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Book of abstracts of the 7^{th} International Conference
on Advanced Computational Methods
in Engineering, ACOMEN 2017
18–22 September 2017.

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Abstract

In this work we derive and analyze an equivalent model for 3D Eddy Current problems with a conductive thin layer of small thickness ϵ. In our model, the conductive sheet is replaced by its mid-surface and their shielding behavior is satisfied by an equivalent transmission conditions on this interface. The transmission conditions are derived asymptotically for vanishing sheet thickness ϵ.

Key words: Asymptotic Expansions, Eddy-Current Problems, Thin Conducting Layers, Transmission Conditions

1 Introduction

We denote by $\Omega = \Omega_-^\epsilon \cup \overline{\Omega_0^\epsilon} \cup \Omega_+^\epsilon \subset \mathbb{R}^3$ the domain of study, where Ω_-^ϵ corresponds to a non-conductive linear material, Ω_+^ϵ the exterior of the structure domain, and Ω_0^ϵ a conductive thin layer of constant thickness ϵ (see figure 1). The discretisation of the conducting sheet by FEM needs a very fine mesh due to the rapid decay of the field under high conductivity. For this, we approximate a new model defined in ϵ-independent domains. Let Σ be a smooth surface, we denote by $[v]_\Sigma$ and $\{v\}_\Sigma$ the jump and mean of v respectively across Σ

$$[v]_\Sigma = v_{|_{\Sigma^+}} - v_{|_{\Sigma^-}}, \quad \{v\}_\Sigma = \frac{1}{2}(v_{|_{\Sigma^+}} + v_{|_{\Sigma^-}}), \quad \text{for} \quad v \in (C^\infty(\Omega_\pm))^3.$$

We consider the eddy current problem as follows

$$\left\{\begin{array}{lcll} \mathrm{curl} H^\epsilon & = & \sigma^\epsilon E^\epsilon + J_0 & \text{in} \quad \Omega \\ \mathrm{curl} E^\epsilon & = & i w \mu_0 H^\epsilon & \text{in} \quad \Omega \\ \mathrm{div}(H^\epsilon) & = & 0 & \text{in} \quad \mathbb{R}^3 \\ [E \times n] = [H \times n] & = & 0 & \text{on} \quad \Gamma_\pm^\epsilon \end{array}\right.$$

$$\text{where} \quad \sigma^\epsilon = \left\{\begin{array}{ll} 0 & \text{in} \quad \Omega_\pm^\epsilon \\ \sigma_0 = \epsilon^{-2}\bar{\sigma} & \text{in} \quad \Omega_0^\epsilon \end{array}\right.$$

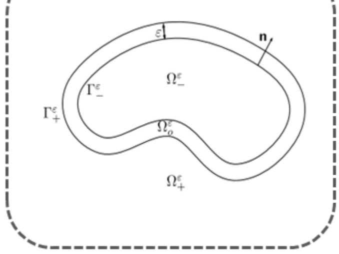

Figure 1: A cross Section of the domain Ω

Let u be a vector field on Γ, then we denote by $\gamma_D u = n \times (u \times n)$, and $\gamma_N u = \mathrm{curl} u \times n$, the Dirichlet and Neumann data, respectively.

2 Multiscale Expansion

Assuming that Γ is a smooth surface, then E^ϵ and H^ϵ can be expanded with an asymptotic expansion in power series of the small parameter ϵ. [1]

$$\begin{array}{lcll} E^\epsilon(x) & \approx & E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \ldots + O(\epsilon^k) & \text{in } \Omega_\pm^\epsilon \\ H^\epsilon(x) & \approx & \mathcal{H}_0(y_\alpha, \frac{h}{\epsilon}) + \epsilon \mathcal{H}_1(y_\alpha, \frac{h}{\epsilon}) + \ldots + O(\epsilon^k) & \text{in } \Omega_0^\epsilon \end{array}$$

Here, $x \in \mathbb{R}^3$ are the cartesian coordinated, and (y_α, h) is the local normal coordinate system, $h \in (-\frac{\epsilon}{2}, \frac{\epsilon}{2})$ is the normal coordinate to Γ. The term $\mathcal{H}_j$ is a profile defined on $\Gamma \times (-\frac{1}{2}, \frac{1}{2})$. The derivation is based on the expansion of the differential operators inside the thin layer Ω_0^ϵ, and the Taylor expansion of $E_j|_{\Gamma_\pm^\epsilon}$ around the mid-surface Γ.

3 Equivalent Model of Order 2

We introduce a problem satisfied by an approximation E_ϵ^k of the expression $E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \ldots + \epsilon^k E_k(x)$ up to a residual term $O(\epsilon^{k+1})$.

The second order approximate solution E_ϵ^1, solves

$$\begin{array}{lcll} \text{curl}(\text{curl} E_\epsilon^1) & = & i\omega\mu J_0 & \text{in} \quad \Omega_\pm \\ \begin{pmatrix} [\gamma_D E_\epsilon^1]_\Gamma \\ \{\gamma_D E_\epsilon^1\}_\Gamma \end{pmatrix} & = & \epsilon \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \begin{pmatrix} \{\gamma_N E_\epsilon^1\}_\Gamma \\ [\gamma_N E_\epsilon^1]_\Gamma \end{pmatrix} & \text{on} \quad \Gamma \end{array}$$

where

$$C_1 = -1 + \frac{2\tanh(\frac{\gamma}{2})}{\gamma} \quad , \quad C_2 = -\frac{1}{4} + \frac{\coth(\frac{\gamma}{2})}{2\gamma}$$
$$\gamma = exp(\tfrac{3i\pi}{4})\sqrt{\omega\mu_0\bar{\sigma}}.$$

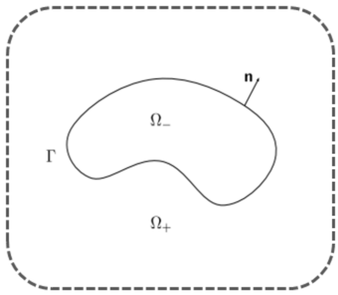

Figure 2: A cross section of the ϵ-independent subdomains

4 Numerical Results

Numerical experiments are performed to assess the accuracy of our model. The results are in particular compared to the model given in [2]. Complementary simulations will be conducted to study the robustness with respect to the sheet conductivity and the convergence of the modelling error.

Acknowledgements

We acknowledge the collaboration of B. BANNWARTH, O. CHADEBEC, and G. MEUNIER who participated in the implementation of the model, and the GDR SEEDS for the financial support.

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