



Trefftz-DG Approximation for the Elasto-Acoustics

Henri Calandra, Elvira Shishenina, Julien Diaz, Hélène Barucq

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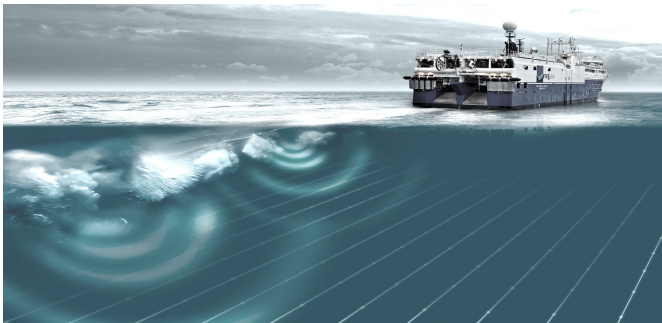
TREFFTZ-DG APPROXIMATION FOR THE ELASTO-ACOUSTICS

WAVES17 | 13th International Conference on Mathematical and Numerical Aspects of Wave Propagation

ABSTRACT

SEISMIC SURVEY

FIGURE 1: Ocean Bottom Seismic (OBS) data acquisition



BASIC NUMERICAL METHODS

TABLE 1: Generic properties of the most widely used numerical methods*

Numerical method	Complex geometries	High-order accuracy and <i>hp</i> -adaptivity	Explicit semi-discrete form	Conservation laws	Elliptic problems
FDM	●	●	●	●	●
FVM					
FEM					
DG-FEM					

* J.S.Hesthaven T.Warburton. *Nodal DG methods. Algorithms, analysis, and applications*. 2007

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FEM	●	●	●	⊙	●
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BASIC NUMERICAL METHODS

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FDM	●	●	●	●	●
FVM	●	●	●	●	⊙
FEM	●	●	●	⊙	●
DG-FEM	●	●	●	●	⊙

* J.S.Hesthaven T.Warburton. *Nodal DG methods. Algorithms, analysis, and applications.* 2007

DISCONTINUOUS GALERKIN METHODS

PROS AND CONS



- Adapted to the complex geometries
- High-order accuracy and hp-adaptivity
- Explicit semi-discrete form
- Conservation laws

DISCONTINUOUS GALERKIN METHODS

PROS AND CONS



- Adapted to the complex geometries
- High-order accuracy and hp-adaptivity
- Explicit semi-discrete form
- Conservation laws



- Higher number of degrees of freedom
comparing to the methods with continuous approximation

TREFFTZ METHOD

DEFINITION *

Given a region of an Euclidean space of some partitions of that region,
a **Trefftz method** is any procedure for solving boundary value problems
of PDE or systems of PDE, on such region,
using solutions of that PDE or its adjoint, defined in its subregions.

* I.Herrera. *Trefftz method: a general theory*. 2000

DISCONTINUOUS GALERKIN METHODS

TIME-HARMONIC FORMULATIONS

O.Cassenat B.Despres. 1998

T.Huttunen P.Monk J.P.Kaipo. 2002

C.Farhat I.Harari U.Hetmaniuk. 2003

R.Tezaur, C.Farhat. 2006

G.Gabard. 2007

TIME-DOMAIN FORMULATIONS

H.Egger F.Kretzschmar S.M.Schnepp T.Weiland. 2014

F.Kretzschmar S.M.Schnepp I.Tsukerman T.Weiland. 2014

F.Kretzschmar A.Moiola I.Perugia. 2015

TREFFTZ METHOD

EXPECTED ADVANTAGES *



Better order of convergence

Flexibility in the choice of basis functions

Low dispersion

Adaptivity and local space-time mesh refinement

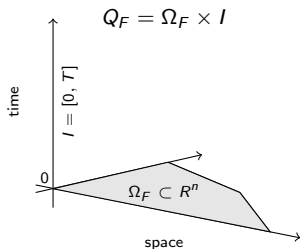
* I.Herrera. *Trefftz method: a general theory*. 2000

MATHEMATICAL FORMULATION

ACOUSTIC SYSTEM

ACOUSTIC SYSTEM

PROBLEM EQUATIONS

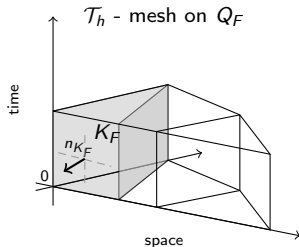


(AS)

$$\left[\begin{array}{ll} \frac{1}{c_F^2 \rho_F} \frac{\partial p}{\partial t} + \operatorname{div} v_F = f, & \text{in } Q_F \\ \rho_F \frac{\partial v_F}{\partial t} + \nabla p = 0, & \text{in } Q_F \\ v_F(\cdot, 0) = v_{F0}, \quad p(\cdot, 0) = p_0, & \text{on } \Omega_F \\ v_F = g_F^D, & \text{on } \partial\Omega_F \times I \end{array} \right.$$

ACOUSTIC SYSTEM

MESHING



$\mathcal{F}_h = \cup_{K_F \in \mathcal{T}_h} \partial K_F$ - mesh skeleton

$K_F \in \mathcal{Q}_F$ ($c_F, \rho_F \equiv \text{const}$ in K_F)

$v_F, p \in H^1(K_F)$

$\omega_F, q \in H^1(K_F)$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

AS

(v_F, p)

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{array}}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\begin{array}{|c|} \hline \text{AS} \\ \hline (v_F, p) \end{array}} \times \begin{array}{|c|} \hline \text{test} \\ \text{functions} \\ \hline (\omega_F, q) \end{array} =$$

SPACE-TIME INTEGRATION ON K_F

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{array}}}_{\text{SPACE-TIME INTEGRATION ON } K_F} = \boxed{\begin{array}{c} \text{VOLUME INTEGRATION TERM} \\ \text{AS} \times (v_F, p) \\ \text{(test functions)} \end{array}}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{array}}}_{\text{SPACE-TIME INTEGRATION ON } K_F} = \boxed{\begin{array}{c} \text{VOLUME INTEGRATION TERM} \\ \text{AS} \times (v_F, p) \\ \text{(test functions)} \end{array}} + \boxed{\begin{array}{c} \text{SURFACE INTEGRATION TERM} \\ \text{(test functions)} \\ (v_F, p) \end{array}}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test functions} \\ (\omega_F, q) \end{array}}}_{\text{SPACE-TIME INTEGRATION ON } K_F} = \underbrace{\boxed{\begin{array}{c} \text{AS} \\ (\text{test functions}) \end{array}} \times (v_F, p)}_{\text{VOLUME INTEGRATION TERM}} + \underbrace{\boxed{\begin{array}{c} (\text{test functions}) \\ (v_F, p) \end{array}}}_{\text{SURFACE INTEGRATION TERM}}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test functions} \\ (\omega_F, q) \end{array}}}_{\text{SPACE-TIME INTEGRATION ON } K_F} = \underbrace{\boxed{\begin{array}{c} \text{test functions in } \mathbf{T} \end{array}}}_{\text{VOLUME INTEGRATION TERM}} + \underbrace{\boxed{\begin{array}{c} (\text{test functions}) \\ (v_F, p) \end{array}}}_{\text{SURFACE INTEGRATION TERM}}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test functions} \\ (\omega_F, q) \end{array}}}_{\text{SPACE-TIME INTEGRATION ON } K_F} = \boxed{\begin{array}{c} \text{VOLUME INTEGRATION TERM} \\ = 0 \end{array}} + \boxed{\begin{array}{c} \text{SURFACE INTEGRATION TERM} \\ (\text{test functions}) \\ (v_F, p) \end{array}}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT K_F

$$\underbrace{\boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}}} \times \boxed{\begin{array}{c} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{array}} = \boxed{\begin{array}{c} \text{(test functions)} \\ (v_F, p) \end{array}}$$

SPACE-TIME INTEGRATION ON K_F SURFACE INTEGRATION TERM

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\begin{array}{c}
 \text{SUM OVER ALL } K_F \\
 \hline
 \boxed{\begin{array}{c} \text{AS} \\ (v_F, p) \end{array}} \times \boxed{\begin{array}{c} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{array}} \\
 \hline
 \text{SPACE-TIME INTEGRATION ON } K_F
 \end{array}
 =
 \begin{array}{c}
 \text{SUM OVER ALL } \partial K_F \\
 \hline
 \text{SURFACE INTEGRATION TERM} \\
 \boxed{\begin{array}{c} (\text{test functions}) \\ (v_F, p) \end{array}}
 \end{array}$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_{K_F} \int_{K_F} \left[p_h \left(\frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_{Fh} \cdot \left(\rho_F \frac{\partial \omega_F}{\partial t} + \nabla q \right) \right] dv \\
 & + \sum_{K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_{Fh} \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_{Fh} q) \cdot n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv
 \end{aligned}$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_{K_F} \int_{K_F} \left[p_h \left(\frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_{Fh} \cdot \left(\rho_F \frac{\partial \omega_F}{\partial t} + \nabla q \right) \right] dv \\
 & + \sum_{K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v \hat{F}_h \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v \hat{F}_h q) \cdot n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv
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 \end{aligned}$$

TREFFTZ SPACE

$$\mathbf{T}_F(\mathcal{T}_h) := \left\{ (\omega_F, q) \in H^1(\mathcal{T}_h)^2 : \forall K_F \in \mathcal{T}_h, \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q = \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F = 0 \right\}$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_{K_F} \int_{K_F} \left[p_h \left(\frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_{Fh} \left(\rho_F \frac{\partial \omega_F}{\partial t} + \nabla q \right) \right] dv \\
 & + \sum_{K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v \hat{F}_h \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v \hat{F}_h q) n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv
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SPACE-TIME DISCRETIZATION IN $\mathbf{T}_F$

$$\sum_{K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v \hat{F}_h \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v \hat{F}_h q) \cdot n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

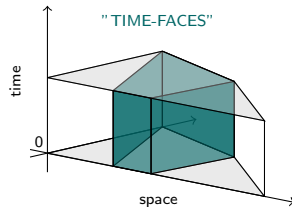
SUM OVER
ALL ∂K_F

SURFACE
INTEGRATION
TERM

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

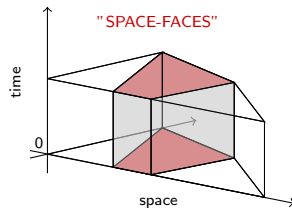
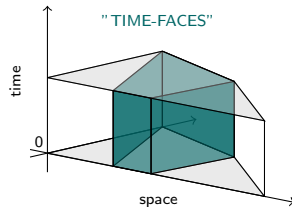
$$\underbrace{\text{SUM THROUGH ALL } \partial K_F}_{\text{SURFACE INTEGRATION TERM}} = \underbrace{\text{SUM THROUGH ALL "TIME-FACES"}}_{\text{SURFACE INTEGRATION TERM}}$$



ACOUSTIC SYSTEM

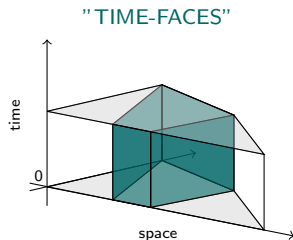
TREFFTZ-DG FORMULATION

$$\underbrace{\text{SUM THROUGH ALL } \partial K_F}_{\text{SURFACE INTEGRATION TERM}} = \underbrace{\text{SUM THROUGH ALL "TIME-FACES"}}_{\text{SURFACE INTEGRATION TERM}} + \underbrace{\text{SUM THROUGH ALL "SPACE-FACES"}}_{\text{SURFACE INTEGRATION TERM}}$$

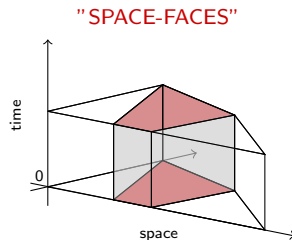


ACOUSTIC SYSTEM

MESH NOTATIONS



internal $\mathcal{F}_h^{I_F}$
 boundary $\mathcal{F}_h^{D_F} (\partial\Omega_F \times [0, T])$



internal $\mathcal{F}_h^{\Omega_F} (t \equiv \text{const.})$
 initial $\mathcal{F}_h^{0_F} (\Omega_F \times \{0\})$
 final $\mathcal{F}_h^{T_F} (\Omega_F \times \{T\})$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\sum_{\partial K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_{Fh} \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_{Fh} q) n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \mathcal{F}_h^{I_F} : \quad \begin{pmatrix} v_{Fh} \\ \hat{p}_h \end{pmatrix} &:= \begin{pmatrix} \{v_{Fh}\} + \beta [\rho_h]_x \\ \{\rho_h\} + \alpha [v_{Fh}]_x \end{pmatrix} \\ \mathcal{F}_h^{D_F} : \quad \begin{pmatrix} v_{Fh} \\ \hat{p}_h \end{pmatrix} &:= \begin{pmatrix} g \hat{D}_F \\ p_h + \alpha (v_{Fh} \cdot n_{K_F}^x - g D_F) \end{pmatrix} \\ \mathcal{F}_h^{\Omega_F} : \quad \begin{pmatrix} v_{Fh} \\ \hat{p}_h \end{pmatrix} &:= \begin{pmatrix} v_{Fh}^- \\ p_h^- \end{pmatrix} \\ \mathcal{F}_h^{T_F} : \quad \begin{pmatrix} v_{Fh} \\ \hat{p}_h \end{pmatrix} &:= \begin{pmatrix} v_{Fh} \\ p_h \end{pmatrix} \\ \mathcal{F}_h^{0_F} : \quad \begin{pmatrix} v_{Fh} \\ \hat{p}_h \end{pmatrix} &:= \begin{pmatrix} v_{F0} \\ p_0 \end{pmatrix} \end{aligned}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

Seek $(v_{Fh}, p_h) \in \mathbf{V}(\mathcal{T}_h) \subset \mathbf{T}_F(\mathcal{T}_h)$ s.t. for all $(\omega_F, q) \in \mathbf{V}(\mathcal{T}_h)$ it holds:

$$\mathcal{A}_{TDG_F}((v_{Fh}, p_h); (\omega_F, q)) = \ell_{TDG_F}(\omega_F, q)$$

ACOUSTIC SYSTEM

$L^2(\mathcal{T}_h)$ NORMS IN T_F

$$\begin{aligned}
 |||(\omega_F, q)|||_{TDGF}^2 &:= \frac{1}{2} \left\| \left(\frac{1}{c_F^2 \rho_F} \right)^{1/2} [\mathbf{q}]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} [\omega_F]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \alpha^{1/2} [\omega_F]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 \\
 &+ \left\| \beta^{1/2} [\mathbf{q}]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 + \frac{1}{2} \left\| \left(\frac{1}{c_F^2 \rho_F} \right)^{1/2} q \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 \\
 &+ \left\| \alpha^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{D_F})}^2
 \end{aligned}$$

ACOUSTIC SYSTEM

$L^2(\mathcal{T}_h)$ NORMS IN T_F

$$\begin{aligned} |||(\omega_F, q)|||_{TDG_F}^2 &:= \frac{1}{2} \left\| \left(\frac{1}{c_F^2 \rho_F} \right)^{1/2} [\mathbf{q}]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} [\omega_F]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \alpha^{1/2} [\omega_F]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 \\ &\quad + \left\| \beta^{1/2} [\mathbf{q}]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 + \frac{1}{2} \left\| \left(\frac{1}{c_F^2 \rho_F} \right)^{1/2} q \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 \\ &\quad + \left\| \alpha^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{D_F})}^2 \end{aligned}$$

$$\begin{aligned} |||(\omega_F, q)|||_{TDG_F^*}^2 &:= |||(\omega_F, q)|||_{TDG_F}^2 \\ &\quad + \left\| \rho_F^{1/2} \omega_F^- \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \left(\frac{1}{c_F^2 \rho_F} \right)^{1/2} q^- \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \beta^{-1/2} \{\omega_F\} \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 \\ &\quad + \left\| \alpha^{-1/2} \{\mathbf{q}\} \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 + \left\| \alpha^{-1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{D_F})}^2 \end{aligned}$$

ACOUSTIC SYSTEM

COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_F}((\omega_F, q); (\omega_F, q)) = |||(\omega_F, q)|||_{TDG_F}^2, \quad \forall (\omega_F, q) \in \mathbf{T}_F(\mathcal{T}_h)$$

ACOUSTIC SYSTEM

COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_F}((\omega_F, q); (\omega_F, q)) = |||(\omega_F, q)|||_{TDG_F}^2, \quad \forall (\omega_F, q) \in \mathbf{T}_F(\mathcal{T}_h)$$

CONTINUITY OF TREFFTZ-DG FORMULATION

$$|\mathcal{A}_{TDG_F}((v_F, p); (\omega_F, q))| \leq 2 |||(v_F, p)|||_{TDG_F^*} |||(\omega_F, q)|||_{TDG_F},$$

$$|\ell_{TDG_F}(\omega_F, q)| \leq \sqrt{2} \left[\|\rho_F^{1/2} v_{f0}\|_{L^2(\mathcal{F}_h^{0F})}^2 + \|(\frac{1}{c_F^2 \rho_F})^{1/2} p_0\|_{L^2(\mathcal{F}_h^{0F})}^2 \right]^{1/2}$$

ACOUSTIC SYSTEM

WELL-POSEDNESS

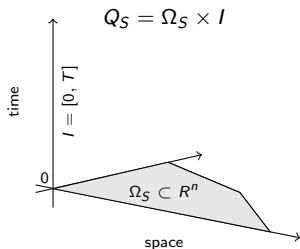
$$|||(v_F - v_{Fh}, p - p_h)|||_{TDG_F} \leq 3 \inf_{(\omega_F, q) \in \mathbf{V}(\mathcal{T}_h)} |||(v_F - \omega_F, p - q)|||_{TDG_F^*}$$

MATHEMATICAL FORMULATION

ELASTODYNAMIC SYSTEM

ELASTODYNAMIC SYSTEM

PROBLEM EQUATIONS

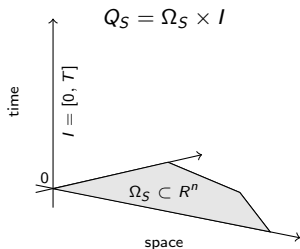


(ES)

$$\left[\begin{array}{ll} \frac{\partial \underline{\underline{\sigma}}}{\partial t} - \underline{\underline{C}} \underline{\underline{\varepsilon}}(v_S) = 0 & \text{in } Q_S \\ \rho_S \frac{\partial v_S}{\partial t} - \operatorname{div} \underline{\underline{\sigma}} = 0 & \text{in } Q_S \\ v_S(\cdot, 0) = v_{S0}, \underline{\underline{\sigma}}(\cdot, 0) = \underline{\underline{\sigma}}_0 & \text{on } \Omega_S \\ v_S = g_{D_S} & \text{on } \partial \Omega_S \times I \end{array} \right.$$

ELASTODYNAMIC SYSTEM

PROBLEM EQUATIONS

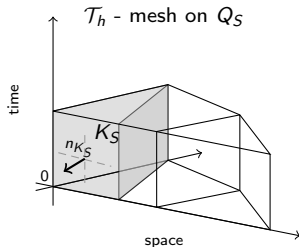


(ES)

$$\left[\begin{array}{ll} \underline{\underline{A}} \frac{\partial \underline{\underline{\sigma}}}{\partial t} - \underline{\underline{\varepsilon}}(v_S) = 0 & \text{in } Q_S \\ \rho_S \frac{\partial v_S}{\partial t} - \operatorname{div} \underline{\underline{\sigma}} = 0 & \text{in } Q_S \\ v_S(\cdot, 0) = v_{S0}, \underline{\underline{\sigma}}(\cdot, 0) = \underline{\underline{\sigma}}_0 & \text{on } \Omega_S \\ v_S = g_{D_S} & \text{on } \partial\Omega_S \times I \end{array} \right.$$

ELASTODYNAMIC SYSTEM

MESHING



$\mathcal{F}_h = \cup_{K_S \in \mathcal{T}_h} \partial K_S$ - mesh skeleton

$K_S \in Q_S$ ($A, \rho_S \equiv \text{const}$ in K_S)

$v_S, \underline{\underline{\sigma}} \in H^1(K_S)$

$\omega_S, \underline{\underline{\xi}} \in H^1(K_S)$

ELASTODYNAMIC SYSTEM

TREFFTZ SPACE

$$\mathbf{T}_S(\mathcal{T}_h) := \left\{ (\omega_S, \underline{\underline{\xi}}) \in H^1(\mathcal{T}_h)^2 : \forall K_S \in \mathcal{T}_h, \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\underline{\xi}} = A \frac{\partial \underline{\underline{\xi}}}{\partial t} - \underline{\underline{\xi}}(\omega_S) = 0 \right\}$$

ELASTODYNAMIC SYSTEM

TREFFTZ SPACE

$$\mathbf{T}_S(\mathcal{T}_h) := \left\{ (\omega_S, \underline{\underline{\xi}}) \subset H^1(\mathcal{T}_h)^2 : \forall K_S \in \mathcal{T}_h, \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\underline{\xi}} = \underline{\underline{A}} \frac{\partial \underline{\underline{\xi}}}{\partial t} - \underline{\underline{\varepsilon}}(\omega_S) = 0 \right\}$$

SPACE-TIME DISCRETIZATION IN T_S

$$\sum_{K_S} \int_{\partial K_S} \left[(\underline{\underline{A}} \underline{\underline{\hat{\sigma}}}_h : \underline{\underline{\xi}} + \rho_S v \hat{S}_h \omega_S) n_{K_S}^t - (v \hat{S}_h \underline{\underline{\xi}} + \underline{\underline{\hat{\sigma}}}_h \omega_S) \cdot n_{K_S}^x \right] ds = 0$$

ELASTODYNAMIC SYSTEM

TREFFTZ-DG FORMULATION

$$\sum_{K_S} \int_{\partial K_S} \left[(\underline{\underline{A}} \hat{\underline{\underline{\sigma}}}_h : \underline{\underline{\xi}} + \rho_S v \hat{S}_h \omega_S) n_{K_S}^t - (v \hat{S}_h \underline{\underline{\xi}} + \hat{\underline{\underline{\sigma}}}_h \omega_S) \cdot n_{K_S}^x \right] ds = 0$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \mathcal{F}_h^{I_S} : \quad \left(\begin{array}{c} v \hat{S}_h \\ \hat{\underline{\underline{\sigma}}}_h \end{array} \right) &:= \left(\begin{array}{c} \{v_{S_h}\} - \delta [\underline{\underline{\sigma}}_h]_x \\ \{\underline{\underline{\sigma}}_h\} - \gamma [v_{S_h}]_x \end{array} \right) \\ \mathcal{F}_h^{D_S} : \quad \left(\begin{array}{c} v \hat{S}_h \\ \hat{\underline{\underline{\sigma}}}_h \end{array} \right) &:= \left(\begin{array}{c} v_{S_h} - \delta (\underline{\underline{\sigma}}_h \cdot n_{K_S}^x - g_{D_S}) \\ g_{D_S} \end{array} \right) \\ \mathcal{F}_h^{\Omega_S} : \quad \left(\begin{array}{c} v \hat{S}_h \\ \hat{\underline{\underline{\sigma}}}_h \end{array} \right) &:= \left(\begin{array}{c} v_{S_h}^- \\ \underline{\underline{\sigma}}_h^- \end{array} \right) \\ \mathcal{F}_h^{T_S} : \quad \left(\begin{array}{c} v \hat{S}_h \\ \hat{\underline{\underline{\sigma}}}_h \end{array} \right) &:= \left(\begin{array}{c} v_{S_h} \\ \underline{\underline{\sigma}}_h \end{array} \right) \\ \mathcal{F}_h^{0_S} : \quad \left(\begin{array}{c} v \hat{S}_h \\ \hat{\underline{\underline{\sigma}}}_h \end{array} \right) &:= \left(\begin{array}{c} v_{S_0} \\ \underline{\underline{\sigma}}_0 \end{array} \right) \end{aligned}$$

ELASTODYNAMIC SYSTEM

TREFFTZ-DG FORMULATION

Seek $(v_{Sh}, \underline{\underline{\sigma}}_h) \in \mathbf{V}(\mathcal{T}_h) \subset \mathbf{T}_S(\mathcal{T}_h)$, s.t. for all $K_S \in \mathcal{T}_h$, $(\omega_S, \underline{\underline{\xi}}) \in \mathbf{V}(\mathcal{T}_h)$ it holds:

$$\mathcal{A}_{TDG_S}((v_{Sh}, \underline{\underline{\sigma}}_h); (\omega_S, \underline{\underline{\xi}})) = \ell_{TDG_S}(\omega_S, \underline{\underline{\xi}})$$

ELASTODYNAMIC SYSTEM

$L^2(\mathcal{T}_h)$ NORMS IN T_S

$$\begin{aligned}
 |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S}^2 := & \frac{1}{2} \|(\underline{\underline{A}})^{1/2} [\underline{\underline{\xi}}]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} [\omega_S]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|\gamma^{1/2} [\omega_S]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 \\
 & + \|\delta^{1/2} [\underline{\underline{\xi}}]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 + \frac{1}{2} \|(\underline{\underline{A}})^{1/2} \underline{\underline{\xi}}\|_{L^2(\mathcal{F}_h^{T_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} \omega_S\|_{L^2(\mathcal{F}_h^{T_S})}^2 \\
 & + \|\delta^{1/2} \underline{\underline{\xi}}\|_{L^2(\mathcal{F}_h^{D_S})}^2
 \end{aligned}$$

ELASTODYNAMIC SYSTEM

$L^2(\mathcal{T}_h)$ NORMS IN T_S

$$\begin{aligned} |||(\omega_S, \underline{\xi})|||_{TDG_S}^2 := & \frac{1}{2} \|(\underline{A})^{1/2} [\underline{\xi}]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} [\omega_S]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|\gamma^{1/2} [\omega_S]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 \\ & + \|\delta^{1/2} [\underline{\xi}]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 + \frac{1}{2} \|(\underline{A})^{1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{T_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} \omega_S\|_{L^2(\mathcal{F}_h^{T_S})}^2 \\ & + \|\delta^{1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{D_S})}^2 \end{aligned}$$

$$\begin{aligned} |||(\omega_S, \underline{\xi})|||_{TDG_S^*}^2 := & |||(\omega_S, \underline{\xi})|||_{TDG_S}^2 \\ & + \|\rho_S^{1/2} \omega_S^-\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|(\underline{A})^{1/2} \underline{\xi}^-\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|\delta^{-1/2} \{\omega_S\}\|_{L^2(\mathcal{F}_h^{I_S})}^2 \\ & + \|\gamma^{-1/2} \{\underline{\xi}\}\|_{L^2(\mathcal{F}_h^{I_S})}^2 + \|\delta^{-1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{D_S})}^2. \end{aligned}$$

ELASTODYNAMIC SYSTEM

COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_S}((\omega_S, \underline{\underline{\xi}}); (\omega_S, \underline{\underline{\xi}})) = |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S}^2, \quad \forall (\omega_S, \underline{\underline{\xi}}) \in \mathbf{T}_S(\mathcal{T}_h)$$

ELASTODYNAMIC SYSTEM

COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_S}((\omega_S, \underline{\underline{\xi}}); (\omega_S, \underline{\underline{\xi}})) = |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S}^2, \quad \forall (\omega_S, \underline{\underline{\xi}}) \in \mathbf{T}_S(\mathcal{T}_h)$$

CONTINUITY OF TREFFTZ-DG FORMULATION

$$|\mathcal{A}_{TDG_S}((v_S, \underline{\underline{\sigma}}); (\omega_S, \underline{\underline{\xi}}))| \leq 2 |||(v_S, \underline{\underline{\sigma}})|||_{TDG_S^*} |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S}$$

$$|\ell_{TDG_S}(\omega_S, \underline{\underline{\xi}})| \leq \sqrt{2} \left[\|\rho_S^{1/2} v_{s0}\|_{L^2(\mathcal{F}_h^0)}^2 + \|\underline{\underline{A}}^{1/2} \underline{\underline{\sigma}}_0\|_{L^2(\mathcal{F}_h^0)}^2 \right]^{1/2}$$

ELASTODYNAMIC SYSTEM

WELL-POSEDNESS

$$|||(v_S - v_{Sh}, \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_h)|||_{TDG_S} \leq 3 \inf_{(\omega_S, \underline{\underline{\xi}}) \in \mathbf{V}(\mathcal{T}_h)} |||(v_S - \omega_S, \underline{\underline{\sigma}} - \underline{\underline{\xi}})|||_{TDG_S^*}$$

MATHEMATICAL FORMULATION

ELASTO-ACOUSTIC SYSTEM

ELASTO-ACOUSTIC SYSTEM

NUMERICAL COUPLING ADVANTAGES



Computing 1 unknown scalar pressure instead of 6 components of stress in fluid

ELASTO-ACOUSTIC SYSTEM

NUMERICAL COUPLING ADVANTAGES

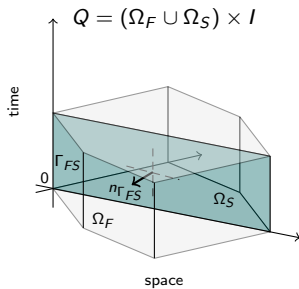


Computing 1 unknown scalar pressure instead of 6 components of stress in fluid

Avoiding the numerical artefacts caused by the slow S - waves

ELASTO-ACOUSTIC SYSTEM

TRANSMISSION CONDITIONS



$$(TC) \quad \begin{cases} v_F n_{\Gamma_{FS}} = v_S n_{\Gamma_{FS}} & \text{on } \Gamma_{FS} \\ \underline{\underline{\sigma}} \cdot n_{\Gamma_{FS}} = -p n_{\Gamma_{FS}} & \text{on } \Gamma_{FS} \end{cases}$$

ELASTO-ACOUSTIC SYSTEM

TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) := \left\{ (\omega_F, q, \omega_S, \underline{\underline{\xi}}) \in H^1(\mathcal{T}_h)^4 : \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q = \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F = 0, \right. \\ \left. \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\underline{\xi}} = \frac{\partial \underline{\underline{A}} \underline{\underline{\xi}}}{\partial t} - \underline{\underline{\varepsilon}}(\omega_S) = 0, \forall K_F, K_S \in \mathcal{T}_h \right\}$$

ELASTO-ACOUSTIC SYSTEM

TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) := \left\{ (\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}}) \in H^1(\mathcal{T}_h)^4 : \rho_F \frac{\partial \omega_F}{\partial t} + \nabla \mathbf{q} = \frac{1}{c_F^2 \rho_F} \frac{\partial \mathbf{q}}{\partial t} + \operatorname{div} \omega_F = 0, \right. \\ \left. \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\underline{\xi}} = \frac{\partial \underline{\underline{A}} \underline{\underline{\xi}}}{\partial t} - \underline{\underline{\varepsilon}}(\omega_S) = 0, \forall K_F, K_S \in \mathcal{T}_h \right\}$$

SPACE-TIME DISCRETIZATION IN T

$$\sum_{K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h \mathbf{q} + \rho_F v_{\hat{F}h} \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_{\hat{F}h} \mathbf{q}) n_{K_F}^x \right] ds \\ + \sum_{K_S} \int_{\partial K_S} \left[(\underline{\underline{A}} \underline{\underline{\sigma}}_h : \underline{\underline{\xi}} + \rho_S v_{\hat{S}h} \omega_S) n_{K_S}^t - (v_{\hat{S}h} \underline{\underline{\xi}} + \underline{\underline{\sigma}}_h \omega_S) \cdot n_{K_S}^x \right] ds = \sum_{K_F} \int_{K_F} f \mathbf{q} dv$$

ELASTO-ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\begin{aligned} & \sum_{K_F} \int_{\partial K_F} \left[\left(\frac{1}{c_F^2 \rho_F} \hat{p}_h \mathbf{q} + \rho_F v_{\hat{F}h} \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_{\hat{F}h} \mathbf{q}) \cdot \mathbf{n}_{K_F}^x \right] ds, \\ & + \sum_{K_S} \int_{\partial K_S} \left[(\underline{\underline{A}} \underline{\underline{\sigma}}_h : \underline{\underline{\xi}} + \rho_S v_{\hat{S}h} \cdot \omega_S) n_{K_S}^t - (\underline{\underline{\xi}} v_{\hat{S}h} + \underline{\underline{\sigma}}_h \omega_S) \cdot \mathbf{n}_{K_S}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv \end{aligned}$$

NUMERICAL FLUXES THROUGH THE FLUID-SOLID INTERFACE

$$\mathcal{F}_h^{FS} : \begin{pmatrix} v_{\hat{F}h} n_{K_F}^x \\ \hat{p}_h \\ v_{\hat{S}h} \\ \underline{\underline{\sigma}}_h \cdot \mathbf{n}_{K_S}^x \end{pmatrix} := \begin{pmatrix} v_{\hat{S}h} n_{K_F}^x \\ p_h + \alpha(v_{\hat{F}h} n_{K_F}^x - v_{\hat{S}h} n_{K_F}^x) \\ v_{\hat{S}h} - \delta(\underline{\underline{\sigma}}_h \cdot \mathbf{n}_{K_S}^x - p_h n_{K_S}^x) \\ -p_h n_{K_S}^x \end{pmatrix}$$

ELASTO-ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

Seek $(v_{Fh}, p_h, v_{Sh}, \underline{\underline{\sigma}}_h) \in \mathbf{V}(\mathcal{T}_h)$: $\forall K_F, K_S \in \mathcal{T}_h, (\omega_F, q, \omega_S, \underline{\underline{\xi}}) \in \mathbf{T}(\mathcal{T}_h)$ it holds:

$$\mathcal{A}_{TDG}((v_{Fh}, p_h, v_{Sh}, \underline{\underline{\sigma}}_h); (\omega_F, q, \omega_S, \underline{\underline{\xi}})) = \ell_{TDG}(\omega_F, q, \omega_S, \underline{\underline{\xi}})$$

ELASTO-ACOUSTIC SYSTEM

$L^2(\mathcal{T}_h)$ NORMS IN T

$$|||(\omega_F, \underline{q}, \omega_S, \underline{\underline{\xi}})|||_{TDG}^2 := |||(\omega_F, \underline{q})|||_{TDG_F}^2 + |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S}^2 + 2\|\delta^{1/2}\underline{\underline{\xi}}\|_{L^2(\mathcal{F}_h^{FS})}^2$$

ELASTO-ACOUSTIC SYSTEM

$L^2(\mathcal{T}_h)$ NORMS IN T

$$|||(\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}})|||_{TDG}^2 := |||(\omega_F, \mathbf{q})|||_{TDG_F}^2 + |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S}^2 + 2\|\delta^{1/2}\underline{\underline{\xi}}\|_{L^2(\mathcal{F}_h^{FS})}^2$$

$$|||(\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}})|||_{TDG^*}^2 := |||(\omega_F, \mathbf{q})|||_{TDG_F^*}^2 + |||(\omega_S, \underline{\underline{\xi}})|||_{TDG_S^*}^2 + \frac{1}{2}\|\delta^{-1/2}\underline{\underline{\xi}}\|_{L^2(\mathcal{F}_h^{FS})}^2$$

ELASTO-ACOUSTIC SYSTEM

COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG}((\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}}); (\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}})) = |||(\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}})|||_{TDG}^2$$

ELASTO-ACOUSTIC SYSTEM

COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG}((\omega_F, q, \omega_S, \underline{\underline{\xi}}); (\omega_F, q, \omega_S, \underline{\underline{\xi}})) = |||(\omega_F, q, \omega_S, \underline{\underline{\xi}})|||_{TDG}^2$$

CONTINUITY OF TREFFTZ-DG FORMULATION

$$|\mathcal{A}_{TDG}((v_F, p, v_S, \underline{\underline{\sigma}}); (\omega_F, q, \omega_S, \underline{\underline{\xi}}))| \leq 2 |||(v_F, p, v_S, \underline{\underline{\sigma}})|||_{TDG} * |||(\omega_F, q, \omega_S, \underline{\underline{\xi}})|||_{TDG}$$

$$\begin{aligned} |\ell_{TDG}(\omega_F, q, \omega_S, \underline{\underline{\xi}})| &\leq \sqrt{2} \left[\|\rho_F^{1/2} v_{F0}\|_{L^2(\mathcal{F}_h^{0F})}^2 + \|(\frac{1}{c_F^2 \rho_F})^{1/2} p_0\|_{L^2(\mathcal{F}_h^{0F})}^2 \right. \\ &\quad \left. + \|\rho_S^{1/2} v_{S0}\|_{L^2(\mathcal{F}_h^{0S})}^2 + \|\underline{\underline{A}}^{1/2} \underline{\underline{\sigma}}_0\|_{L^2(\mathcal{F}_h^{0S})}^2 \right]^{1/2} \end{aligned}$$

ELASTO-ACOUSTIC SYSTEM

WELL-POSEDNESS

$$|||(\mathbf{v}_F - \mathbf{v}_{Fh}, \mathbf{p} - \mathbf{p}_h, \mathbf{v}_S - \mathbf{v}_{Sh}, \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_h)|||_{TDG}$$

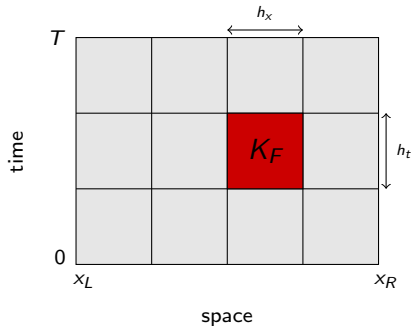
$$\leq 3 \inf_{(\omega_F, \mathbf{q}, \omega_S, \underline{\underline{\xi}}) \in \mathbf{V}(\mathcal{T}_h)} |||(\mathbf{v}_F - \omega_F, \mathbf{p} - \mathbf{q}, \mathbf{v}_S - \omega_S, \underline{\underline{\sigma}} - \underline{\underline{\xi}})|||_{TDG^*}.$$

IMPLEMENTATION OF THE ALGORITHM

1D ACOUSTIC SYSTEM EXAMPLE

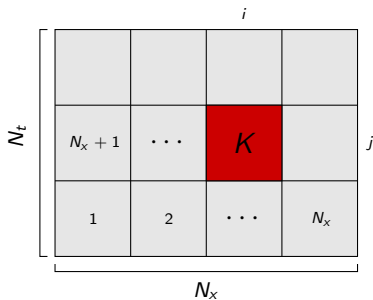
IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

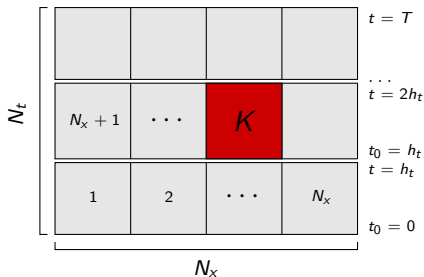
MESH AND ELEMENT NUMBERING



$$K = (j - 1) \times N_x + i$$

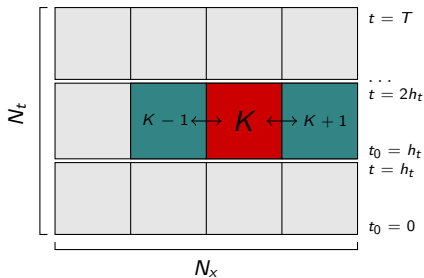
IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



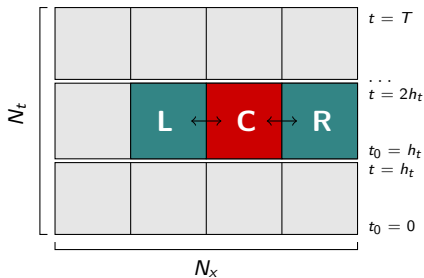
IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX (1^{ST} LAYER)

$$\begin{bmatrix}
 \text{C} & \text{R} & & \dots & & \text{L} \\
 \text{L} & \text{C} & \text{R} & & & \\
 & \text{L} & \text{C} & \text{R} & & \\
 \vdots & & & \ddots & & \vdots \\
 & & & & \text{L} & \text{C} & \text{R} \\
 & & & & & \text{L} & \text{C} \\
 \text{R} & & \dots & & & &
 \end{bmatrix} \times \begin{bmatrix} \text{U}_1 \\ \text{U}_2 \\ \text{U}_3 \\ \vdots \\ \text{U}_{N_x-1} \\ \text{U}_{N_x} \end{bmatrix} = \begin{bmatrix} \text{U}_1^0 \\ \text{U}_2^0 \\ \text{U}_3^0 \\ \vdots \\ \text{U}_{N_x-1}^0 \\ \text{U}_{N_x}^0 \end{bmatrix}$$

IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX (1^{ST} LAYER)

The diagram illustrates the global matrix equation for the 1st layer. It features a large teal square matrix labeled M enclosed in square brackets. To its right is a multiplication symbol $\times$, followed by a tall, narrow gray rectangular vector labeled U , also enclosed in square brackets. This is followed by an equals sign $=$, and then another tall, narrow gray rectangular vector labeled U^0 , enclosed in square brackets. The vectors U and U^0 are significantly narrower than the matrix M .

$$\begin{bmatrix} M \end{bmatrix} \times \begin{bmatrix} U \end{bmatrix} = \begin{bmatrix} U^0 \end{bmatrix}$$

IMPLEMENTATION OF THE ALGORITHM

1. INITIATION

COMPUTE

$$\begin{bmatrix} \mathbf{M} \end{bmatrix} ; \begin{bmatrix} \mathbf{M}^{-1} \end{bmatrix} ; \begin{bmatrix} \mathbf{U}^0 \end{bmatrix}$$

2. PROPAGATION

FOR $t = 1 : N_t$

$$\begin{bmatrix} \mathbf{U} \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{-1} \end{bmatrix} \times \begin{bmatrix} \mathbf{U}^0 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{U}^0 \end{bmatrix} = \begin{bmatrix} \text{projection operator} \end{bmatrix} \times \begin{bmatrix} \mathbf{U} \end{bmatrix}$$

ENDFOR

IMPLEMENTATION OF THE ALGORITHM

POLYNOMIAL BASIS (ORDER 3)

$$\phi_1^v = 0$$

$$\phi_2^v = 1$$

$$\phi_3^v = x$$

$$\phi_4^v = c_F t$$

$$\phi_5^v = -\frac{x^2}{2} - \frac{c_F^2 t^2}{2}$$

$$\phi_6^v = -c_F x t$$

$$\phi_7^v = -\frac{x^3}{6} - \frac{x c_F^2 t^2}{2}$$

$$\phi_8^v = -\frac{c_F^3 t^3}{6} - \frac{x^2 c_F t}{2}$$

$$\phi_1^p = -c_F$$

$$\phi_2^p = 0$$

$$\phi_3^p = -c_F^2 t$$

$$\phi_4^p = -c_F x$$

$$\phi_5^p = c_F^2 x t$$

$$\phi_6^p = c_F \left(\frac{x^2}{2} + \frac{c_F^2 t^2}{2} \right)$$

$$\phi_7^p = c_F \left(\frac{c_F^3 t^3}{6} + \frac{x^2 c_F t}{2} \right)$$

$$\phi_8^p = c_F \left(\frac{x^3}{6} + \frac{x c_F^2 t^2}{2} \right)$$

IMPLEMENTATION OF THE ALGORITHM

NUMERICAL TESTS

FIGURE 1: 1D Acoustic system.

Exact and numerical velocities $v_F(x, t = 1)$.

Periodical boundaries. $\alpha = \beta = 0$

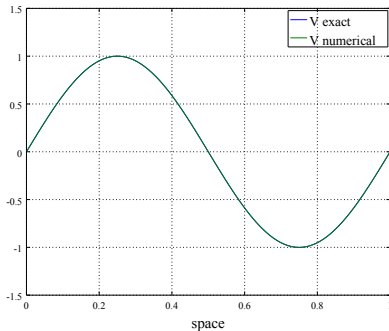
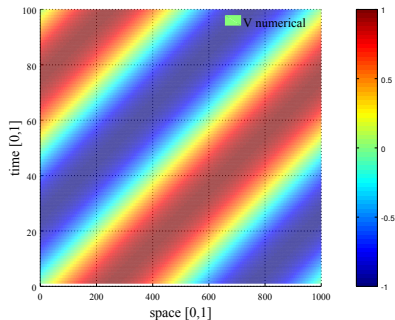


FIGURE 2: 1D Acoustic system.

Numerical velocity $v_F(x, t)$.

Periodical boundaries. $\alpha = \beta = 0$



IMPLEMENTATION OF THE ALGORITHM

NUMERICAL TESTS

FIGURE 3: 1D Acoustic system.

Exact and numerical velocities $v_F(x, t = 1)$.

Periodical boundaries. $\alpha = \beta = 0$

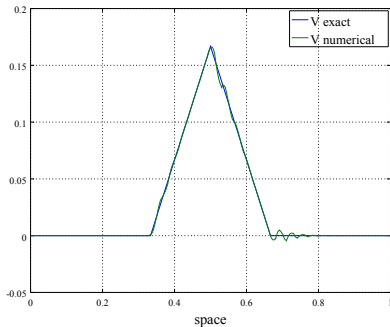
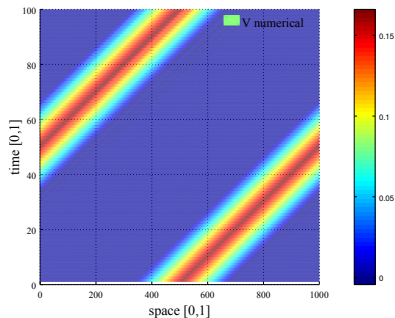


FIGURE 4: 1D Acoustic system.

Numerical velocity $v_F(x, t)$.

Periodical boundaries. $\alpha = \beta = 0$



IMPLEMENTATION OF THE ALGORITHM

NUMERICAL TESTS

FIGURE 5: 1D Acoustic system.

Exact and numerical velocities $v_F(x, t = 1)$.

Periodical boundaries. $\alpha = \beta = 0.5$

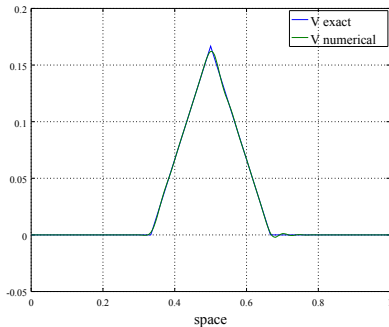
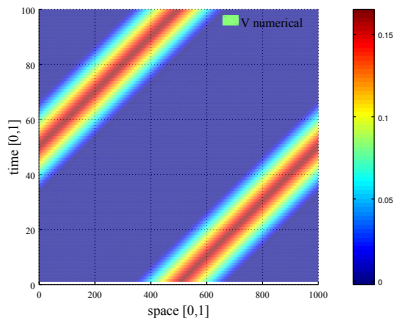


FIGURE 6: 1D Acoustic system.

Numerical velocity $v_F(x, t)$.

Periodical boundaries. $\alpha = \beta = 0.5$



IMPLEMENTATION OF THE ALGORITHM

NUMERICAL TESTS

FIGURE 7: 1D Acoustic system.

Convergence of velocity v_F in function of cell size h .
Periodical boundaries. $\alpha = \beta = 0.0$

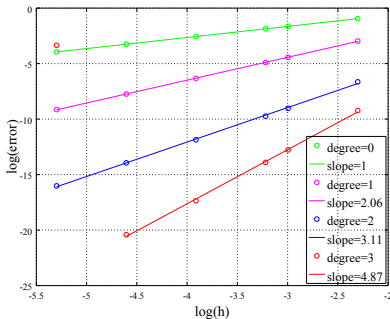
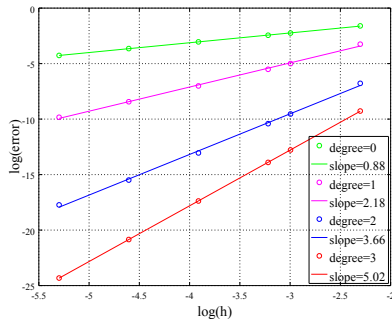


FIGURE 8: 1D Acoustic system.

Convergence of velocity v_F in function of cell size h .
Periodical boundaries. $\alpha = \beta = 0.5$



IMPLEMENTATION OF THE ALGORITHM

NUMERICAL TESTS

FIGURE 9: 1D Acoustic system.

Numerical error in function of ratio h_t/h_x .

Periodical boundaries. $\alpha = \beta = 0.5$

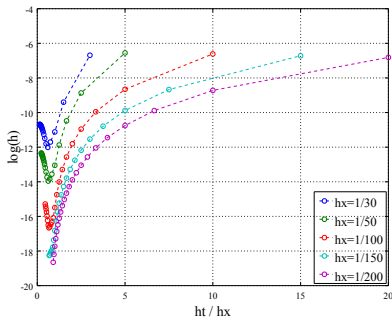
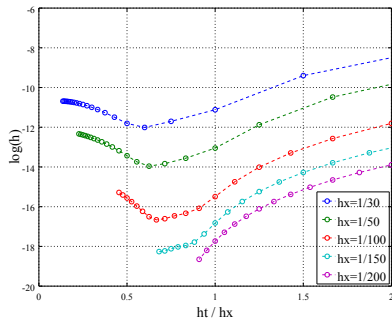


FIGURE 10: 1D Acoustic system.

Numerical error in function of ratio h_t/h_x .

Error minimum corresponds to $h_t/h_x = 1/(1 + \alpha)$



NUMERICAL RESULTS

2D SIMULATION

NUMERICAL RESULTS

2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

FIGURE 11: 2D Acoustic system.

Convergence of velocity v_F in function of cell size h

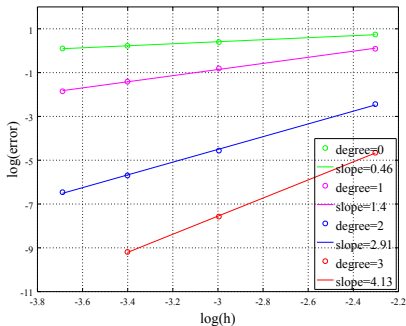
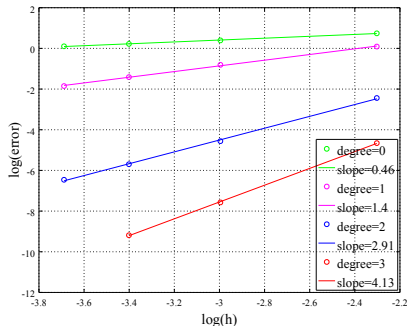


FIGURE 12: 2D Elastodynamic system.

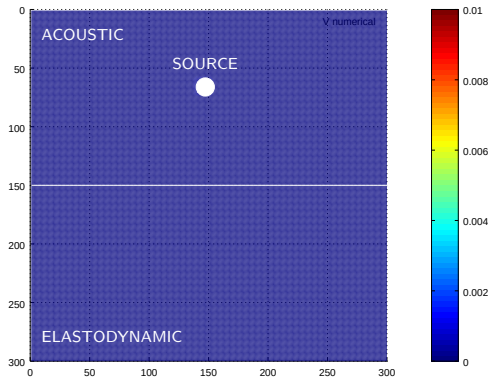
Convergence of velocity v_S in function of cell size h



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

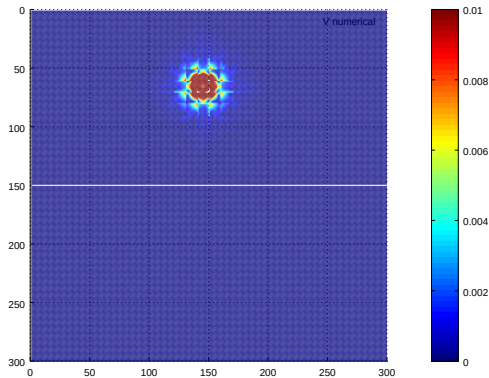
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.0)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

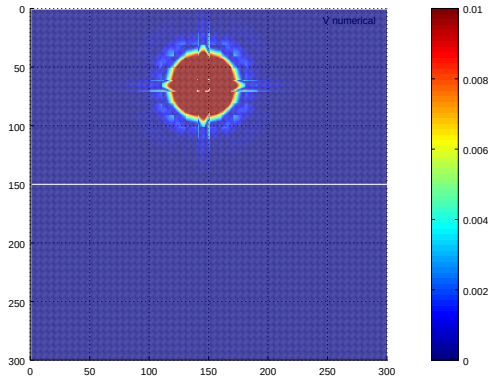
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.233)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

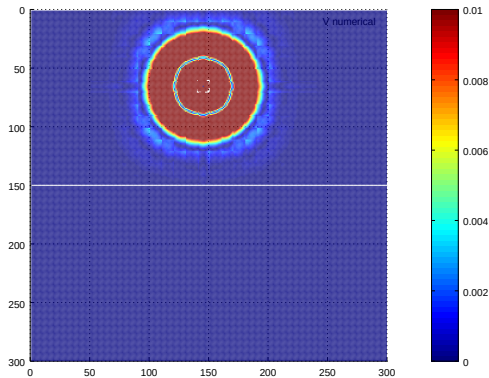
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.300)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.367)$. Dirichlet boundaries

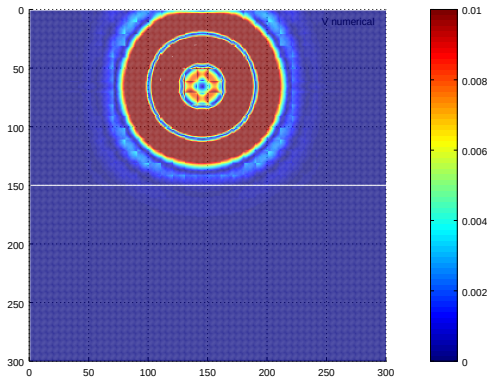


NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

FIGURE 13: 2D Elasto-acoustic system.

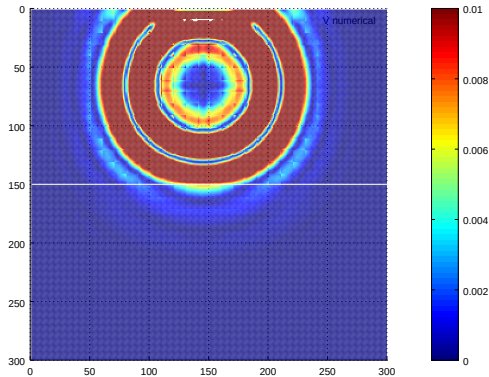
Numerical velocity $v(x, 0.433)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

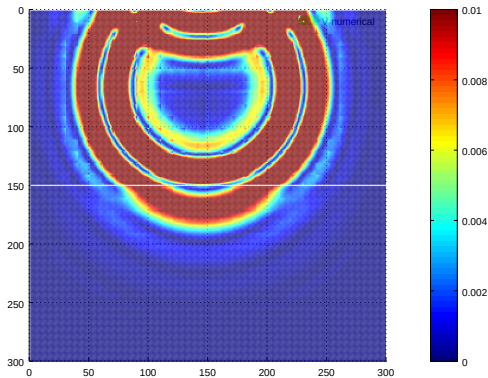
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.500)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

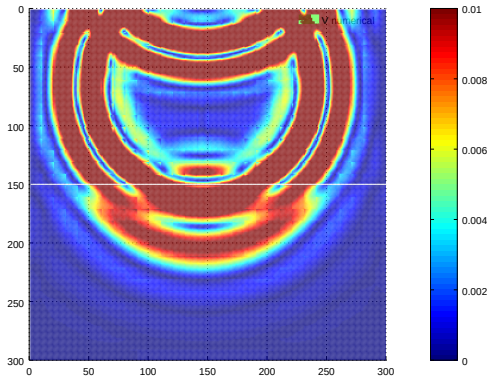
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.567)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

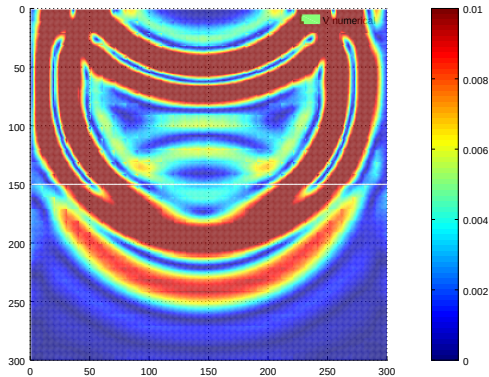
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.633)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

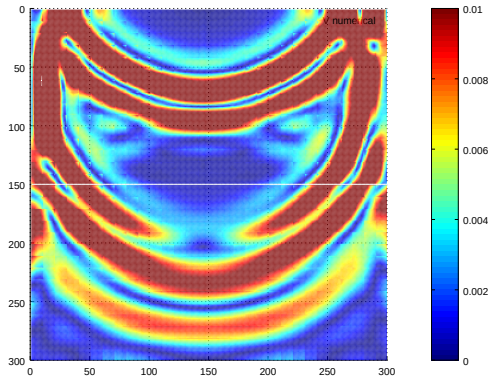
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.700)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

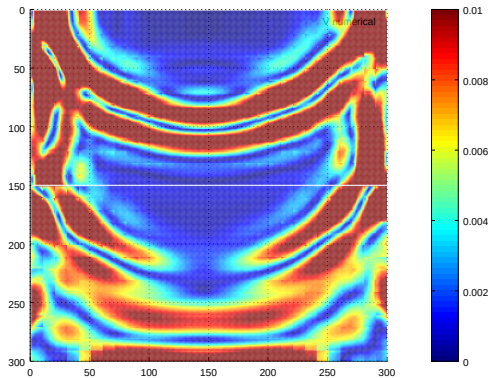
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.767)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

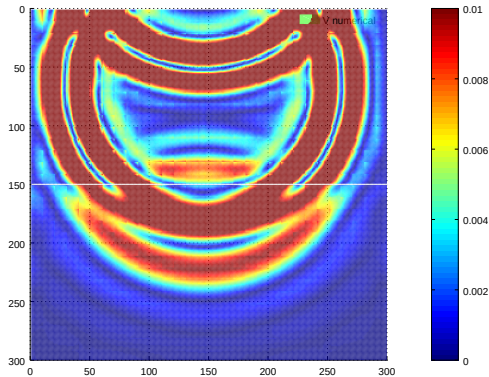
FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.833)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

FIGURE 13: 2D Elasto-acoustic system.
Numerical velocity $v(x, 0.667)$. Dirichlet boundaries



NUMERICAL RESULTS

2D ELASTO-ACOUSTIC SYSTEM

FIGURE 14: 2D Elasto-acoustic system.

Numerical and analytical* seismograms for v_x .
Source (150,75), receiver (195, 210)

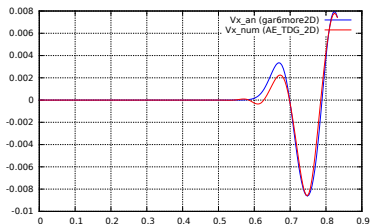
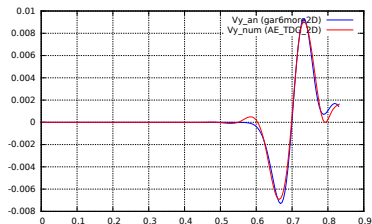


FIGURE 15: 2D Elasto-acoustic system.

Numerical and analytical* seismograms for v_y .
Source (150,75), receiver (195, 210)



* J.Diaz. *Gar6more2D*.

CONCLUSION

CONCLUSION



Implementation of the method for 2D acoustic, elastic, elasto-acoustic systems

Numerical tests, validation of the results by analytical solutions

Analytical and numerical study of efficiency of the method

Change-over between time-layers study (initial conditions)

CONCLUSION

ON-GOING WORK AND PERSPECTIVES



Change-over between time-layers study (source)

Alternative for the global matrix inversion

Boundary conditions

Hybrid Trefftz-DG + FVM coupling for the elasto-acoustics

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THANK YOU FOR ATTENTION!
