An improved Branch-Cut-and-Price algorithm for heterogeneous vehicle routing problems

Artur Alves Pessoa, Ruslan Sadykov, Eduardo Uchoa

An improved Branch-Cut-and-Price Algorithm for Heterogeneous Vehicle Routing Problems

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Verolog 2017
Amsterdam, July 11

Heterogeneous Vehicle Routing

Set I of n customers, each $i \in I$ with demand d_i.

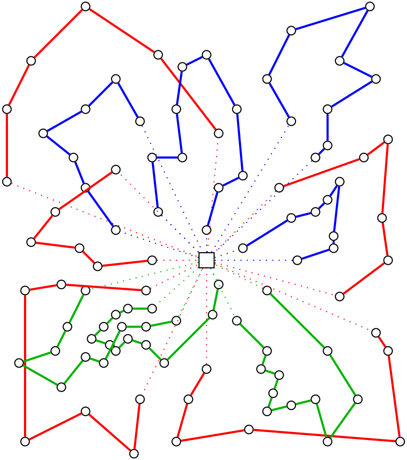
Set U of vehicle types, each $u \in U$ has a depot with K_u vehicles of capacity Q_u, with fixed cost f_u and travel costs c_a^u for each edge a.

Objective: minimize the total fixed and travel cost.

Variants

- Multi-depot VRP
- Site-dependent VRP

Instance HVRPFV-20-100



6 , 4 , 3 

Optimum **4760.68** (BKS 4761.26)

Set partitioning (master) formulation

- R_u — set of $q-$routes feasible for a vehicle of type u
- a_i^r — number of times that customer i appears in route r.
- c_r — cost of route r.
- Binary variable $\lambda_u^r = 1$ if and only if a vehicle of type u uses route r

$$
\begin{aligned}
\min \sum_{u \in U} \sum_{r \in R_u} c_r \lambda_r & & \\
\sum_{u \in U} \sum_{r \in R_u} a_i^r \lambda_r &= 1, \quad \forall i \in I, \\
\sum_{r \in R_u} \lambda_r &\leq K_u, \quad \forall u \in U, \\
\lambda_r &\in \{0,1\}, \quad \forall r \in R_u, \forall u \in U.
\end{aligned}
$$

Pricing subproblem for a vehicle type

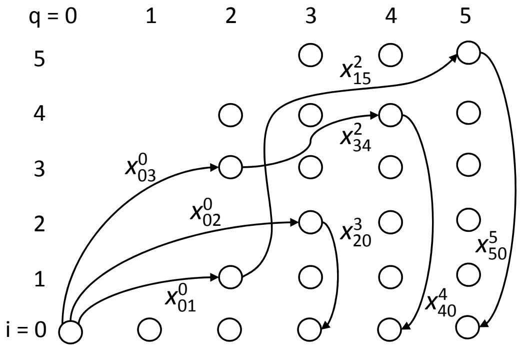
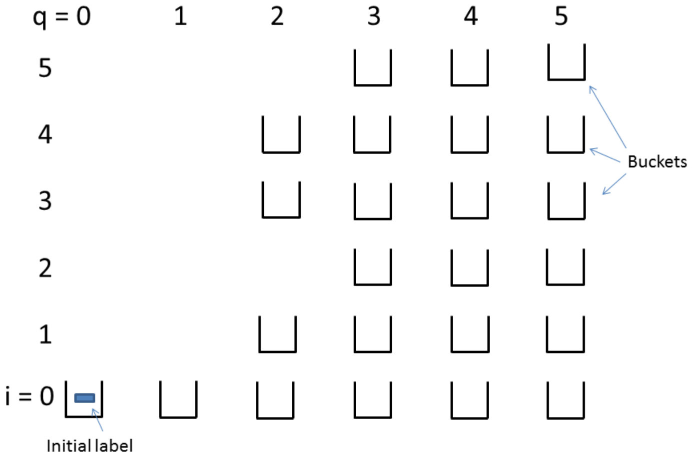


Figure: $|I| = Q = 5$, $d_1 = d_3 = d_4 = 2$, $d_2 = d_5 = 3$; routes 0-1-5-0, 0-2-0, 0-3-4-0 are shown

The labeling algorithm



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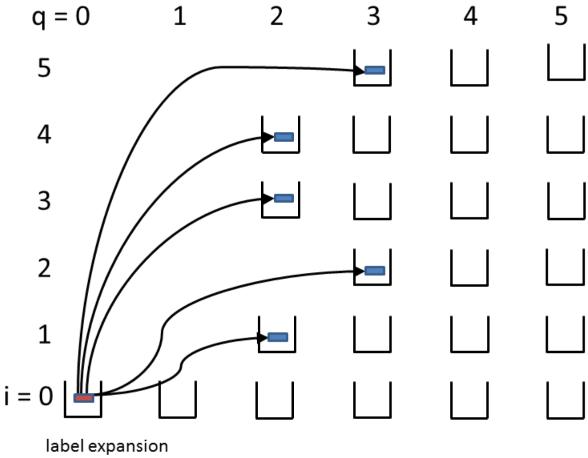

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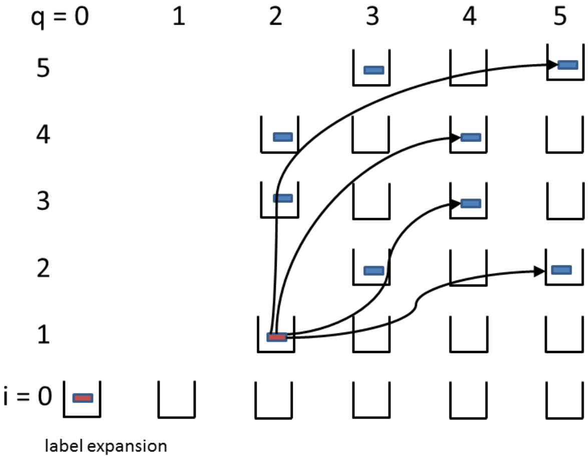

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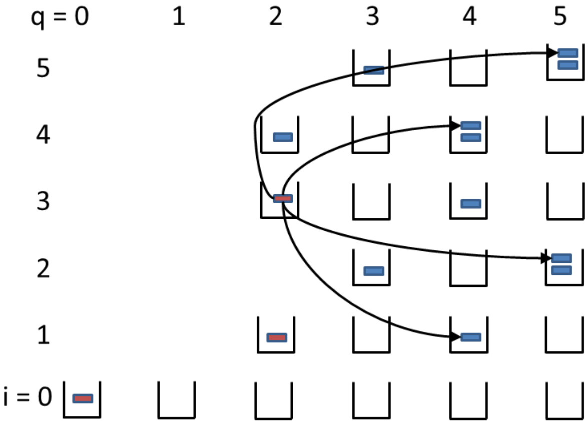
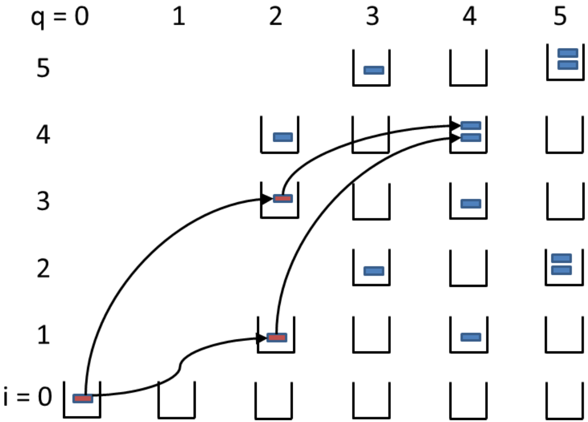


Do both labels need to be kept in bucket (4,4)?

The labeling algorithm



The labels represent partial paths 0-1-4 and 0-3-4

Subset Row Cuts (SRCs)

Given $C \subseteq V_+$ and a multiplier p, the (C, p)-Subset Row Cut is:

$$\sum_{u \in U} \sum_{r \in R_u} \left\lfloor p \sum_{i \in C} a_i^r \right\rfloor \lambda_r \leq \lfloor p|C| \rfloor$$

Special case of **Chvátal-Gomory rank-1 cuts** obtained by rounding of $|C|$ constraints in the master

Each cut adds an additional resource in the shortest path pricing problem

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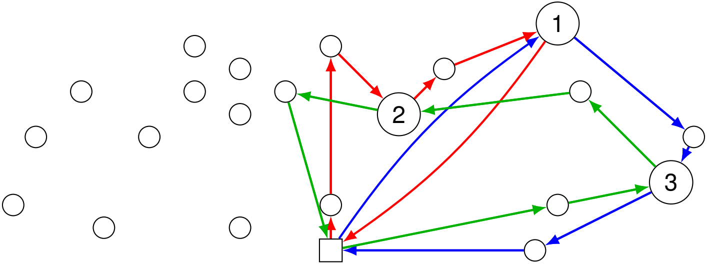
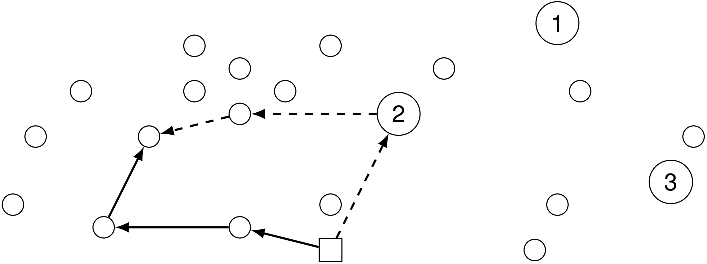
Mads Jepsen and Bjorn Petersen and Simon Spoorendonk and David Pisinger (2008).
Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows.
Operations Research, 56(2):497–511.

Example of 3-Subset Row Cut, $|C| = 3$, $p = \frac{1}{2}$



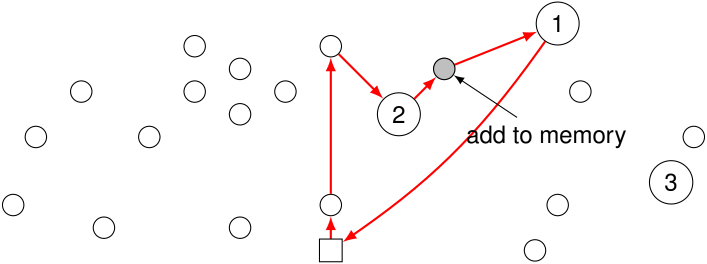
If $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, and $\lambda_3 = 0.5$, cut $C = \{1, 2, 3\}$ is violated.

Example of 3-Subset Row Cut, $|C| = 3$, $p = \frac{1}{2}$



Less possibilities for domination after adding the cut.

Example of 3-Subset Row Cut, $|C| = 3$, $p = \frac{1}{2}$



Concept of limited memory cuts [Pecin et al., 2017b].

Pecin, D., Pessoa, A., Poggi, M., and Uchoa, E. (2017a).
Improved branch-cut-and-price for capacitated vehicle routing.
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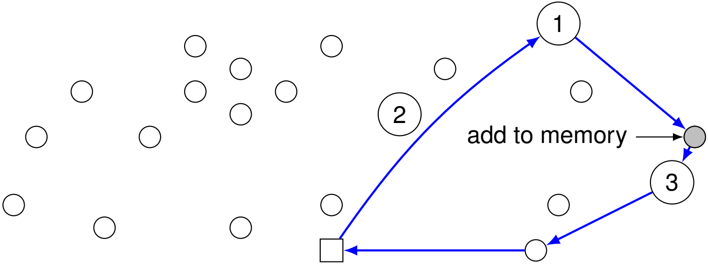

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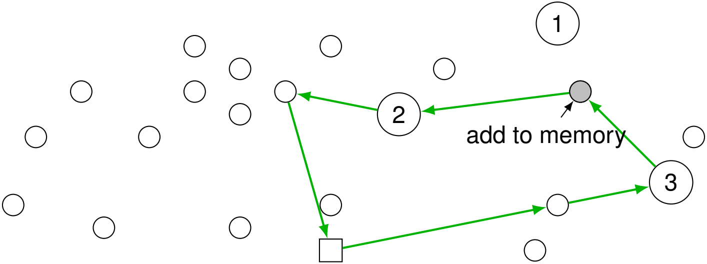

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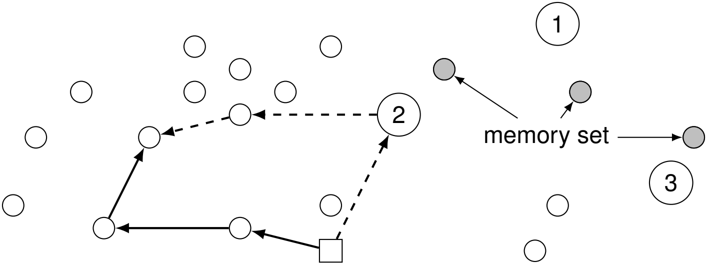
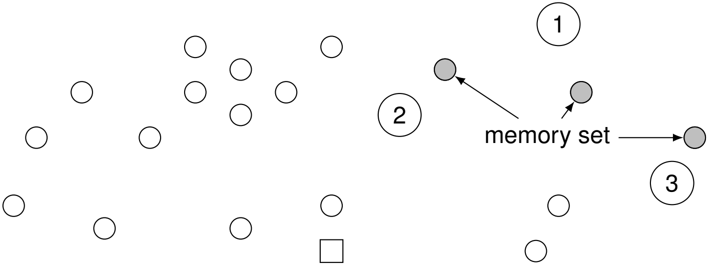


Dashed partial path "forgot" the cut (cut state in the label is 0)
$\Rightarrow$ larger domination probability.

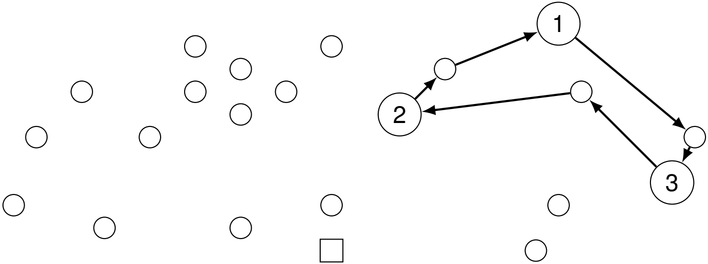
Example of 3-Subset Row Cut, $|C| = 3$, $p = \frac{1}{2}$



Different memory sets

- Node memory

Example of 3-Subset Row Cut, $|C| = 3$, $p = \frac{1}{2}$



Different memory sets

- Node memory
- Arc memory [Pecin et al., 2017a]

Pecin, D., Contardo, C., Desaulniers, G., and Uchoa, E. (2017a).
New enhancements for the exact solution of the vehicle routing problem with time windows.
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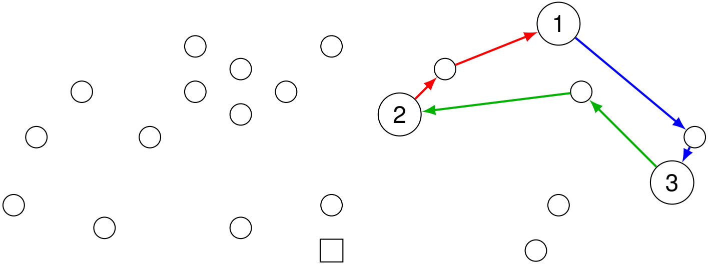

Different memory sets

- Node memory
- Arc memory [Pecin et al., 2017a]
- Subproblem dependent memory (this work)

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Arbitrary cuts of Rank 1

Rounding using a vector p instead of single value:

$$\sum_{u \in U} \sum_{r \in R_u} \left\lfloor \sum_{i \in C} p_i a_i^r \right\rfloor \lambda_r \leq \left\lfloor \sum_{i \in C} p_i \right\rfloor$$

All facet-defining vectors p for cuts up to 5 rows [Pecin et al., 2017c]:

- $|C| = 1$, $p = \left(\frac{1}{2}\right)$
- $|C| = 3$, $p = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
- $|C| = 4$, $p = \left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
- $|C| = 5$, $p = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
- $|C| = 5$, $p = \left(\frac{2}{4}, \frac{2}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$
- $|C| = 5$, $p = \left(\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$
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- $|C| = 5$, $p = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
- $|C| = 5$, $p = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
- $|C| = 5$, $p = \left(\frac{3}{4}, \frac{3}{4}, \frac{2}{4}, \frac{2}{4}, \frac{1}{4}\right)$

Pecin, D., Pessoa, A., Poggi, M., Uchoa, E., and Santos, H. (2017).
Limited memory rank-1 cuts for vehicle routing problems.
Operations Research Letters, 45(3):206 – 209.

Extended Capacity Cuts

Definition

An Extended Capacity Cut (ECC) [Pessoa et al., 2009] over subset C of customers is any inequality valid for $P(C)$, the polyhedron given by the convex hull of the $0-1$ solutions of

$$\sum_{u \in U} \left(\sum_{a \in \delta_u^-(C)} \sum_{q=1}^{Q} d x_a^q - \sum_{a \in \delta_u^+(C)} \sum_{q=0}^{Q-1} d x_a^q \right) = d(C)$$

Pessoa, Artur and Uchoa, Eduardo and Poggi, Marcus (2009).
A robust branch-cut-and-price algorithm for the heterogeneous fleet vehicle routing problem.
Networks, 54(4):167–177.

Homogeneous Extended Capacity Cuts

$$y^{m,q} = \sum_{a^q \in \delta_u^-(C)} x_a^q, \quad z^q = \sum_{a^q \in \delta_u^+(C)} x_a^{m,q}, \quad (q = 0, \ldots, Q).$$

Definition

A Homogeneous Extended Capacity Cut (HECC) over set C of customers is any inequality valid for the polyhedron given by the convex hull of the integral solutions of

$$\sum_{u \in U} \left(\sum_{q=1}^{Q} dy^{m,q} - \sum_{q=0}^{Q-1} dz^{m,q} \right) = d(C). \tag{1}$$

Separation

- Cuts obtained by applying integer rounding of (1).
- Heuristic separation of [Pessoa et al., 2009] is used.

Labeling algorithm enhancements

- *ng*-routes to impose partial elementarity [Baldacci et al., 2011].
- Bi-directional labelling [Righini and Salani, 2006]
- Reduced cost fixing of subproblem arc variables x [Irnich et al., 2010]

Baldacci, R., Mingozzi, A., and Roberti, R. (2011).
New route relaxation and pricing strategies for the vehicle routing problem.
Operations Research, 59(5):1269–1283.

Righini, G. and Salani, M. (2006).
Symmetry helps: Bounded bi-directional dynamic programming for the elementary shortest path problem with resource constraints.
Discrete Optimization, 3(3):255 – 273.

Irnich, S., Desaulniers, G., Desrosiers, J., and Hadjar, A. (2010).
Path-reduced costs for eliminating arcs in routing and scheduling.
INFORMS Journal on Computing, 22(2):297–313.

Elementary routes enumeration

We try to enumerate all elementary routes whose reduced cost is smaller than the current gap [Baldacci et al., 2008], possibly to a pool [Contardo and Martinelli, 2014].

This work contributions

- **Subproblem dependent** enumeration
- If succeeded, a subproblem passes to the enumerated state:
  - Pricing is performed by **inspection**
  - **Cut coefficients** of columns in the master are **lifted**

Baldacci, R., Christofides, N., and Mingozzi, A. (2008).
An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts.
Mathematical Programming, 115:351–385.

Lifting of cuts in enumerated state

Rank-1 cuts

Limited memory is extended to full memory

Homogeneous Extended Capacity Cuts

Integer rounding of

$$\sum_{u \in EU} \sum_{r \in R_u} d_r(C)\lambda_r + \sum_{u \in U \setminus EU} \left(\sum_{q=1}^{Q} dy^{m,q} - \sum_{q=0}^{Q-1} dz^{m,q} \right) = d(C).$$

For a particular rounding multiplier and $EU = U$ (all subproblems are in the enumerated state), equivalent to Strong Capacity Cuts [Baldacci et al., 2008].

Other enhancements

- Heuristic pricing (keeping one label per bucket)
- Automatic dual price smoothing stabilization [Pessoa et al., 2017].
- Rollback mechanism [Pecin et al., 2017b]

Pessoa, A., Sadykov, R., Uchoa, E., and Vanderbeck, F. (2017).
Automation and combination of linear-programming based stabilization techniques in column generation.
INFORMS Journal on Computing, (Forthcoming).

Branching

Strong branching [Pecin et al., 2017b]

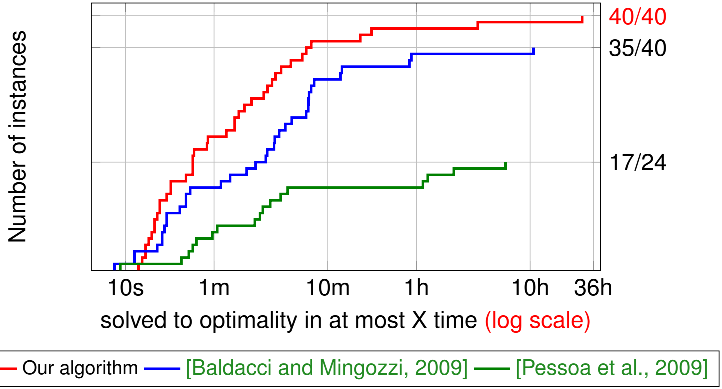
- Multi-strategy
- Branching history (pseudo-costs)
- Multi-phase

Branching strategies

- Number of vehicles
- Assignment of customers to vehicle types
- Participation of arcs in routes

Results for classic Heterogeneous VRP instances

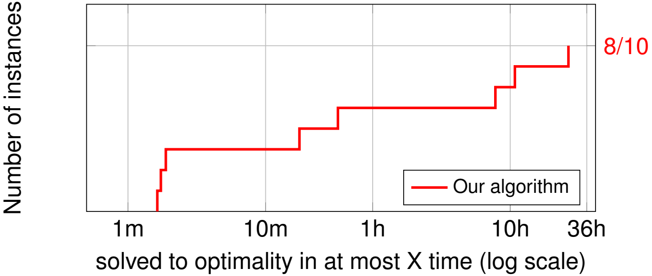
40 instances with 50-100 customers by [Taillard, E. D., 1999]



Baldacci, R. and Mingozzi, A. (2009).
A unified exact method for solving different classes of vehicle routing problems.
Mathematical Programming, 120(2):347–380.

Results for larger Heterogeneous VRP instances

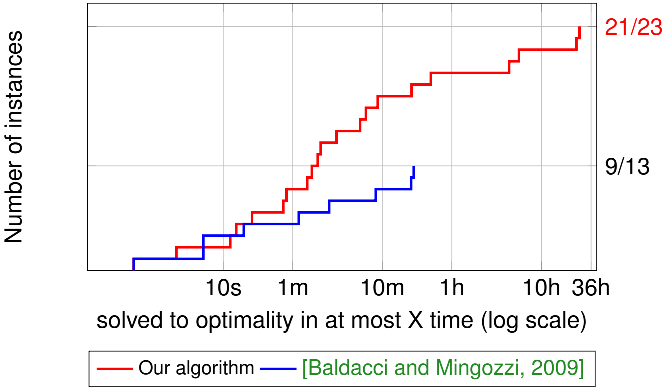
10 instances with 100-200 customers [Brandao, 2011]



Brandao, J. (2011).
A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem.
Computers and Operations Research, 38(1):140 – 151.

Results for standard Site-Dependent VRP instances

Instances with 27-324 customers by [Cordeau and Laporte, 2001]

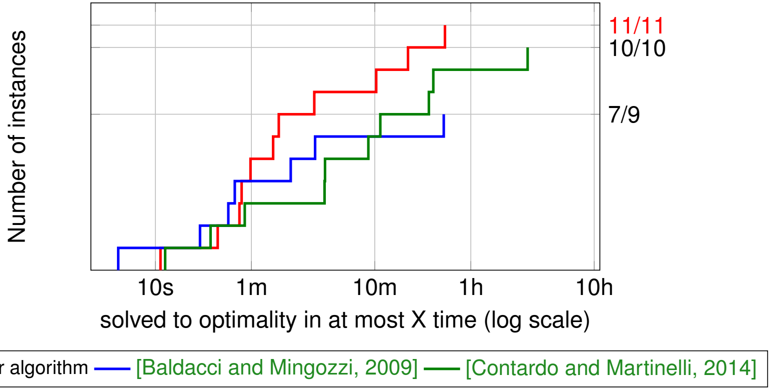


Largest solved instance has 216 customers

[Baldacci and Mingozzi, 2009] solved only 1 of 5 instances with 100 customers and more

Results for standard Multi-Depot VRP instances

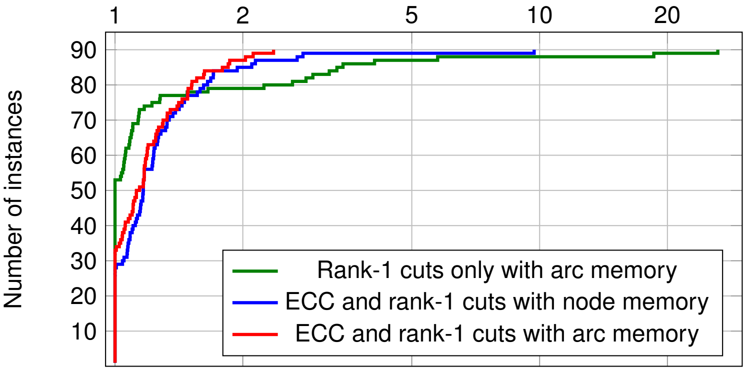
Instances with 50–360 customers [Cordeau et al., 1997]



Contardo, C. and Martinelli, R. (2014).
A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints.
Discrete Optimization, 12:129 – 146.

Impact of Extended Capacity Cuts

Cuts	Rank-1 cuts memory	Root gap	Root time	Nodes num.	Solved	Total time
R1C only	arc, sp.dep	0.323%	93	67.8	87/90	181
R1C+ECC	node, sp.dep	0.133%	106	38.7	86/90	178
R1C+ECC	arc, sp.dep	0.105%	113	29.6	88/90	170



for which variant is at most X times slower than the best

Improved Best Known Solutions

Problem	Instance	Size	Previous BKS	Reference	Improved value
HVRP	BrandaoN1fsmd	150	2212.77	[SPUS12]	2211.63
	BrandaoN1hd	150	2234.13	[S16]	2233.90
	BrandaoN2fsmd	199	2823.75	[SPUS12]	2810.20
	BrandaoN2hd	199	2859.82	[S16]	2851.94
	c100_20fsmf	100	4032.81	[SPUS12]	4029.61
	c100_20hvrp	100	4761.26	[SPUS12]	4760.68
MDVRP	n200-k16-3-80	200	1757.86	[BM09]	1756.48
SDVRP	p16	216	3393.55	[CM12]	3393.31
	p18	324	4751.27	[CM12]	4747.75[1]
	p21	209	1263.71	[CM12]	1260.01

[1] optimality is not proved, other values are optimal

[SPUS12] [Subramanian et al., 2012]

[S16] [Subramanian, 2016]

[BM09] [Baldacci and Mingozzi, 2009]

[CM12] [Cordeau and Maischberger, 2012]

Contributions

- Large computational improvement over the state-of-the-art algorithms for the problem
- Showed importance of Extended Capacity Cuts
- New concept of subproblem dependent memory for rank-1 cuts
- New concept of “enumerated state” for pricing subproblems
- New family of lifted Extended Capacity Cuts

Case of multiple and “non-discretizable” resources

Presentation at IFORS in Quebec next week

A Bucket Graph Based Labelling Algorithm for the RCSPP with Applications to Vehicle Routing

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A glimpse of the results

- Solved 5/9 open VRPTW instances of [Gehring and Homberger, 2002] with 200 customers
- Solved 6/7 distance-constrained CVRP instances of [Christofides et al., 1979] (CMT) with up to 200 customers
- Solved all 22 distance-constrained MDVRP instances of [Cordeau et al., 1997] with up to 288 customers
- Solved 7/10 distance-constrained SDVRP instances of [Cordeau and Laporte, 2001] with up to 216 customers
- Solved 56/96 “nightmare” HFVRP instances of [Duhamel et al., 2011] with up to 186 customers.

References I

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Combinatorial Optimization, chapter The vehicle routing problem, pages 315–338.
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Cordeau, J.-F., Gendreau, M., and Laporte, G. (1997).
A tabu search heuristic for periodic and multi-depot vehicle routing problems.
Networks, 30(2):105–119.

Cordeau, J.-F. and Laporte, G. (2001).
A tabu search algorithm for the site dependent vehicle routing problem with time windows.
INFOR: Information Systems and Operational Research, 39(3):292–298.

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A parallel iterated tabu search heuristic for vehicle routing problems.
Computers and Operations Research, 39(9):2033 – 2050.

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Efficient frameworks for greedy split and new depth first search split procedures for routing problems.
Computers and Operations Research, 38(4):723 – 739.

Gehring, H. and Homberger, J. (2002).
Parallelization of a two-phase metaheuristic for routing problems with time windows.
Journal of Heuristics, 8(3):251–276.

Irnich, S., Desaulniers, G., Desrosiers, J., and Hadjar, A. (2010).
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A hybrid algorithm for the heterogeneous fleet vehicle routing problem.
European Journal of Operational Research, 221(2):285 – 295.

Taillard, E. D. (1999).
A heuristic column generation method for the heterogeneous fleet vrp.
RAIRO-Oper. Res., 33(1):1–14.