



# Combining Free choice and Time in Petri Nets

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# Combining Free choice and Time in Petri Nets

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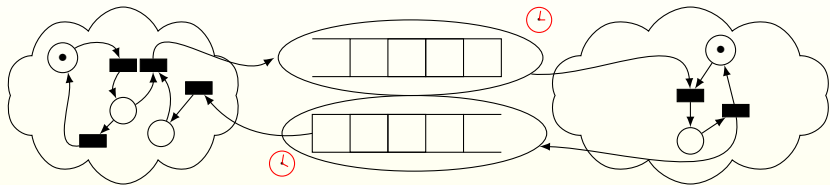
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TRENDS Sept. 9<sup>th</sup> 2017

[TIME'16]

# Motivation 1 : Modeling issues

Model Time constrained **unbounded** concurrent systems



## Desired features

- Latency : messages take **at least 10 ms** to reach their destination
- Timeout/urgency : a message not consumed **after 200ms** is lost
- Rates : a message is received **every  $\gamma$  t.u.,...**

# Motivation 2 : Verification

## Standard questions

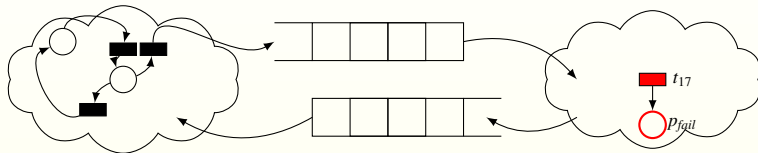
- **Reachability** : is marking  $M$  reachable from initial marking  $M_0$  ?
- **Coverability** : Given a marking  $M$ , is there a marking  $M'$  reachable from  $M_0$  such that  $M'(p) > M(p)$  for every place  $p$  ?
- **Boundedness** : is there a bound  $K$  such that for every reachable marking, every place  $p$ ,  $M(p) \leq K$  ?
- **Firability** : is there an execution in which transition  $t$  is fired ?

## Objectives

- Decidability for these questions
- Efficient algorithms

# Motivation 3 : Robustness

$\varphi_{17}$  : Transition  $t_{17}$  (a major failure) is not firable.



What if :

- time is measured with some imprecision
- clocks tend to have some drift/jitter/delay, ...

**Robustness : reasoning with idealized time representation**

Assume a class of properties  $\Phi$ , a model for time imprecision  $\llbracket \cdot \rrbracket_\delta$

Given a model  $\mathcal{M}$ , and a value  $\delta \in \mathbb{R}$ , check that :

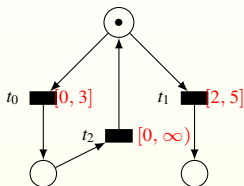
$$\forall \varphi \in \Phi, \mathcal{M} \models \varphi \iff \llbracket \mathcal{M} \rrbracket_\delta \models \varphi$$

Given a model  $\mathcal{M}$ , check if :

$$\exists \Delta, \forall \delta \leq \Delta, \forall \varphi \in \Phi, \mathcal{M} \models \varphi \iff \llbracket \mathcal{M} \rrbracket_\delta \models \varphi$$

# Time vs Timed Petri nets

*Time Petri nets* [Merlin74]

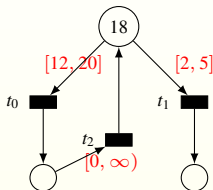


Pros :

- time
- Urgency
- unbounded places
- Expressive power

Cons : Undecidability

*Timed Petri nets* [Walter83]



Pros :

- time
- unbounded places
- Ages
- WSTS  $\equiv$  decidability of coverability, boundedness,...

Cons : no urgency

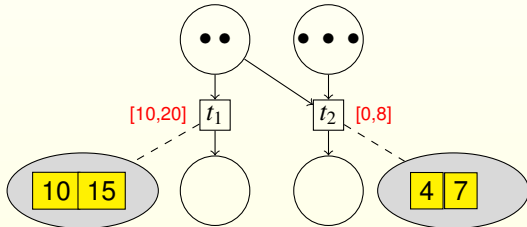
- Free-choice Multiserver Time Petri nets
- Processes and their relation to untimed nets
- Firability
- Robustness

# TPN Multiserver Semantics : Configuration

## Configuration (Threshold Semantics)

$$C = (M, mem)$$

- $M$  : marking, enables transitions **several** times
- $mem$  : remembers for each **enabling instance** of a transition for how long it has been enabled

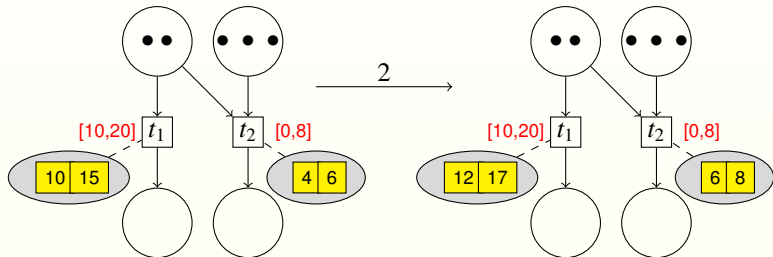


$$M(p_0) = 2; M(p_1) = 3; M(p_3) = 0; M(p_4) = 0$$

$$mem(t_1) = \{10; 15\} \quad mem(t_2) = \{4; 7\}$$



# TPN Multiserver Semantics : timed move



Timed Move :  $C \xrightarrow{\delta} C'$

Let a duration  $\delta$  elapse = update memorized durations

Urgency of a transition  $t$  in  $C = (M, mem)$

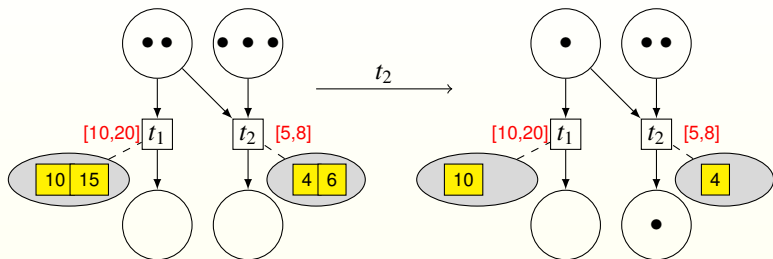
Let  $t$  be a transition such that :

- $I(t) = [l, u]$
- $t$  has been enabled for  $u$  t.u,                    i.e.,  $\max mem(t) = u$

then,  $t$  is **urgent**.

Time **cannot** progress, a **discrete** move must occur.

# TPN Multiserver Semantics : discrete move



Discrete firing :  $C \xrightarrow{t_i} C'$

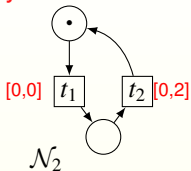
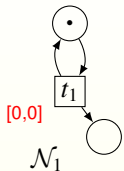
- $I(t_i) = [l, u], \max(\text{mem}(t_i)) > l$  ( $t_i$  need not be urgent)
- modification of memory for transitions in competition with  $t_i$
- Other enabling instances remain untouched

Behaviors of a net

- Labeled Transition System :  $(\mathcal{C}, \longrightarrow, C_0)$
- $\text{Lang}(\mathcal{N}) \subseteq T \times \mathbb{R}$  :  
set of timed words of  $\mathcal{N}$  :  $w = (t_1, d_1)(t_2, d_2) \dots$

# Restrictions (to obtain decidable classes)

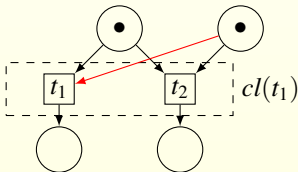
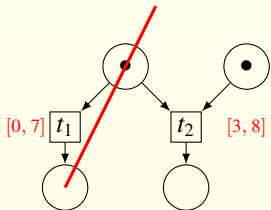
Forbid nets that can **force zero-delay behaviors**.



## Free choice PN and free choice TPN

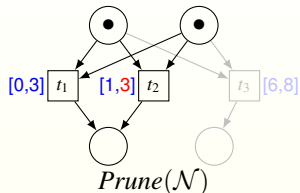
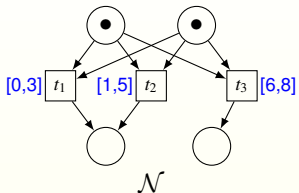
(FC-PN)  $\mathcal{U} = (P, T, F)$  is **free choice** if  $\forall t, t' \in T, \bullet t \cap \bullet t' \neq \emptyset \implies \bullet t = \bullet t'$ .  
(FC-TPN)  $\mathcal{N} = (\mathcal{U}, M_0, I)$  is a **free choice TPN** if  $Untime(\mathcal{N}) = \mathcal{U}$  is free choice.

The **cluster** of transition  $t$  is  $Cl(t) = \{t' \in T \mid \bullet t \cap \bullet t' \neq \emptyset\}$



# Pruning FC-TPN (a.k.a normalization [Chatain13])

Some transitions in FC-TPNs will obviously never fire !



To obtain a Pruned FC-TPN

- remove unfirable transitions :

$$t : I(t) = [a, b] \wedge \exists t', I(t') = [c, d] \text{ with } d < a$$

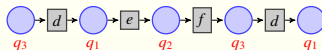
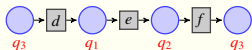
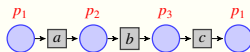
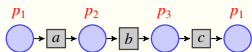
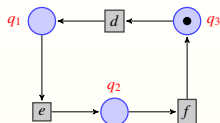
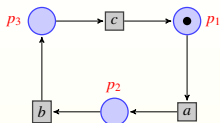
- Associate to remaining transition possible values for intervals

$$I(t) = [a, b] \Rightarrow I'(t) = [a, \max_{t' \in Cl(t)} (lft(t'))]$$

**Pruning Lemma** : Let  $\mathcal{N}$  be a **FC-TPN**, then

the transition systems associated with  $\mathcal{N}$  and  $Prune(\mathcal{N})$  are isomorphic.  
(not true outside FC-TPNs)

# Causal processes



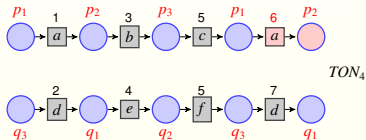
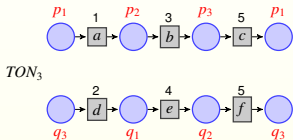
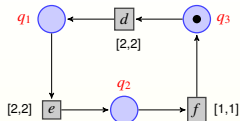
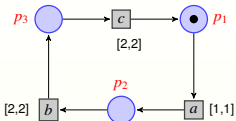
Untimed causal processes  $ON_1$  and  $ON_2$

## Causal processes

- A partial order representation of executions
- **Principle** : Unfold the net by glueing transitions/places occurrences one after another starting from the initially marked places

*Still an untimed setting !*

# Timed causal processes



## Advantages of Timed Causal processes

- Partial order timed representation of timed executions
- **URGENCY** is considered : if  $d$  occurs at date 7 in  $TON$ , all urgent transitions before  $d$  also occur in  $TON$ .

## Relation between Timed causal Processes and timed languages

Let  $\mathcal{P}$  be the set of timed causal processes of  $\mathcal{N}$ . Then,

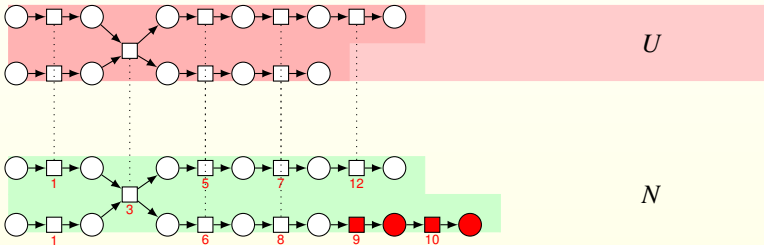
$$Lang(\mathcal{N}) = \bigcup_{N \in \mathcal{P}} Lang(N)$$

# Properties of Free Choice TPN

## Theorem 1 : Inclusion of untimed prefixes

Let  $\mathcal{N} = (\mathcal{U}, M_0, I)$  be a **pruned FC-TPN** (w.o. forced 0-delay sequences).  
 $\mathcal{U}$  be an (untimed) causal process of  $\mathcal{U} = \text{Untime}(\mathcal{N})$ .

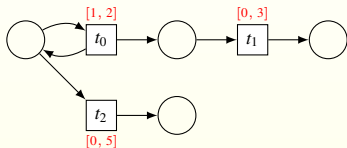
Then there exists a timed causal process  $N$  of  $\mathcal{N}$  such that  $\mathcal{U} \leq \text{Untime}(N)$ .



## Theorem 2 : Fireability

Let  $\mathcal{N} = (\mathcal{U}, M_0, I)$  be an FC-TPN (w.o. forced 0-delay).  
Checking fireability of a transition  $t \in T$  in  $\mathcal{N}$  is **decidable**.

Proof idea : Firability  $\sim$  coverability in untimed nets [Rack78]



## Theorem 3 : Termination

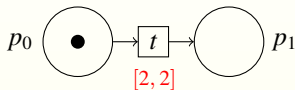
Let  $\mathcal{N} = (\mathcal{U}, M_0, I)$  be an FC-TPN (w.o. forced 0-delay). It is decidable if  $\mathcal{N}$  terminates.

### **Proof Idea :**

an infinite run of  $\mathcal{U} = \text{Untime}(\text{Prune}(\mathcal{N}))$  has a timed counterpart in  $\mathcal{N}$  and conversely. (and termination is decidable in untimed Petri nets)

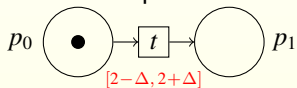


A major drawback of timed models :

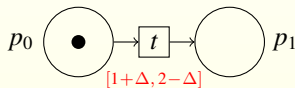


```
public class MyProgram {  
    throws InterruptedException {  
        Myprogram prog=new MyProgram();  
        //Pause for 2 seconds  
        Thread.sleep(2000);  
        prog.t();  
    }  
}
```

What the implementation might really do :



*Guard enlargement*



*Guard Shrinking*

Form now :  $\mathcal{N}_\delta = \mathcal{N}$  with enlarged guards

## Definition (robustness problems for TPNs)

Given a TPN  $\mathcal{N}$ , does there exist  $\Delta \in \mathbb{Q}_{>0}$  such that  $\forall \delta \leq \Delta$

- $\text{Fireable}(\mathcal{N}) = \text{Fireable}(\mathcal{N}_\delta)$ ?
- $\mathcal{N}$  terminates iff  $\mathcal{N}_\delta$  terminates
- $\mathcal{N}$  is bounded iff  $\mathcal{N}_\delta$  is bounded...

Note : Firability, termination, boundedness are not a priori robust/non robust properties of the whole class of FC-TPNs.

## Theorem 4

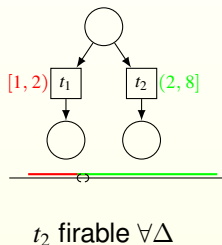
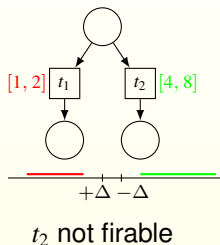
Let  $\mathcal{N}$  be a FC-TPN without forced 0-delay time firing sequences. Then robustness of fireability is decidable. If  $\mathcal{N}$  has robust fireability, bound  $\Delta$  can be effectively computed.

## Theorem 5

Let  $\mathcal{N}$  be a FC-TPN without forced 0-delay time firing sequences. Then it is decidable whether termination is a robust property of  $\mathcal{N}$ .

# Proof idea

If  $\text{Prune}(\mathcal{N})$  and  $\text{Prune}(\mathcal{N}_\Delta)$  have the same clusters, then  $\Delta$ -enlargement of  $\mathcal{N}$  does not modify firable transitions.



Check for each accessible cluster  $C$  whether intervals can be enlarged by some  $\Delta_C$  without changing firable transitions

Contributions so far :

- A FC-Multiserver PN variant with its process semantics
- Decidable firability, termination
- Decidability of robustness of firability, termination, wrt enlargement (and shrinking)

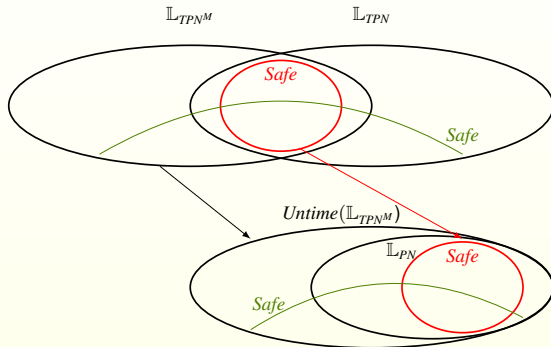
Open questions :

- decidability of Coverability, reachability and boundedness for FC-TPNs ?
- Robustness of more properties ?
- same issues without multi-enabledness ?
- Expressiveness of FC-TPNs ?

$\mathbb{L}_{TPNM}$  : Timed languages expressible with Time Petri nets (Multiserver)

$\mathbb{L}_{TPN}$  : Timed languages expressible with Time Petri nets

$\mathbb{L}_{PN}$  : Untimed languages expressible with Petri nets



(Thm. 1) In free choice nets, timed / untimed processes tightly related  
What about **Languages** ?

$$\mathbb{L}_{PN} \subseteq \text{Untime}(\mathbb{L}_{TPNM}) \quad \mathbb{L}_{PN \cap \text{Safe}} = \text{Untime}(\mathbb{L}_{TPNM \cap \text{Safe}}) \quad \mathbb{L}_{PN} \neq \text{Untime}(\mathbb{L}_{TPNM}??)$$

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