Combining Free choice and Time in Petri Nets

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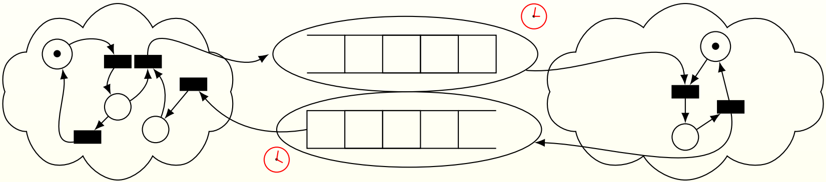
[TIME'16]

Motivation 1 : Modeling issues

Model Time constrained unbounded concurrent systems



Desired features

- Latency : messages take at least 10 ms to reach their destination
- Timeout/urgency : a message not consumed after 200ms is lost
- Rates : a message is received every γ t.u,...

Motivation 2 : Verification

Standard questions

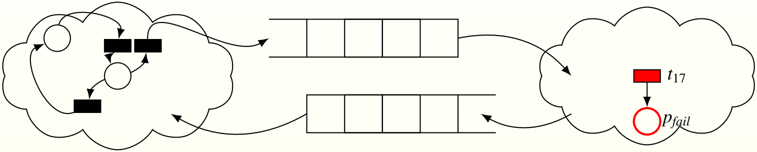
- Reachability : is marking M reachable from initial marking M_0 ?
- Coverability : Given a marking M, is there a marking M' reachable from M_0 such that $M'(p) > M(p)$ for every place p ?
- Boundedness : is there a bound K such that for every reachable marking, every place p, $M(p) \leq K$?
- Firability : is there an execution in which transition t is fired ?

Objectives

- Decidability for these questions
- Efficient algorithms

Motivation 3 : Robustness

φ_{17} : Transition t_{17} (a major failure) is not firable.



What if :

- time is measured with some imprecision
- clocks tend to have some drift/jitter/delay, ...

Robustness : reasonning with idealized time representation

Assume a class of properties Φ, a model for time imprecision $[\![.]\!]_\delta$

Given a model $\mathcal{M}$, and a value $\delta \in \mathbb{R}$, check that :

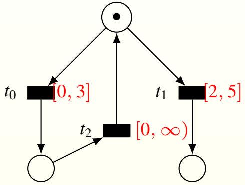
$$\forall \varphi \in \Phi, \mathcal{M} \models \varphi \Longleftrightarrow [\![\mathcal{M}]\!]_\delta \models \varphi$$

Given a model $\mathcal{M}$, check if :

$$\exists \Delta, \forall \delta \leq \Delta, \forall \varphi \in \Phi, \mathcal{M} \models \varphi \Longleftrightarrow [\![\mathcal{M}]\!]_\delta \models \varphi$$

Time vs Timed Petri nets

Time Petri nets [Merlin74]

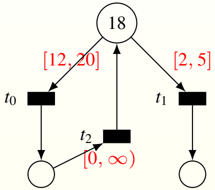


Pros :

- time
- Urgency
- unbounded places
- Expressive power

Cons : Undecidability

Timed Petri nets [Walter83]



Pros :

- time
- unbounded places
- Ages
- WSTS $\equiv$ decidability of coverability, boundedness,...

Cons : no urgency

Outline

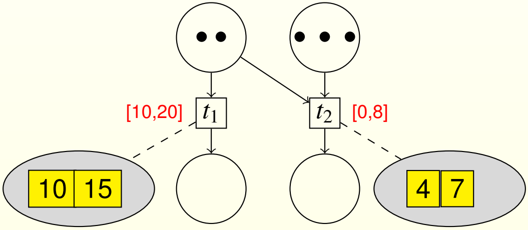
- Free-choice Multiserver Time Petri nets
- Processes and their relation to untimed nets
- Firability
- Robustness

TPN Multiserver Semantics : Configuration

Configuration (Threshold Semantics)

$$C = (M, mem)$$

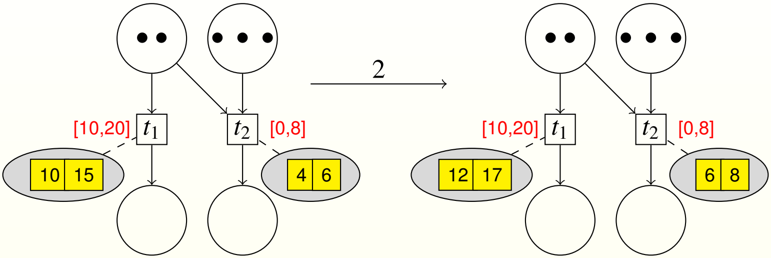
- M : marking, enables transitions several times
- mem : remembers for each enabling instance of a transition for how long it has been enabled



$M(p_0) = 2; M(p_1) = 3; M(p_3) = 0; M(p_4) = 0$

$mem(t_1) = \{10; 15\}$ $mem(t_2) = \{4; 7\}$

TPN Multiserver Semantics : timed move



Timed Move : $C \xrightarrow{\delta} C'$

Let a duration δ elapse = update memorized durations

Urgency of a transition t in $C = (M, mem)$

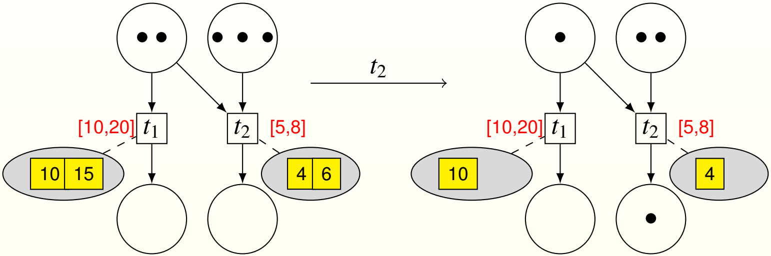
Let t be a transition such that :

- $I(t) = [l, u]$
- t has been enabled for u t.u, i.e., $\max mem(t) = u$

then, t is urgent.

Time **cannot** progress, a discrete move must occur.

TPN Multiserver Semantics : discrete move



Discrete firing : $C \xrightarrow{t_i} C'$

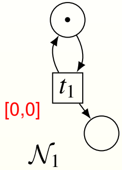
- $I(t_i) = [l, u], \max(mem(t_i)) > l$ (t_i need not be urgent)
- modification of memory for transitions in competition with t_i
- Other enabling instances remain untouched

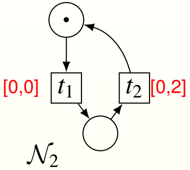
Behaviors of a net

- Labeled Transition System : $(\mathcal{C}, \longrightarrow, C_0)$
- *Lang*$(\mathcal{N}) \subseteq T \times \mathbb{R}$:
  set of timed words of $\mathcal{N}$: $w = (t_1, d_1)(t_2, d2)\dots$

Restrictions (to obtain decidable classes)

Forbid nets that can force zero-delay behaviors.

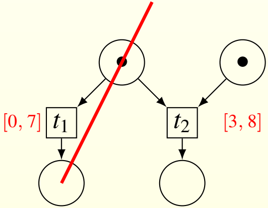


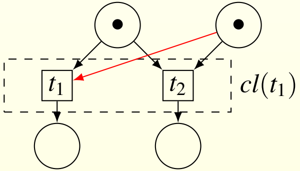


Free choice PN and free choice TPN

(FC-PN) $\mathcal{U} = (P, T, F)$ is free choice if $\forall t, t' \in T, {}^\bullet t \cap {}^\bullet t' \neq \emptyset \implies {}^\bullet t = {}^\bullet t'$.
(FC-TPN) $\mathcal{N} = (\mathcal{U}, M_0, I)$ is a free choice TPN if *Untime*$(\mathcal{N}) = \mathcal{U}$ is free choice.

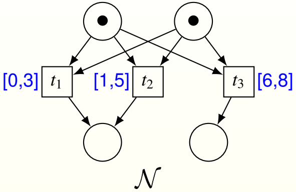
The cluster of transition t is $Cl(t) = \{t' \in T \mid {}^\bullet t \cap {}^\bullet t' \neq \emptyset\}$

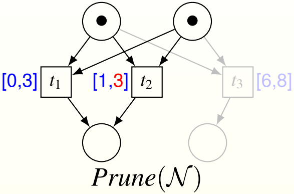




Pruning FC-TPN (a.k.a normalization [Chatain13])

Some transitions in FC-TPNs will obviously never fire !





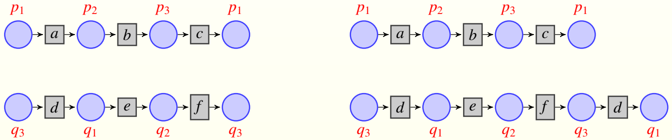
To obtain a Pruned FC-TPN

- remove unfirable transitions :
  $t : I(t) = [a, b] \wedge \exists t', I(t') = [c, d]$ with $d < a$
- Associate to remaining transition possible values for intervals
  $I(t) = [a, b] \Rightarrow I'(t) = [a, \max_{t \in Cl(t)} (lft(t'))]$

Pruning Lemma : Let $\mathcal{N}$ be a FC-TPN, then

the transition systems associated with $\mathcal{N}$ and *Prune*$(\mathcal{N}))$ are isomorphic.
(not true outside FC-TPNs)

Causal processes



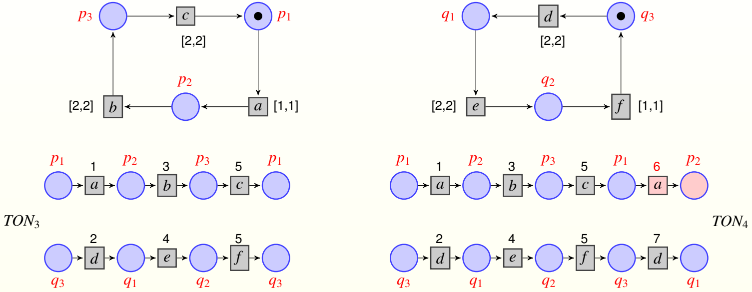
Untimed causal processes ON_1 and ON_2

Causal processes

- A partial order representation of executions
- **Principle :** Unfold the net by glueing transitions/places occurrences one after another starting from the initially marked places

Still an untimed setting !

Timed causal processes



Advantages of Timed Causal processes

- Partial order timed representation of timed executions
- URGENCY is considered : if d occurs at date 7 in TON, all urgent transitions before d also occur in TON.

Relation between Timed causal Processes and timed languages

Let $\mathcal{P}$ be the set of timed causal processes of $\mathcal{N}$. Then,

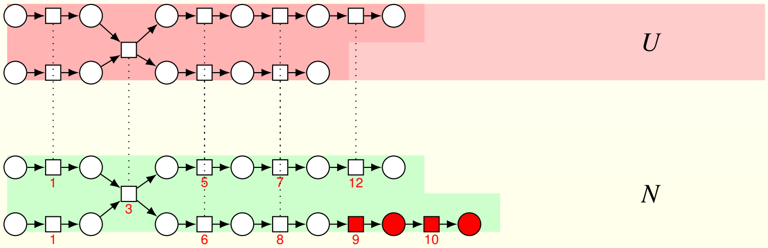
$$Lang(\mathcal{N}) = \bigcup_{N \in \mathcal{P}} Lang(N)$$

Properties of Free Choice TPN

Theorem 1 : Inclusion of untimed prefixes

Let $\mathcal{N} = (\mathcal{U}, M_0, I)$ be a pruned FC-TPN (w.o. forced 0-delay sequences).
U be an (untimed) causal process of $\mathcal{U} = \mathit{Untime}(\mathcal{N})$.

Then there exists a timed causal process N of $\mathcal{N}$ such that $U \leq \mathit{Untime}(N)$.

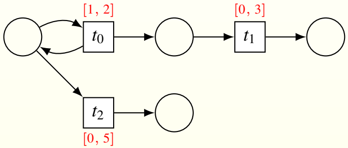


Decidability

Theorem 2 : Fireability

Let $\mathcal{N} = (\mathcal{U}, M_0, I)$ be an FC-TPN (w.o. forced 0-delay).
Checking fireability of a transition $t \in T$ in $\mathcal{N}$ is decidable.

Proof idea :Firability $\sim$ coverability in untimed nets [Rack78]



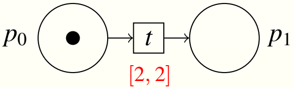
Theorem 3 : Termination

Let $\mathcal{N} = (\mathcal{U}, M_0, I)$ be an FC-TPN (w.o. forced 0-delay). It is decidable if $\mathcal{N}$ terminates.

Proof Idea :
an infinite run of $\mathcal{U} = \mathit{Untime}(\mathit{Prune}(\mathcal{N}))$ has a timed counterpart in $\mathcal{N}$ and conversely. `(and termination is decidable in untimed Petri nets)`

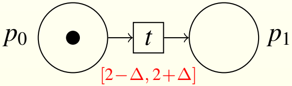
Robustness

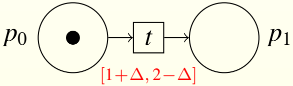
A major drawback of timed models :




```
public class MyProgram {
     throws InterruptedException {
          Myprogram prog=new MyProgram();
          //Pause for 2 seconds
          Thread.sleep(2000);
          prog.t();
     }
}
```

What the implementation might really do :





Guard enlargement

Guard Shrinking

Form now : $\mathcal{N}_\delta = \mathcal{N}$ with enlarged guards

Robustness

Definition (robustness problems for TPNs)

Given a TPN $\mathcal{N}$, does there exist $\Delta \in \mathbb{Q}_{>0}$ such that $\forall \delta \leq \Delta$

- *Fireable*$(\mathcal{N}) =$ *Fireable*$(\mathcal{N}_\delta)$?
- $\mathcal{N}$ terminates iff $\mathcal{N}_\delta$ terminates
- $\mathcal{N}$ is bounded iff $\mathcal{N}_\delta$ is bounded...

Note : Firability, termination, boundedness are not a priori robust/non robust properties of the whole class of FC-TPNs.

Theorem 4

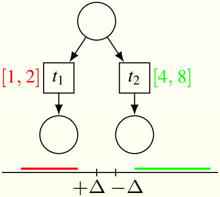
Let $\mathcal{N}$ be a FC-TPN without forced 0-delay time firing sequences. Then robustness of fireability is decidable. If $\mathcal{N}$ has robust fireability, bound Δ can Be effectvely computed.

Theorem 5

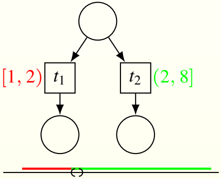
Let $\mathcal{N}$ be a FC-TPN without forced 0-delay time firing sequences. Then it is decidable whether temination is a robust property of $\mathcal{N}$.

Proof idea

If $Prune(\mathcal{N})$ and $Prune(\mathcal{N}_\Delta)$ have the same clusters, then Δ-enlargement of $\mathcal{N}$ does not modify firable transitions.



t_2 not firable



t_2 firable $\forall\Delta$

Check for each accessible cluster C whether intervals can be enlarged by some Δ_C without changing firable transitions

Conclusion

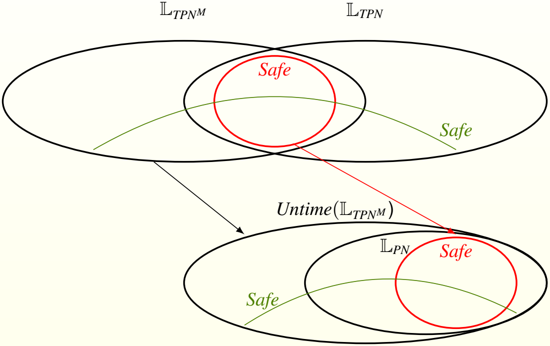
Contributions so far :

- A FC-Multiserver PN variant with its process semantics
- Decidable firability, termination
- Decidability of robustness of firability, termination, wrt enlargement (and shrinking)

Open questions :

- decidability of Coverabillity, reachability and boundedness for FC-TPNs ?
- Robustness of more properties ?
- same issues without multi-enabledness ?
- Expressiveness of FC-TPNs ?

$\mathbb{L}_{TPN^M}$: Timed languages expressible with Time Petri nets (Multiserver)
$\mathbb{L}_{TPN}$: Timed languages expressible with Time Petri nets
$\mathbb{L}_{PN}$: Untimed languages expressible with Petri nets



(Thm. 1) In free choice nets, timed / untimed processes tightly related
What about Languages ?

$\mathbb{L}_{PN} \subseteq Untime(\mathbb{L}_{TPN^M})$ $\quad$ $\mathbb{L}_{PN \cap Safe} = Untime(\mathbb{L}_{TPN^M \cap Safe})$ $\quad$ $\mathbb{L}_{PN} \neq Untime(\mathbb{L}_{TPN^M}??)$

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