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Quantifier elimination for robot positioning with landmarks of uncertain position

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We consider a mobile robot equipped with a camera that moves in a 2D environment. We aim to compute a domain for the pose (position and orientation) $\mathbf{r} = (x, y, \theta)$ in which we are sure the robot is situated [1, 3]. The robot knows landmarks positions $X_w(x_w, y_w)$ in the world frame, and measures their projection u in the image (represented in normalized coordinates, assuming known camera calibration parameters). The perspective projection of a world point in the camera frame is given as follows:

$$\begin{cases} u &= f(\mathbf{r}, X_w) = \frac{1}{z_c} ((x_w - x) \cos(\theta) - (y_w - y) \sin(\theta)) \\ z_c &= (x_w - x) \sin(\theta) + (y_w - y) \cos(\theta) > 0 \end{cases} \quad (1)$$

Problem The problem amounts in determining the set of all poses that are consistent with 1-D image measurement and its 2-D world corresponding point which is known with a bounded error. Assuming the image measurement uncertainty is represented by the interval $[u]$ and the landmarks positions are known to be inside the box $[X_w]$, the pose domain is the solution of the following quantified set inversion [2] problem:

$$S = \{\mathbf{r} \in \mathbb{R}^2 \times [0, 2\pi], \exists X_w \in [X_w], f(\mathbf{r}, X_w) \in [u], z_c > 0\} \quad (2)$$

Since the expression of f in Eq. 1 is not well posed when z_c gets close to 0, the constraints $f(\mathbf{r}, X_w) \in [u], z_c > 0$ can be re-written as two inequalities:

$$g : \begin{cases} g_1 : (x_w - x)(\cos(\theta) - \underline{u} \sin(\theta)) - (y_w - y)(\sin(\theta) + \underline{u} \cos(\theta)) > 0 \\ g_2 : (x_w - x)(\cos(\theta) - \bar{u} \sin(\theta)) - (y_w - y)(\sin(\theta) + \bar{u} \cos(\theta)) < 0 \end{cases}$$

where \underline{u} and \bar{u} denote the lower bounds and the upper bound of $[u]$.

Quantified set inversion with a branch and bound algorithm s.a. SIVIA requires bisecting also on the quantified variables (otherwise, parts of the domain may remain undetermined). We therefore need to eliminate the quantifier (\exists) to avoid bisecting on the landmark position box $[X_w]$.

Quantifier free equations We adopt a geometric approach to perform the elimination. Let $\mathcal{C} = \{(x_w, y_w), (\underline{x}_w, \underline{y}_w), (\overline{x}_w, \overline{y}_w), (\overline{x}_w, \underline{y}_w)\}$, the set of corners formed by the box $[X_w]$. The pose is found consistent if g_1 is verified by at least one corner from \mathcal{C} , g_2 is also verified by at least one corner, and the box $[X_w]$ is not behind the camera.

$$S = \left\{ \mathbf{r} \in \mathbb{R}^2 \times [0, 2\pi], \bigvee_{X_w \in \mathcal{C}} g_1(\mathbf{r}, X_w, [u]), \bigvee_{X_w \in \mathcal{C}} g_2(\mathbf{r}, X_w, [u]), \neg \text{behind}(\mathbf{r}, X_w) \right\}$$

The first two conditions can also be written as:

$$\begin{cases} \max_{X_w \in \mathcal{C}} ((x_w - x)(\cos(\theta) - \underline{u} \sin(\theta)) - (y_w - y)(\sin(\theta) + \underline{u} \cos(\theta))) > 0 \\ \min_{X_w \in \mathcal{C}} ((x_w - x)(\cos(\theta) - \overline{u} \sin(\theta)) - (y_w - y)(\sin(\theta) + \overline{u} \cos(\theta))) < 0 \end{cases}$$

The \neg behind constraint expressing the fact that the camera is front-looking is given by:

$$\begin{cases} \cos \alpha \geq 0 \implies x < \overline{x}_w \\ \cos \alpha \leq 0 \implies x > \underline{x}_w \\ \sin \alpha \geq 0 \implies y < \overline{y}_w \\ \sin \alpha \leq 0 \implies y > \underline{y}_w \end{cases} \Leftrightarrow \begin{cases} \cos \theta - u \sin \theta < 0 \quad \vee \quad x < \overline{x}_w \\ \cos \theta - u \sin \theta > 0 \quad \vee \quad x > \underline{x}_w \\ \sin \theta + u \cos \theta > 0 \quad \vee \quad y > \overline{y}_w \\ \sin \theta + u \cos \theta < 0 \quad \vee \quad y < \underline{y}_w \end{cases}$$

with α the observed ray angle in the world frame:

Results An example with 3 uncertain landmarks is depicted in Fig. 1.

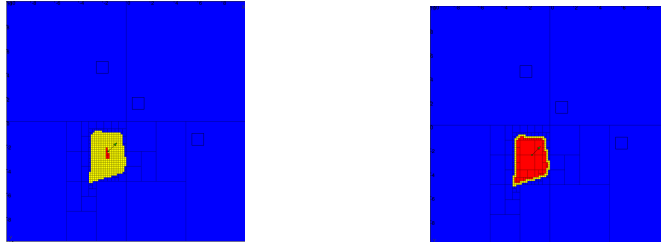


Figure 1: Pose domain in X-Y plane without and with quantifier elimination.

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