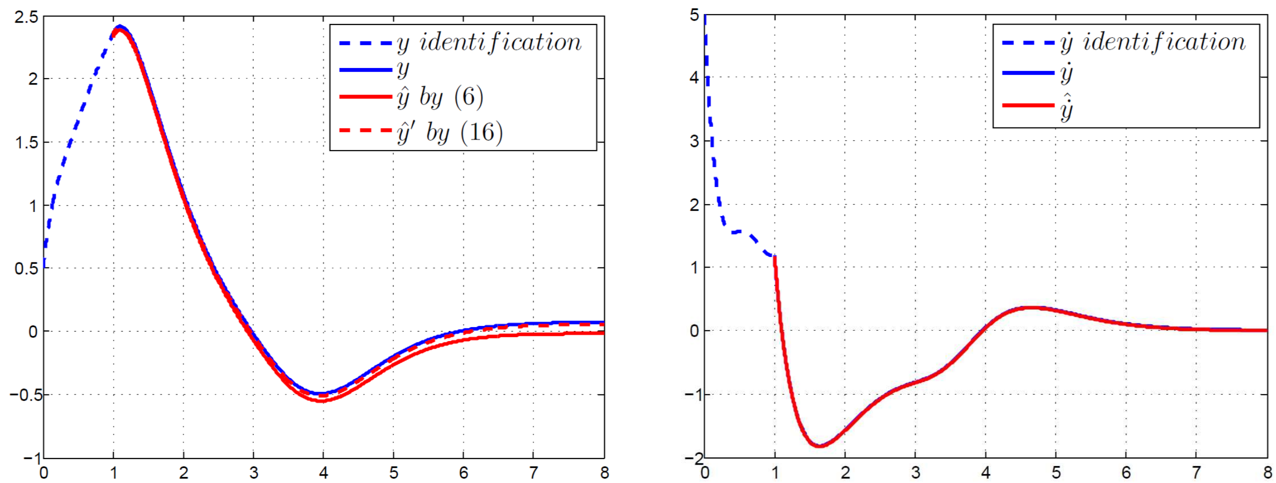

Fig. 2. Position and velocity for adaptive controller

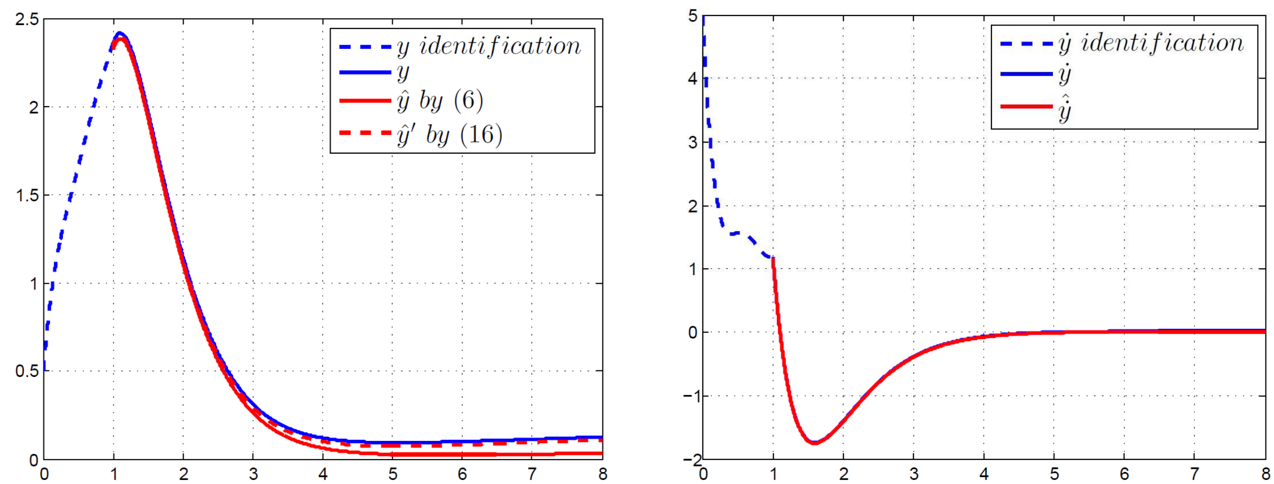

Fig. 3. Position and velocity for robust controller

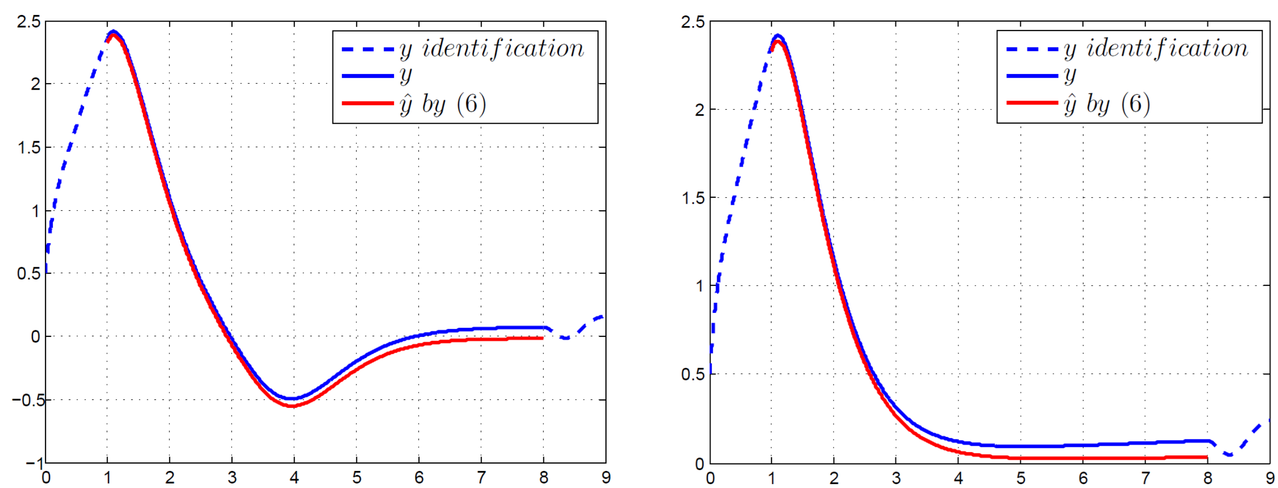

Fig. 4. Position behavior with periodical identification (left one with adaptive controller, right one with robust controller)

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