Maximizing microalgae productivity by shading outdoor cultures

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HAL Id: hal-01666463
https://hal.inria.fr/hal-01666463

Submitted on 15 Jan 2018

Maximizing microalgae productivity by shading outdoor cultures [⋆]

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Abstract: Outdoor microalgae cultures can undergo a photoinhibitory process that can result in a loss in biomass productivity. This loss can be reduced by shading the culture such that the incident photon flux decreases. Based on a simple model of light-limited growth, we look for a control strategy to shadow the culture in order to maximize the biomass productivity. The strategy results in a feedback control that depends on the microalgae strain, the microalgae concentration, and the incident light. In the case that the incident light and the loss rate vary periodically in time, we give conditions for the existence of a positive periodic solution that is globally stable. We show the performance of the feedback control by means of numerical simulations.

Keywords: Dynamics and control; Industrial biotechnology; Photoinhibition; Biomass productivity; Microalgae

1. INTRODUCTION

Light supply is one of the most important factors affecting microalgae growth. Howewer, excess light can result in photoinhibition, that is, a decrease in the rate of photosynthesis due to high light intensities (Long et al. (1994)). In outdoor microalgae cultures, photoinhibition may cause a loss in biomass productivity in the midday, even in high dense cultures (Qiang et al. (1996)). Indeed, Vonshak and Guy (1992) demonstrated that by shading dense *Spirulina platensis* cultures grown in outdoor and protecting them from full exposure to solar radiation higher productivities could be achieved. This shows that by shading adequately outdoor cultures, the biomass productivity can increase. In this context, shading systems are used nowadays for plant cultures in greenhouse (Gent (2007))

This paper is concerned with the optimization of biomass productivity in outdoor raceway ponds by shading microalgae. In the literature, we can find some works where the incident light is directly modulated. For example, Mairet and Bernard (2016) modulate the incident light in order to maintain a constant light at the bottom of the culture. However, we are interested in cultures illuminated with natural light, where we cannot directly control the incident light.

In our approach, we model the microalgae growth following the work of Huisman and Weissing (1994). In order to include the process of shading the culture, we consider a control variable $u \in [0, 1]$ indicating the percentage of the incident photon flux arriving to the culture surface. We will refer to u as *exposure factor*. Then, we construct a feedback control for the exposure factor that depends on the microalgae strain, the microalgae concentration, and the incident light. In the case that the growth rate function is a Haldane-type function (see Eq. (2)), this control depends on the light intensities at the bottom and at the top of the culture (see Eq. (19)). Regardless of the feasibility of using this control in real systems, we can determine the loss in productivity that can be attributed to high light intensities.

This article is organized as follows. In section 2, we describe a model for light-limited cultures. In section 3, we construct a feedback control for the exposure factor that maximizes the biomass productivity. Finally, by numerical simulations, we show the performance of the control strategy in section 4.

2. MODELLING LIGHT-LIMITED GROWTH OF MICROALGAE

Let us consider a phototrophic continuous culture of depth L in which microalgae grow (whose concentration will be denoted by x). We assume that the system is well mixed such that the concentrations are homogeneous. Mixing does not prevent the formation of a vertical light gradient: the light decreases progressively in moving deeper into the culture medium due to light absorption and scattering by microalgae (Bernard et al. (2016)). Let us assume that the light decrease in the water can be described by

the Lambert-Beer law. Thus, at depth $z \in [0, L]$, the corresponding light intensity $I(x, I_{in}, z)$ satisfies

$$I(x, I_{in}, z) = I_{in}e^{-axz}, \qquad (1)$$

where I_{in} is the incident light, and $a > 0$ is the specific light attenuation coefficient of the microalgae.

We assume that nutrients and carbon dioxide are in ample supply, so that the growth rate is only affected by the light i.e the specific growth rate $\mu : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a function of the light intensity I. Additionally, we assume that

Hypothesis 1. μ is differentiable, there is $I^* > 0$ such that $\mu'(I) > 0$ for all $I \in [0, I^*)$ and $\mu'(I) < 0$ for all $I \in (I^*, \infty)$, $\mu(0) = 0$, and $\mu(I)/I$ is bounded on $(0, \infty]$.

I^* corresponds to the irradiance at which growth rate is maximal i.e. at higher irradiances than I^* photoinhibition occurs. Several models for the specific growth rate in the literature fullfill this hypothesis (Han et al. (2000); Peeters and Eilers (1978); Platt (1980); Yeh et al. (2010)). For instance, Bernard and Rémond (2012) presented the following Haldane-type model

$$\mu(I) = \mu_{max} \frac{I}{I + \frac{\mu_{max}}{\alpha}\left(\frac{I}{I^*} - 1\right)^2} \qquad (2)$$

where α is the initial slope of the light response curve, and μ_{max} is the specific maximum growth rate. Figure 1 shows the function μ given in (2) calibrated for *Chlorella vulgaris* (see paramaters in Table 1) with experimental data obtained by Yeh et al. (2010).

Table 1. Kinetic parameters of μ in (2).

Parameter	Value	Unit
μ_{max}	1.63	d^{-1}
I^*	87.2	$\mu mol\, m^{-2}\, s^{-1}$
α	0.027	$\mu mol^{-1}\, m^2\, s\, d^{-1}$

An interesting property of any Haldane-type model [2], that will be used later, is that

$$\mu(I_1) = \mu(I_2) \text{ if, and only if, } I_1 = I_2 \text{ or } I_1I_2 = I^{*2}. \quad (3)$$

Following Huisman and Weissing (1994), we compute the total growth function g in the reactor by integrating the local growth over depth $g(\cdot) := \frac{x}{L}\int_0^L \mu(I(x, I_{in}, z))dz$. By doing the change of variable $I = I(x, I_{in}, z)$, we rewrite the total growth rate

$$g(\cdot) = \frac{1}{aL}\int_{I_{out}(xL, I_{in})}^{I_{in}} \frac{\mu(I)}{I}dI, \qquad (4)$$

where $I_{out}(xL, I_{in})$ is used for indicating the light intensity at the bottom of the reactor i.e. $I_{out}(xL, I_{in}) = I(x, I_{in}, L)$.

Considering a dilution rate D and a mortality rate m, the dynamic evolution of the microalgae concentration x is given by

$$\dot{x} = g(\cdot) - (D + m)x. \qquad (5)$$

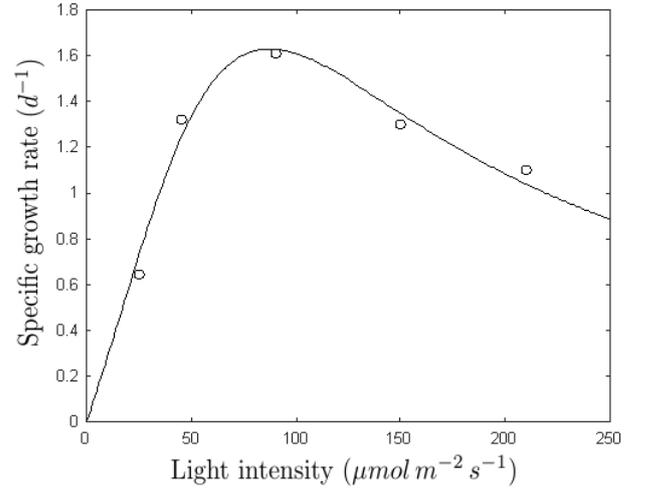[2] By a Haldane-type model, we mean $\mu(I) = \frac{I}{aI^2 + bI + c}$ with $a, c > 0$.

Fig. 1. Haldane function given in (2). The parameters are given in Table 1.

By doing the change of variable $X = xL$, we rewrite Eq. (5) in terms of X;

$$\dot{X} = G(X, I_{in}) - (D + m)X, \qquad (6)$$

with

$$G(X, I_{in}) := Lg(\cdot) = \frac{1}{a}\int_{I_{out}(X, I_{in})}^{I_{in}} \frac{\mu(I)}{I}dI. \qquad (7)$$

X corresponds to the areal microalgal concentration.

From expression (7) we can compute the derivatives of G with respect to X for each $I_{in} > 0$;

$$\frac{\partial}{\partial X}G(X, I_{in}) = \mu(I_{out}(X, I_{in})), \qquad (8)$$

and

$$\frac{\partial^2}{\partial X^2}G(X, I_{in}) = -aI_{out}(X, I_{in})\mu'(I_{out}(X, I_{in})). \quad (9)$$

We can see that (8) remains positive on $[0, \infty)$ while (9) changes its sign at

$$\bar{X}(I_{in}) := \max\left(0, \frac{1}{a}\ln\left(\frac{I_{in}}{I^*}\right)\right). \qquad (10)$$

Then, we can state the following lemma.

Proposition 2. (Properties of G)

a) $G(X, 0) = 0$ for any $X \geq 0$,
b) $G(\cdot, I_{in})$ is strictly increasing on $[0, \infty)$, strictly convex on $[0, \bar{X}(I_{in}))$, and strictly concave on $(\bar{X}(I_{in}), \infty)$ for any $I_{in} > 0$.
c) G is Lipschitz.

Proof. For the part c), we have that

$$\frac{\partial}{\partial I_{in}}G(X, I_{in}) = \frac{1}{aI_{in}}[\mu(I_{in}) - \mu(I_{in}e^{-aX})]. \qquad (11)$$

From Hypothesis 1, we have that the partial derivatives of G are bounded, thus, G is Lipschitz. □

3. CONTROLLING THE EXPOSURE FACTOR

The problem of optimizing the biomass productivity can be stated as the following optimal control problem

$$\max P = \int_{t_i}^{t_f} D(t)X(t)dt,$$
$$s.t. \quad \dot{X}(t) = G(X, u(t)I_{in}(t)) - (D(t) + m(t))X,$$
$$X(0) = X_0 , \ u(t) \in [0,1], \tag{12}$$

where P is the areal biomass productivity on the interval of time $[t_i, t_f]$, and X_0 is the initial areal microalgal concentration. The exposure factor u (defined in the introduction) is our control variable. We assume that I_{in}, D, and m are continuous functions of time.

We note that the objective function P does not depend directly on u and is increasing in X (point-wise order). Therefore, if a function $\epsilon : \mathbb{R}^2_+ \longrightarrow \mathbb{R}_+$ satisfies

$$G(X, \epsilon(X, I_{in})I_{in}) = \max\{G(X, \delta I_{in}) ; \delta \in [0,1]\}, \tag{13}$$

for all $X, I_{in} \geq 0$, and $u = \epsilon(X, I_{in})$ satisfies the restrictions of (12), then, u is a solution of (12), and ϵ is an optimal feedback control.

To construct a function ϵ satisfying condition (13), we need a description of the monotonicity of G with respect to I_{in}. For this reason, we state the following lemma (see Eq. (11)).

Lemma 3. Let $X > 0$ be given, and let $h : \mathbb{R}_+ \longrightarrow \mathbb{R}$ be the function defined by

$$h(I) = \mu(I) - \mu(Ie^{-aX}). \tag{14}$$

Then, there exists $\hat{I} \in (I^*, I^*e^{aX})$ such that $h(\hat{I}) = 0$, h is positive on $(0, \hat{I})$, and h is negative on $(\hat{I}, \infty)$.

Proof. Since μ is strictly increasing on $(0, I^*]$, we have that

$$h(I) > 0 \text{ for all } I \in (0, I^*]. \tag{15}$$

In the same way, since μ is strictly decreasing on $[I^*, \infty)$, we have that

$$h(I) < 0 \text{ for all } I \in [I^*e^{aX}, \infty). \tag{16}$$

From (15) and (16), we conclude that there exists $\hat{I} \in J := (I^*, I^*e^{aX})$ such that $h(\hat{I}) = 0$. Now let $I \in J$ be given, then $Ie^{-aX} < I^* < I$, which implies

$$h'(I) = \mu'(I) - e^{-aX}\mu'(Ie^{-aX}) < 0,$$

from where h is strictly decreasing on J. Combining this result with the inequalities (15) and (16), we conclude that h is positive on $(0, \hat{I})$ and negative on $(\hat{I}, \infty)$. $\square$

Given a microalgae concentration $X > 0$, we have that (see Eq.(11) and (14))

$$\frac{\partial}{\partial I_{in}} G(X, I_{in}) = \frac{1}{aLI_{in}} h(I_{in}). \tag{17}$$

According to Lemma 3, there exists $\mathcal{I}(X) > 0$ such that $G(X, \cdot)$ is strictly increasing on $(0, \mathcal{I}(X))$ and is strictly

decreasing on $(\mathcal{I}(X), \infty)$. This shows that, if the incident light I_{in} is greater than $\mathcal{I}(X)$, we should reduce I_{in} with a factor $\epsilon(X, I_{in}) := \mathcal{I}(X)/I_{in}$ in order to illuminate the system with optimal light for the total growth rate. Conversely, if I_{in} is lower than $\mathcal{I}(X)$, since $G(X, \cdot)$ is strictly increasing on $(0, \mathcal{I}(X))$ any decrease in the incident light will decrease the total growth rate, so we should take $\epsilon(X, I_{in}) := 1$. Thus, we have defined ϵ for any $X, I_{in} > 0$ by

$$\epsilon(X, I_{in}) = \begin{cases} 1 & if \ I_{in} \leq \mathcal{I}(X), \\ \dfrac{\mathcal{I}(X)}{I_{in}} & if \ I_{in} > \mathcal{I}(X). \end{cases} \tag{18}$$

Now we extend the definition of ϵ to $\mathbb{R}^2_+$. From Lemma 3, we have that $I^* < \mathcal{I}(X) < I^*e^{aX}$, which implies that $\lim_{X \to 0^+} \mathcal{I}(X) = I^*$. Then we define $\mathcal{I}(0) := I^*$. This extends the definition of ϵ to $X = 0$. With respect to I_{in}, since $\epsilon(X, I_{in}) = 1$ for any $I_{in} < I^*$, we define $\epsilon(X, 0) := 1$ for any $X \in \mathbb{R}_+$.

In the case that μ is given by Eq. (2) (or by any Haldane-type model), it can be shown that $\mathcal{I}(X) = I^*e^{aX/2}$ (see (3)). Thus, we can write

$$\epsilon(X, I_{in}) := \begin{cases} 1 & if \ I_{in}I_{out} \leq I^{*2}, \\ \dfrac{I^*}{\sqrt{I_{in}I_{out}}} & if \ I_{in}I_{out} > I^{*2}, \end{cases} \tag{19}$$

with $I_{out} = I_{in}e^{-aX}$. Expression (19) shows that ϵ can be determined from the light intensities at the top I_{in} and at the bottom I_{out} of the reactor.

Now we have to prove that the differential equation in (12) admits a unique solution on $[t_i, t_f]$ when $u = \epsilon(X, I_{in})$. For this purpose, we state the following lemma.

Lemma 4. It is held

a) $\mathcal{I}$ is strictly increasing.
b) $\mathcal{I}'(X) \leq a\mathcal{I}(X)$ for all $X > 0$
c) $\lim_{X \to \infty} \mathcal{I}(X) = \infty$.

Proof. Let $X > 0$ be given. From the definition of $\mathcal{I}(X)$ we have that

$$\mu(\mathcal{I}(X)) = \mu(\mathcal{I}(X)e^{-aX}). \tag{20}$$

By using the implicit function theorem, we obtain that

$$\mathcal{I}'(X) = \frac{a\mathcal{I}(X)e^{-aX}}{\gamma(X) + e^{-aX}}, \tag{21}$$

with $\gamma(X) = \frac{-\mu'(\mathcal{I}(X))}{\mu'(\mathcal{I}(X)e^{-aX})}$. From Eq. (20) we have that $\mathcal{I}(X)e^{-aX} < I^* < \mathcal{I}(X)$, from where it is clear that $\gamma(X) > 0$. Thus, from Eq. (21) we obtain easily the parts a) and b).

For the part c), since $\mathcal{I}$ is strictly increasing, $\lim_{X \to \infty} \mathcal{I}(X) = \infty$ or $\lim_{X \to \infty} \mathcal{I}(X) = l > I^*$. If $\lim_{X \to \infty} \mathcal{I}(X) = l$, since μ is a continuous function, we can take the limit when $X \to \infty$ in Eq. (20) and conclude that $\mu(l) = 0$, which contradicts the assumptions over μ. Hence, $\lim_{X \to \infty} \mathcal{I}(X) = \infty$. $\square$

Lemma 4 shows that the function $\mathcal{I} : [0, \infty) \longrightarrow [I^*, \infty)$ is a bijection. As usual, we will denote $\mathcal{I}^{-1}$ the inverse function of $\mathcal{I}$. Now, we can write ϵ in the following way

$$\epsilon(X, I_{in}) := \begin{cases} 1 & if \ \hat{X}(I_{in}) \leq X, \\ \dfrac{\mathcal{I}(X)}{I_{in}} & if \ \hat{X}(I_{in}) > X, \end{cases} \qquad (22)$$

where

$$\hat{X}(I_{in}) := \begin{cases} 0 & if \ I_{in} \leq I^*, \\ \dfrac{\mathcal{I}^{-1}(I_{in})}{aL} & if \ I_{in} > I^*. \end{cases} \qquad (23)$$

Expression (22) shows that when the microalgae concentration is higher than the threshold $\hat{X}(I_{in})$ the culture must be under full exposure to solar radiation. This is consistent with the assumption that in high dense cultures the light intensity is a limiting factor (Burlew (1953)).

We define the function $G_\epsilon : \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+$ by $G_\epsilon(X, I_{in}) := G(X, \epsilon(X, I_{in})I_{in})$ for all $X, I_{in} \in \mathbb{R}_+$. Thus, when $u = \epsilon(X, I_{in})$, the ODE in (12) can be written

$$\dot{X} = G_\epsilon(X, I_{in}) - (m + D)X. \qquad (24)$$

From expression (22), we obtain that $\frac{\partial}{\partial X}\epsilon(X, I_{in}) = 0$ when $\hat{X}(I_{in}) < X$, while for $\hat{X}(I_{in}) > X$, by using Lemma 4, we have that

$$\frac{\mathcal{I}'(X)}{I_{in}} \leq a\frac{\mathcal{I}(X)}{I_{in}} \leq a\frac{\mathcal{I}(\hat{X}(I_{in}))}{I_{in}} = a. \qquad (25)$$

Following this result, ϵ is Lipschitz with respect to X with Lipschitz constant a. Since G is a Lipschitz function, it follows directly that G_ϵ is Lipschitz with respect to X. Thus, Eq. (24) admits a unique solution on $[t_i, t_f]$ for any non-negative initial condition.

Figure 2 shows a comparison of $G(\cdot, I_{in})$ and $G_\epsilon(\cdot, I_{in})$. We can see that G_ϵ is strictly concave on $(0, \infty)$, therefore, in the autonomous case, Eq. (24) admits at most one positive equilibrium which is globally stable on $(0, \infty)$. On the other hand, the non-controlled system may face bistability: the washout can be locally stable (Gerla et al. (2011)). In the following lemma we prove that G_ϵ is strictly concave.

Proposition 5. $G_\epsilon(\cdot, I_{in})$ is strictly increasing and strictly concave for any $I_{in} > 0$.

Proof. We prove that G_ϵ is strictly concave in the case when μ is given by (2) and $I_{in} > I^*$. In that case, $\hat{X}(I_{in}) = \frac{2}{aL}ln(I_{in}/I^*)$ and $\bar{X}(I_{in}) = \frac{1}{aL}ln(I_{in}/I^*)$. Thus $\hat{X}(I_{in}) > \bar{X}(I_{in})$. Therefore, by Proposition 2 part b), $G(\cdot, I_{in})$ is strictly concave on $J = (\hat{X}(I_{in}), \infty)$. According to (22), $G(X, I_{in}) = G_\epsilon(X, I_{in})$ for all $X \in J$, hence $G_\epsilon(\cdot, I_{in})$ is strictly concave on J.

Now let $X \in (0, \hat{X}(I_{in}))$ be given. According to (22) we have that

$$G_\epsilon(X, I_{in}) = \frac{1}{a}\int\limits_{I^*e^{-aX/2}}^{I^*e^{aX/2}} \frac{\mu(I)}{I}dI, \qquad (26)$$
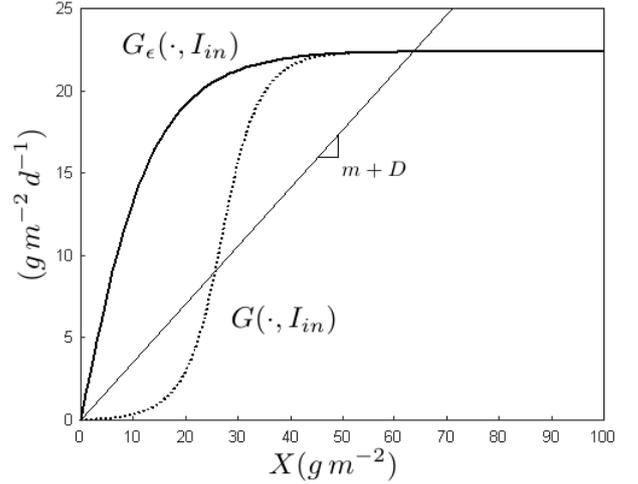

Fig. 2. Comparison of the total growth functions with shading $G(\cdot, I_{in})$ (continuous line) and without shading $G_\epsilon(\cdot, I_{in})$ (dotted line). μ is given by (2) with kinetic parameters from Table 1, and $I_{in} = 2000\mu mol \, m^{-2} \, s^{-1}$.

from where we obtain

$$\frac{\partial}{\partial X}G_\epsilon(X, I_{in}) = \frac{1}{2}\left[\mu(I^*e^{aX/2}) + \mu(I^*e^{-aX/2})\right]. \qquad (27)$$

By using (3), we obtain that $\frac{\partial}{\partial X}G_\epsilon(X, I_{in}) = \mu(I^*e^{aX/2})$. Then, we obtain that

$$\frac{\partial^2}{\partial X^2}G_\epsilon(X, I_{in}) = \frac{a}{2}I^*e^{aX/2}\mu'(I^*e^{aX/2}). \qquad (28)$$

Since $I^*e^{aX/2} > I^*$, from Hypothesis 1, we have that $\mu'(I^*e^{aX/2}) < 0$. From where we obtain that $G_\epsilon(\cdot, I_{in})$ is strictly concave on $(0, \hat{X}(I_{in}))$. $\square$

In outdoor cultures, the light source varies with a light phase (day) and a dark phase (night). Thus, in a first approach, we can assume that I_{in} is ω-periodic with $\omega > 0$ the length of the day. In the following proposition, we prove that the periodic fluctuations lead the controlled culture to a periodic state that is attained by any initial biomass concentration i.e. solutions of Eq. (24) converges to an ω-periodic solution.

Proposition 6. Assume that I_{in}, D and m are ω-periodic with I_{in} and m not identically zero. Let $f_\epsilon : \mathbb{R}_+^2 \longrightarrow \mathbb{R}$ be the function defined by $f_\epsilon(t, X) = G_\epsilon(X, I_{in}(t))/X - m - D$. If

$$\int\limits_0^\omega f_\epsilon(t, 0)dt > 0, \qquad (29)$$

then there exists a unique ω-periodic solution X^* to the differential equation (24) which is globally stable on $\mathbb{R}_+\backslash\{0\}$.

Proof. Since I_{in} is continuous and not identically zero, there exists an open interval $J \subset [0, \omega]$ such that $I_{in}(t) > 0$ for all $t \in J$. From Proposition 5, we have that $G_\epsilon(\cdot, I_{in}(t))$ is strictly concave for any $t \in J$. Following the same arguments as in Proposition 3 in Mawhin (1987), we conclude that Eq. (24) has at most one positive ω-periodic solution. Now, we note that for any $R > 0$

$$\int_0^\omega f_\epsilon(t,R)dt \le \frac{\omega}{aR}\int_0^{I_{max}}\frac{\mu(I)}{I}dI - \int_0^\omega [D(t)+m(t)]dt, \quad (30)$$

with $I_{max} = \max_{t\in[0,\omega]} I_{in}(t)$. Since m is not identically zero and continuous, we can find $R > 0$ such that $\int_0^\omega f_\epsilon(t,R)dt < 0$. This result together with (29) implies the result of the proposition (same arguments as in Corollary 2 in Zanolin (1992)).   □

4. SIMULATIONS

In this section we evaluate the performance of our control strategy; we determine and compare numerically the productivities and the solutions of the controlled culture (modelled by Eq. (24)) and the non-controlled culture (modelled by Eq. (6)). We consider that μ is given by Eq. (2), with the kinetic parameters given in Table 1. We consider that the incident light varies according to $I_{in}(t) = I_{max}\max\{0,\sin(2\pi t)\}^2$ with $I_{max} = 2000\,\mu mol\,m^{-2}\,s^{-1}$. The attenuation coefficient of microalgae and the mortality rate are taken to be $a = 0.2\,m^2\,g^{-1}$ and $m = 0.1\,d^{-1}$ respectively.

4.1 Evaluation of the biomass productivity at different constant dilution rates

Here, we evaluate the biomass productivities at different constant dilution rates in a period of 30 days with an initial microalgae concentration of $10\,g\,m^{-2}$. Figure 3 shows a plot of these evaluations. At small dilution rates, the productivities are the same, in contrast with high dilution rates where the productivity of the controlled culture is clearly higher. This suggest that we can reach higher productivities even with small reactors. The maximal productivity in the controlled culture is a 20.3% higher than the maximal productivity in the non-controlled case.
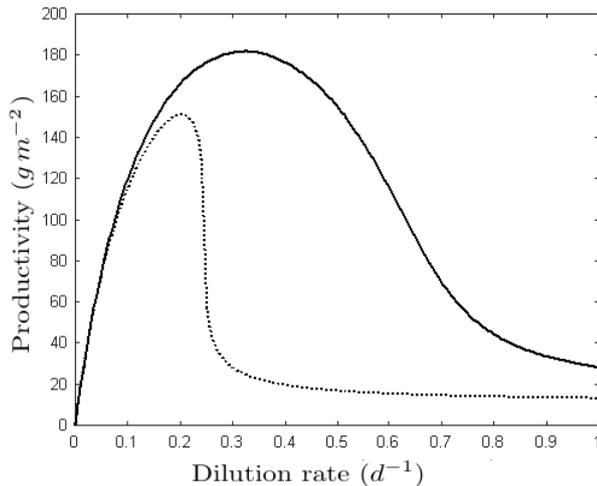

Fig. 3. Biomass productivities of the controlled culture (continuous line) and the non-controlled culture (dotted line) at different dilution rates.

4.2 Reaching rapidly a high density by shading

Figure 4A shows the evolution of the microalgae concentrations for an initial concentration $X_0 = 5g\,m^{-2}$ and a dilution rate $D = 0.13\,d^{-1}$. The controlled culture is clearly denser during the first 25 days. After that, both cultures reach the same periodic state. The productivities over the 30 days are $131.4g\,m^{-2}$ and $82.4\,g\,m^{-2}$ for the controlled and the non-controlled cultures respectively. Figure 4B shows that the exposure factor is regulated only during the first seven days, then, the microalgae population is dense enough $(X > \hat{X}(I_{in}))$ to protect itself from high light intensities. With such a strategy, two weeks are gained in the culturing.
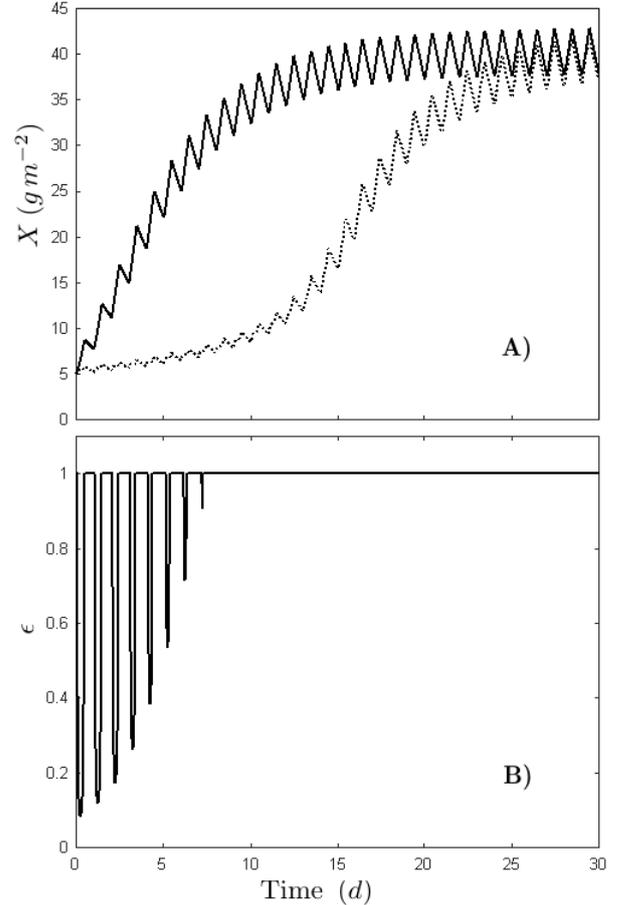

Fig. 4. **A)** Comparison of the evolution of biomass concentration of the controlled culture (continuous line) and the non-controlled culture (dotted line) at a dilution rate $D = 0.13d^{-1}$. **B)** Evolution of the exposure factor.

4.3 Shading the culture to avoid washout

Consider a dilution rate $D = 0.3\,d^{-1}$. If we start at an initial concentration $X_0 = 10g\,m^{-2}$, the controlled culture converges to a periodic state while the non-controlled culture washouts as it is shown in Figure 5A. According to Proposition 6, the controlled culture will reach the same periodic state, regardless of the initial biomass concentration. The washout in the non-controlled culture is due to photoinhibition. This washout can be avoided by starting at a higher initial concentration $(X_0 = 17\,g\,m^{-2})$. In that case, the non-controlled culture reaches a positive periodic state. However, this periodic state is lower than that of the controlled culture. Figure 5B shows that in this case, shading is necessary during the whole culture.
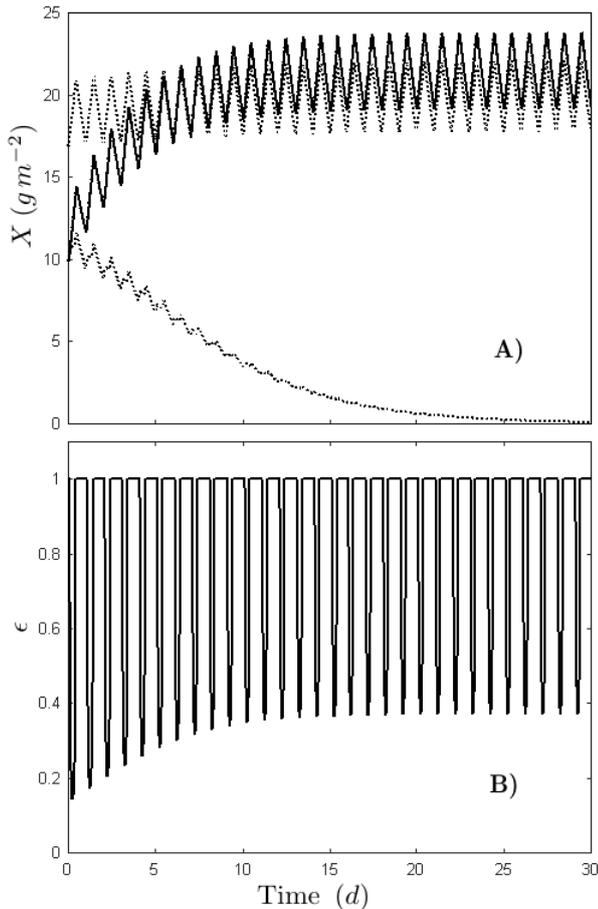
Fig. 5. **A)** Comparison of the evolution of biomass concentration of the controlled culture (continuous line) and the non-controlled culture (dotted line) for different initial concentrations at a dilution rate $D = 0.3d^{-1}$. **B)** Evolution of the exposure factor.

5. CONCLUSION

We determined a control strategy to shadow a culture illuminated with natural light in order to maximize the biomass productivity. This control strategy also allows to avoid washout due to photoinhibition and to gain time in culturing.

As a future work, the effect of shadowing for decreasing temperature must also be studied. It results in avoiding culture over warming, and thus even more strongly stimulating productivity. This problem deserves to be studied, and the temperature dynamics must be included in the analysis.

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