State estimation for a fish population via a nonlinear observer

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Abstract

This paper focuses on the construction of an exponential observer for an age-structured model of an exploited fish population in order to get an estimation of the number of fishes by age class. The considered model is nonlinear and involves a stock-recruitment function which is not well known. The observer that we construct is independent of this function.

Key words: discrete-time systems, estimation, nonlinear control systems, observer, population dynamics.

1 Introduction

We consider a population of exploited fish which is structured in n age classes ($n \geq 2$); under some assumptions on the population, we can represent the dynamics of the population by the following system of difference equations:

$$\left\{ \begin{array}{rcl} x_1(t+1) &=& f(\sum_{i=1}^{n} b_i x_i(t)) \\ x_2(t+1) &=& x_1(t)\exp(-M_1 - q_1 E(t)) \\ \vdots & & \vdots \\ x_n(t+1) &=& x_{n-1}(t)\exp(-M_{n-1} - q_{n-1}E(t)) \end{array} \right. \quad (1)$$

where $f : \mathbb{R}_+ \to \mathbb{R}_+$ is the stock-recruitment function. It is a continuous map such that $f(0) = 0$ and

- b_i is the number of individuals produced by individuals of the i^{th} age class;
- M_i is the natural mortality of the individuals of the i^{th} age class;
- q_i is the catchability of the individuals of the i^{th} age class;
- $E(t)$ is the fishing effort at time t and is regarded as an input.

Several authors have proposed different kind of functions f (see [1, 4, 6, 7]. The most quoted mathematical expressions of the nonlinear recruitment function f are (β being a positive parameter):

$$\begin{array}{ll} \textit{Beverton and Holt} & f(x) = \dfrac{x}{1+\beta x} \ ; \\ \textit{Ricker} & f(x) = e^{-\beta x} \ ; \\ \textit{Powerfunction} & f(x) = x^{-\beta} \ ; \\ \textit{Shepherd} & f(x) = \dfrac{x}{1+\beta x^c} \ , \ (c > 0). \end{array}$$

Our aim is to construct an observer, that is to say, an auxiliary system of difference equations whose state $z(t)$ gives an estimate of the state $x(t)$ of system (1). More precisely we shall have $\lim\limits_{t \to +\infty} (z(t) - x(t)) = 0$ with an exponential rate of convergence, i.e, there exists $\alpha < 1$ such that, for all $t \in \mathbb{N}$ and for all initial conditions $(x(0), z(0))$, one has

$$\mid z(t) - x(t) \mid \leq \alpha^t \mid z(0) - x(0) \mid .$$

We wish to do this job without using a precise expression of the function f: we shall use only some minimal assumption on this function. One can notice that although the theory of observer design for linear systems is a well developped field, its analogous part for nonlinear systems is an intriguing subject, and is still receiving considerable attention by many researchers. The design of state observers for discret-time systems has been studied in many articles. One can cite, for example, [2, 3] where the problem has been addressed for general systems but the given observers are only local and here we rae interested in the construction of a global observer. In [5], the observer design is done in the context of solving simultaneous nonlinear equations by using Newton's algorithm. Here we give a new procedure that simplifies the observer design for the considered system.

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2 Design of an exponential observer

System (1) can be rewritten with standard control notations:

$$\left\{\begin{array}{rcl} x(t+1) & = & F\Big(x(t),u(t)\Big) \ ; \\ y(t) & = & h\Big(x(t),u(t)\Big) \ ; \end{array}\right. \tag{2}$$

where $x(t)\in\Omega=\mathbb{R}_+^n$, is the state of the system, $u(t)=E(t)\in\mathbb{R}_+$, is the control (here it is the fishing effort) and $y(t)\in\mathbb{R}_+$, is the measurable output of the system.

We assume that we can measure the quantity of catched fishes which is expressed by :

$$y(t) \;=\; \sum_{i=1}^{n-1} x_i(t)\,e^{-M_i}(1-\exp(-q_iE(t))).$$

We suppose that the fishing effort is subject to the constraints:

$$0<E_{\mathrm{m}}\le E(t)\le E_{\mathrm{M}}$$

(this assumption is reasonable because if we don't catch any fish, we can't get any information on the population).

Below, we expose a system that we claim to be an observer for (1), for simplicity of exposition, the construction is made in the 3-dimensional case (3 age classes).

$$\begin{cases} z_1(t+1)=\dfrac{y(t+1)-(e^{-M_2}-v_2(t+1))v_1(t)z_1(t)}{e^{-M_1}-v_1(t+1)} \\ z_2(t+1)=v_1(t)z_1(t) \\ z_3(t+1)=v_2(t)z_2(t) \end{cases} \tag{3}$$

where we put $v_i(t)=\exp(-M_i-q_iE(t))$.

Proposition 2.1 *Assume that* $E_{\mathrm{m}}\ge \ln 2/q_1$, *then system (3) is an observer for system (1).*

Proof. Letting $e(t)=x(t)-z(t)$, we have:

$$\left\{\begin{array}{rcl} e_1(t+1) & = & x_1(t+1)-z_1(t+1) \\ & = & -\dfrac{e^{-M_2}-v_2(t+1)}{e^{-M_1}-v_1(t+1)}v_1(t)e_1(t) \\ e_2(t+1) & = & v_1(t)e_1(t) \\ e_3(t+1) & = & v_2(t)e_2(t) \end{array}\right. \tag{4}$$

thus:

$$\begin{array}{rcl} \mid e_1(t+1)\mid & = & \dfrac{1-e^{-q_2E(t+1)}}{1-e^{-q_1E(t+1)}}v_1(t)\mid e_1(t)\mid \\ & \le & \dfrac{1-e^{-q_2E_{\mathrm{M}}}}{1-e^{-q_1E_{\mathrm{m}}}}e^{-M_2}e^{-M_1-q_1E_{\mathrm{m}}}\mid e_1(t)\mid \end{array}$$

now since $E_{\mathrm{m}}\ge\ln 2/q_1$, $e^{-q_1E_{\mathrm{m}}}/(1-e^{-q_1E_{\mathrm{m}}})\le 1$ which implies

$$\mid e_1(t+1)\mid \;\le\; \alpha\mid e_1(t)\mid$$

where $\alpha=e^{-M_2}(1-e^{-q_2E_{\mathrm{M}}})<1$. This proves the exponential convergence of $e_1(t)$ to zero and from (4), it is clear that the same is true for $e_2(t)$ and $e_3(t)$.

Remark. There is an alternative proof which uses the following candidate Lyapunov function $V(e)=e^TPe$, with:

$$P=\begin{pmatrix} \frac{3}{1-\alpha^2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Where, $\alpha=e^{-M_2}(1-e^{-q_2E_{\mathrm{M}}})<1$.

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