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Asymptotic Modelling for 3D Eddy Current Problems with a Conductive Thin Layer

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Abstract

In this work we derive and analyze an equivalent model for 3D Eddy Current problems with a conductive thin layer of small thickness ϵ. In our model, the conductive sheet is replaced by its mid-surface and their shielding behavior is satisfied by an equivalent transmission conditions on this interface. The transmission conditions are derived asymptotically for vanishing sheet thickness ϵ.

Key words: Asymptotic Expansions, Eddy-Current Problems, Thin Conducting Layers, Transmission Conditions

1 Introduction

We denote by $\Omega = \Omega^\epsilon_- \cup \overline{\Omega^\epsilon_0} \cup \Omega^\epsilon_+ \subset \mathbb{R}^3$ the domain of study, where Ω^ϵ_- corresponds to a non-conductive linear material, Ω^ϵ_+ the exterior of the structure domain, and Ω^ϵ_0 a conductive thin layer of constant thickness ϵ (see figure 1). The discretisation of the conducting sheet by FEM needs a very fine mesh due to the rapid decay of the field under high conductivity. For this, we approximate a new model defined in ϵ-independent domains. Let Σ be a smooth surface, we denote by $[v]_\Sigma$ and $\{v\}_\Sigma$ the jump and mean of v respectively across Σ

$$[v]_\Sigma = v_{|_{\Sigma^+}} - v_{|_{\Sigma^-}}, \quad \{v\}_\Sigma = \frac{1}{2}(v_{|_{\Sigma^+}} + v_{|_{\Sigma^-}}), \quad \text{for } v \in (C^\infty(\Omega_\pm))^3.$$

We consider the eddy current problem as follows

$$\begin{cases} \text{curl}H^\epsilon &= \sigma^\epsilon E^\epsilon + J_0 & \text{in} & \Omega \\ \text{curl}E^\epsilon &= iw\mu_0 H^\epsilon & \text{in} & \Omega \\ \text{div}(H^\epsilon) &= 0 & \text{in} & \mathbb{R}^3 \\ [E \times n] = [H \times n] &= 0 & \text{on} & \Gamma^\epsilon_\pm \end{cases}$$

$$\text{where } \sigma^\epsilon = \begin{cases} 0 & \text{in} \quad \Omega^\epsilon_\pm \\ \sigma_0 = \epsilon^{-2}\bar{\sigma} & \text{in} \quad \Omega^\epsilon_0 \end{cases}$$

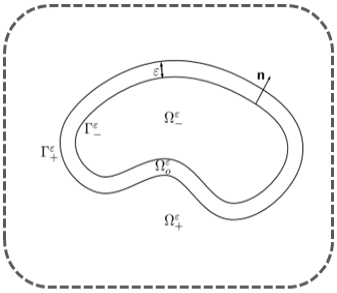

Figure 1: A cross Section of the domain Ω

Let u be a vector field on Γ, then we denote by $\gamma_D u = n \times (u \times n)$, and $\gamma_N u = \text{curl}u \times n$, the Dirichlet and Neumann data, respectively.

2 Multiscale Expansion

Assuming that Γ is a smooth surface, then E^ϵ and H^ϵ can be expanded with an asymptotic expansion in power series of the small parameter ϵ. [1]

$$\begin{array}{rcll} E^\epsilon(x) & \approx & E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \ldots + O(\epsilon^k) & \text{in } \Omega^\epsilon_\pm \\ H^\epsilon(x) & \approx & \boldsymbol{\mathcal{H}}_0(y_\alpha, \frac{h}{\epsilon}) + \epsilon \boldsymbol{\mathcal{H}}_1(y_\alpha, \frac{h}{\epsilon}) + \ldots + O(\epsilon^k) & \text{in } \Omega^\epsilon_0 \end{array}$$

Here, $x \in \mathbb{R}^3$ are the cartesian coordinated, and (y_α, h) is the local normal coordinate system, $h \in (-\frac{\epsilon}{2}, \frac{\epsilon}{2})$ is the normal coordinate to Γ. The term $\boldsymbol{\mathcal{H}}_j$ is a profile defined on $\Gamma \times (-\frac{1}{2}, \frac{1}{2})$. The derivation is based on the expansion of the differential operators inside the thin layer Ω^ϵ_0, and the Taylor expansion of $E_j|_{\Gamma^\epsilon_\pm}$ around the mid-surface Γ.

3 Equivalent Model of Order 2

We introduce a problem satisfied by an approximation E^k_ϵ of the expression $E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \ldots + \epsilon^k E_k(x)$ up to a residual term $O(\epsilon^{k+1})$.

The second order approximate solution E^1_ϵ, solves

$$\begin{array}{rcll} \text{curl}(\text{curl} E^1_\epsilon) & = & i\omega\mu J_0 & \text{in } \Omega_\pm \\ \begin{pmatrix} [\gamma_D E^1_\epsilon]_\Gamma \\ \{\gamma_D E^1_\epsilon\}_\Gamma \end{pmatrix} & = & \epsilon \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \begin{pmatrix} \{\gamma_N E^1_\epsilon\}_\Gamma \\ [\gamma_N E^1_\epsilon]_\Gamma \end{pmatrix} & \text{on } \Gamma \end{array}$$

where

$$C_1 = -1 + \frac{2\tanh(\frac{\gamma}{2})}{\gamma} \quad , \quad C_2 = -\frac{1}{4} + \frac{\coth(\frac{\gamma}{2})}{2\gamma}$$
$$\gamma = exp(\tfrac{3i\pi}{4})\sqrt{\omega\mu_0\bar{\sigma}}.$$

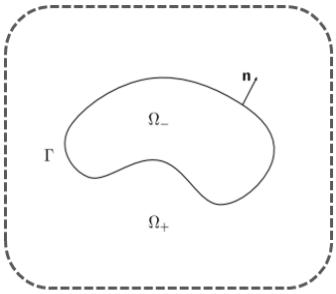

Figure 2: A cross section of the ϵ-independent subdomains

4 Numerical Results

Numerical experiments are performed to assess the accuracy of our model. The results are in particular compared to the model given in [2]. Complementary simulations will be conducted to study the robustness with respect to the sheet conductivity and the convergence of the modelling error.

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