A simple observer for nonlinear discrete-time systems, application to a population dynamics model

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Abstract

A constructive dead-beat observer design method for nonlinear discrete system is given and illustrated with an application to a model of population dynamics.

Key words: difference equations, nonlinear systems, observer, population dynamics.

1 Introduction

The design of state feedback needs to have the system states available. While in some cases this can be achieved by a direct measurement, in general either the complexity required to perform accurate measurement or the very nature of the system becomes a hindrance to such an approach; this is particularly true for the biological systems.
A common solution to this problem is to design an auxiliary system, called "state observer", which gives an estimate of the true state using only the input and the output of the original systeme.
Although the theory of observers design for linear systems is a well developped field, its analogous part for nonlinear systems is an intriguing subject, and still receiving considerable attention by many researchers.
In this paper, we investigate the design of a state observer for a nonlinear discrete system in the form:

$$\left\{\begin{array}{rcl} x_1(k+1) & = & x_2(k) \\ x_2(k+1) & = & x_3(k) \\ \vdots & & \vdots \\ x_n(k+1) & = & f\Big(x_1(k),\dots,x_n(k)\Big) \\ y(k) & = & x_1(k) \end{array}\right. \quad (1)$$

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where $x(k)=\Big(x_1(k),\dots,x_n(k)\Big)\in \mathbb{R}^n$ is the state of the system at time k, $y(k)\in\mathbb{R}$ is the measurable output of the system and f is a smooth function defined on $\mathbb{R}^n$.
This form of system is the analogous of the canonical form for continuous systems of Gauthier-Hammouri-Othman (see [3]) and can be obtained, after diffeomorphism, from general nonlinear discrete systems under some conditions on the observability. The problem of obtaining canonical forms for such systems has been investigated by Nijmeier in [6]. Moreover, notice that some systems naturally arise in form (1), this is the case for some biological systems as we will see later.

2 Main result Design of the observer

The design of state observers for discret-time systems has been studied in many articles. In [1, 7], the problem has been addressed for more general systems but the result is local. In [5], the observer design is done in the context of solving simultaneous nonlinear equations by using Newton's algorithm. Here we give a new and simple procedure that allows to design a dead-beat observer for system (1).
To system (1) we associate the following dynamical system:

$$\left\{\begin{array}{rcl} z_1(k+1) & = & z_2(k) \\ z_2(k+1) & = & z_3(k) \\ \vdots & & \vdots \\ z_n(k+1) & = & f\Big(y(k),w_2(k),\dots,w_n(k)\Big) \end{array}\right. \quad (2)$$

where $y(t)$ is the output of sytem (1) at time t and the w_i are defined as follows

$$\left\{\begin{array}{l} w_2(k) \quad = f\Big(y(k-n+1),\dots,y(k)\Big) \\ w_{l+1}(k) = f\Big(y(k-n+l),\dots,y(k),w_2(k),..,w_l(k)\Big) \\ \qquad\qquad \text{for } l=2,\dots,n-1 \end{array}\right.$$

Theorem 2.1 *The dynamical system (2) is a dead-beat observer for system (1)*

Sketch of the proof. Reasoning by induction, it is easily proven that $w_l(k) = x_l(k)$, from which it follows that $z_n(k+1) = x_n(k+1)$ for all $k \geq n-1$. From this equality and the form of system (2), we obviously have $z_l(k) = x_l(k)$ for all $l = 1, \ldots, n$ and all $k \geq n-1$ which proves that (2) is a dead-beat observer for (1).

3 Application to a model of population dynamics

We consider a population which has a well defined age structure. Let $N_i(k)$, for $i = 1, 2, \ldots, n-1$, be the number of young individuals of age i in the population at time k and $N_n(k)$ is number of mature and reproductive individuals (of age n and over) at time k. Under some assumptions on the population, the dynamics of the population can be represented by the following discrete-time system (see [2]):

$$\left\{\begin{array}{lcl} N_1(k+1) & = & F\Big(N_n(k)\Big)N_n(k) \\ N_2(k+1) & = & S_1 N_1(k) \\ \vdots & & \vdots \\ N_n(k+1) & = & S_{n-1}N_{n-1}(k) + S_n N_n(k). \end{array}\right. \quad (3)$$

Where S_i, for $i = 1, \ldots, n-1$, is the survival rate of individuals of age i and S_n is the survival rate of reproductive individuals of age n and over. $F\left(N_n(k)\right)$ is the density dependent fecundity function of reproductive individuals at time k. The function F is ,in general, nonlinear. For fish population, many expressions have been proposed. For instance (see [4]):
$F(N) = \frac{N}{1+\beta N}$, $\beta > 0$ in the Beverton and Holt model
$F(N) = e^{-\beta N}$, $\beta > 0$, $c > 0$ in the Ricker model.
$F(N) = \frac{N}{1+\beta N^c}$, $\beta > 0$ in The Shepherd model.
In general it is not possible to measure all the variables $N_i(k)$ online. There is only partial information on the population which is available. This information can be written, with standard control theory notations, $y(k) = h\left(N(k)\right)$ and will be called the output of the system (3). We shall suppose that the population considered is an exploited fish population and that we measure the quantity of catched mature fishes so that we have $y(k) = q_n N_n(k)$, where q_n is the catchability of the mature individuals. This parameter is supposed to be known.
We shall show that the result of Section 2 can apply in order to construct an observer that can be able to give an estimation of the variables $N_i(k)$, for $i = 1, \ldots, n$. For the sake of simplicity, the construction is made for 3 age classes (The construction is the same for arbitrary number of age classes but the calculus are longer). We begin by making the following change of coordinates: $x_1 = y = q_3 N_3$, $x_2 = q_3(S_2 N_2 + S_3 N_3)$, $x_3 = q_3(S_1 S_2 N_1 + S_2 S_3 N_2 + S_3^2 N_3)$. With the new coordinates, system (3) is given by:

$$\left\{\begin{array}{ll} x_1(k+1) = x_2(k) & \\ x_2(k+1) = x_3(k) & \\ x_3(k+1) = q_3 S_3 x_3(k) + S_1 S_2 x_1(k) F\left(\frac{x_1(k)}{q_3}\right) & \\ \qquad\qquad = f\Big(x_1(k), x_2(k), x_3(k)\Big) & \\ y(k) \qquad = x_1(k). & \end{array}\right. \quad (4)$$

This system has the same form as system (1). Hence, if we apply the formula (2) then we get the following dead-beat observer for (4):

$$\left\{\begin{array}{lcl} z_1(k+1) & = & z_2(k) \\ z_2(k+1) & = & z_3(k) \\ z_3(k+1) & = & S_3\, y(k) + S_1 S_2\, y(k) F\left(\frac{y(k)}{q_3}\right) \\ & & + S_1 S_2 S_3\, y(k-1) F\left(\frac{y(k-1)}{q_3}\right) \\ & & + \; S_1 S_2 S_3^2\, y(k-2) F\left(\frac{y(k-1)}{q_3}\right). \end{array}\right. \quad (5)$$

From this we deduce the following observer for the original system :

$$\left\{\begin{array}{l} \hat{N}_1(k+1) = -S_3 \hat{N}_1(k) - \frac{S_3^2}{S_1}\hat{N}_2(k) - \frac{S_3^3}{S_1 S_2}\hat{N}_3(k) \\ + \frac{S_3}{q S_1 S_2}\, y(k) + \frac{1}{q}\, y(k) F\left(\frac{y(k)}{q_3}\right) \\ + \frac{S_3}{q}\, y(k-1) F\left(\frac{y(k-1)}{q_3}\right) + \frac{S_3^2}{q}\, y(k-2) F\left(\frac{y(k-1)}{q_3}\right) \\ \hat{N}_2(k+1) = S_1 \hat{N}_1(k) \\ \hat{N}_3(k+1) = S_2 \hat{N}_2(k) + S_3 \hat{N}_3(k). \end{array}\right.$$

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