# Balance-Enforced Multi-Level Algorithm for Multi-Criteria Graph Partitioning

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# Balance-Enforced Multi-Level Algorithm for Multi-Criteria Graph Partitioning

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# Outline


1. Objective
   - Context
   - Example & Model
   - State of the art
2. Approach
   - The multi-level framework
   - Scotch multi-criteria algorithm
3. Results
4. Perspective
   - Improving robustness
5. Conclusion

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# Context

In the field of numerical simulation, one challenge is to reduce the restitution time.

We consider simulations:

- Running on distributed memory architectures
- Using 2D or 3D meshes
- Coupling several physics (multi-physics simulations)

Running these simulations efficiently needs to:

1. Balance the workloads of each processor for each phase of the computation
2. Overlap or minimize communications
3. Take care of memory accesses
4. Exploit full processor characteristics

We focus on the first 2 items.

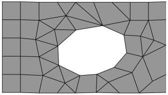
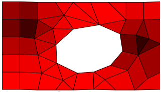
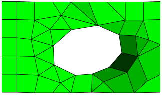
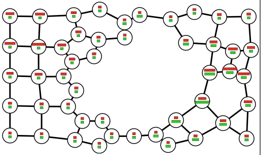
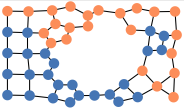
# Example

**Algorithm 1** Example of the temporal loop of a multi-physics simulation
(2 computing phases: $f$ and $g$, coupling the variables $X$ and $Y$)

**for** $t \in [0, t_{end} - 1]$ **do**
$X(t+1) \leftarrow f(t+1, X(t), Y(t))$ # First computing phase
$Y(t+1) \leftarrow g(t+1, X(t+1), Y(t))$ # Second computing phase
**end for**

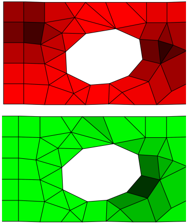
Objectives:

1. Balance the workloads of each processor *for each phase*
2. Minimize communications between processors

| Input | | Model | Output |
|---|---|---|---|
| Geometry | Weights | Multi-criteria Dual Graph (1 color = 1 criterion) | Graph/Mesh Partition (1 color = 1 part) |
|  | → Weights for phase 1:  → Weights for phase 2:  |  |  imbalance c1: 1.9% <br> imbalance c2: 2.9% <br> edgecut: 11 |

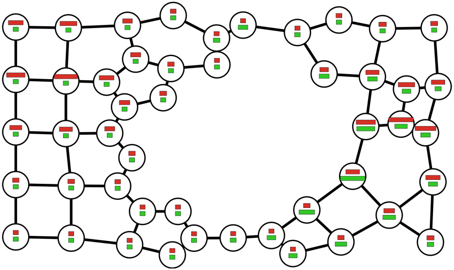
# Model

Mesh



$\rightarrow$

Dual Graph $G = (V, E)$



## Problem : Graph partitioning

We search for a partition $P$ of $V$ respecting:

1. some **constraints**: load-balancing
   (imbalance of $P$ is for all criteria smaller than a given tolerance)
2. an **objective**: minimize the edgecut
   (sum of the weights of the edges cut by the partition).

NP-Hard Problem.

Main existing software:

| Software | Representations | Multi-criteria | Origin | Limitations for us |
|---|---|---|---|---|
| Scotch | Topological | No | INRIA | No multi-criteria (yet) |
| MeTiS | Topological | Yes | University of Minnesota | Does not often meet the balance constraints |
| PaToH | Topological | Yes | Bilkent University | No parallel version<br>No open-source version<br>Does not always meet the balance constraints |
| Zoltan | Geometric<br>Topological | Yes<br>No | Sandia National Laboratories | Geometric representations are inefficient |

⇒ Lack of efficient multi-criteria partitioning tools.

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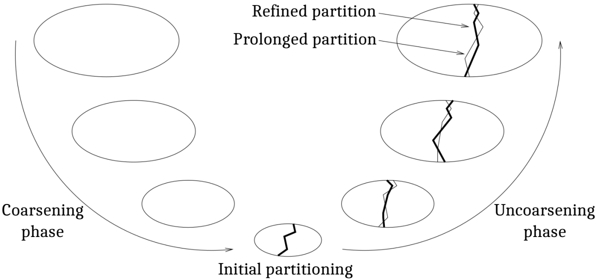
A 3-phases algorithm:

1. Coarsening

2. Initial partitioning of the coarsened hypergraph

3. Uncoarsening and refinement

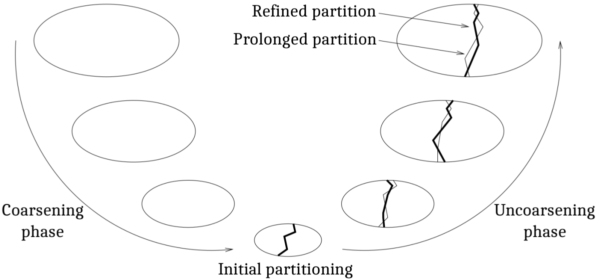


# Our approach: Multi-level multi-criteria algorithm

A 3-phases algorithm:

1. Coarsening
   → (Ongoing) Selection of a matching policy improving the quality of the returned solutions
2. **Initial partitioning of the coarsened hypergraph**
   → New algorithm focusing on balance constraints
3. **Uncoarsening and refinement**
   → Adapted Fiduccia-Mattheyses algorithm

Implemented in Scotch



# Scotch multi-criteria
## *Currently implemented algorithm*

- Coarsening:
  - Classical algorithm: Heavy-Edge-Matching
- Initial Partitioning:
  - Local optimization of the balance
  - Enforce balance for all the constraints
  - Presented in SIAM CSC 2016
- Refinement:
  - Fiduccia-Mattheyses algorithm: local optimization of the edgecut with hill-climbing
  - Moves that unbalance the partition more than the tolerance are forbidden

$\Rightarrow$ Finding a balanced partition of the coarsest graph guarantees that the returned partition will respect the balance constraints.

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# Instances and Parameters

Comparison between

- MeTiS (5.1.0)
- PaToH (3.2)
- Scotch (6.1.0 - not yet released multi-criteria version)

Instances considered:

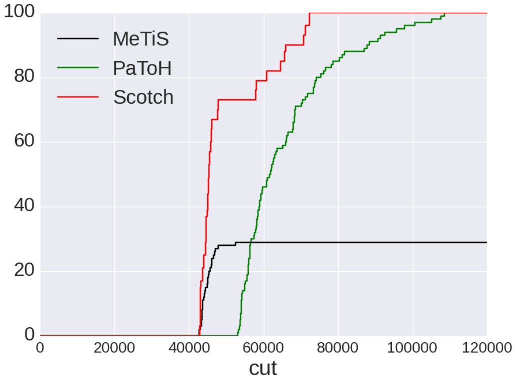
| Name | Geometry | Vertices | Edges | Criteria | Edge weights |
|---|---|---|---|---|---|
| mushroom_1 | 2D | $22800$ | $45253$ | $3$ | $\sqrt{w_i + w_j}$ |
| mushroom_2 | 2D | $22800$ | $45253$ | $3$ | independant |
| onera | 3D | $85567$ | $166817$ | $3$ | $\sqrt{w_i + w_j}$ |
| wave | 3D | $156317$ | $1059331$ | $3$ | $\sqrt{w_i + w_j}$ |

Partitioning parameters:

- Imbalance tolerance $5\%$
- Number of parts: $2$
- Each algorithm is executed $100$ times on each instance with random seeds.

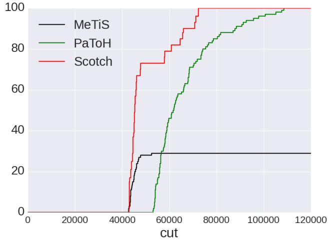
$f_{algo}(x) =$ number of valid solutions of $algo$ of cut $\leq x$.

- We want to minimize the cut of each returned solution: left is better




mushroom_1

# Results
## *2 - Comparison on the quality of the returned solutions*

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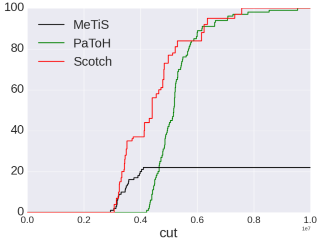


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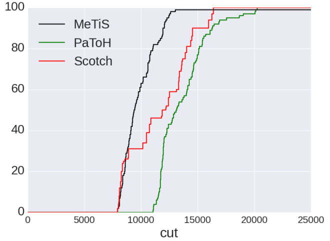


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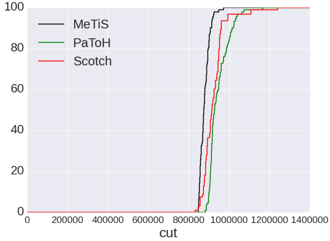


wave

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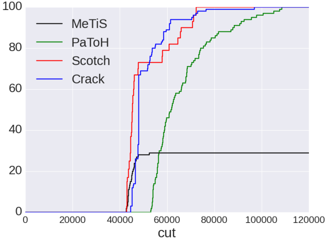
## Improving robustness

- Python prototype: Crack
- Allows to test different parameters on the multi-level
- An example: settings of the coarsening parameters
  - Heavy-Edge-Matching
  - Particular order of the vertices:
    sorted by increasing degrees
  - Do not allow to match 2 vertices if the resulting vertex is overweighted

# Comparison of Crack on the previous instances

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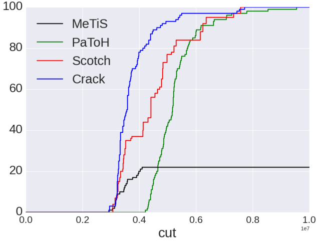


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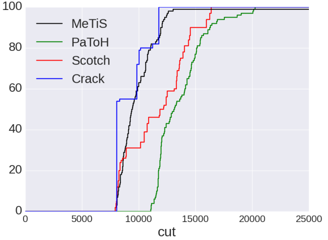


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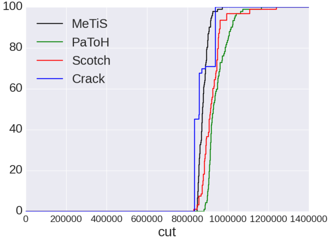


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# Conclusion

Summary:

- Scotch next release will support multi-criteria graph partitioning
- This new version implements a new initial partitioning and sets the refinement phase to guaranty that the returned solution will respect the balance constraints
- Scotch results are promising

Perspectives:

- Enforce the algorithm robustness by the study of variations of the algorithms
- Validation on more instances
  Validation on a simulation code
- Set up of a parallel version of the algorithms

Thank you

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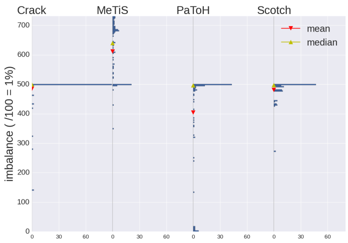
# Imbalance distributions

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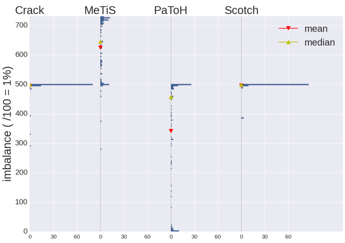


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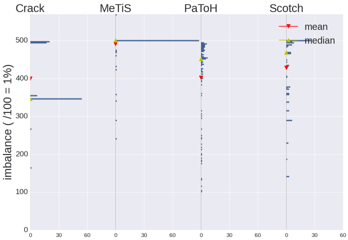


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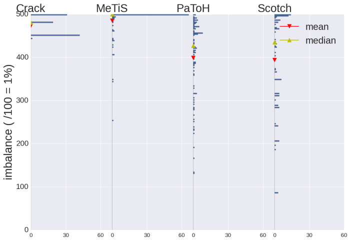


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