A convex optimisation approach to Youla's broadband matching theory

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A CONVEX OPTIMISATION APPROACH TO YOULA'S BROADBAND MATCHING THEORY

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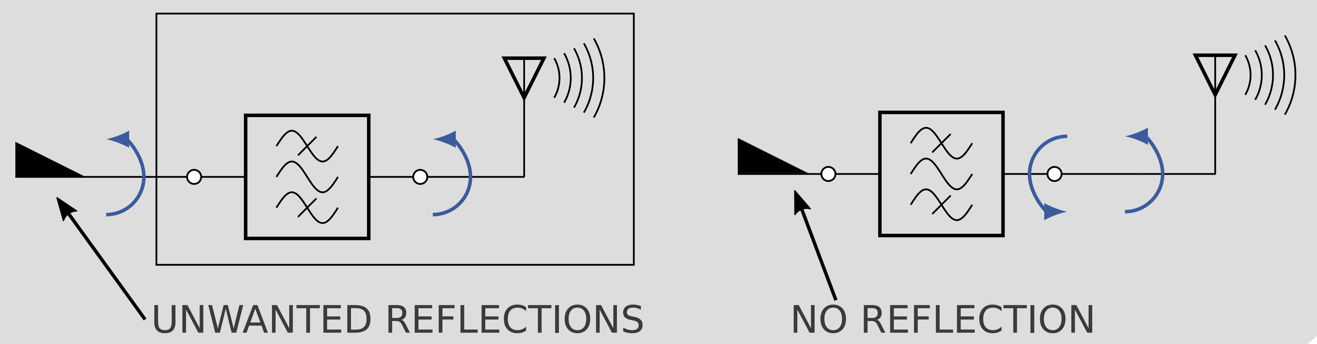
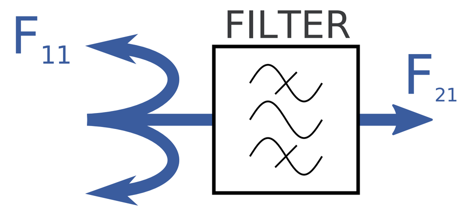
INTRODUCTION

UNMATCHED LOAD
Reflection of the load goes through the filter

MATCHING FILTER
Filter input signal & reject reflections



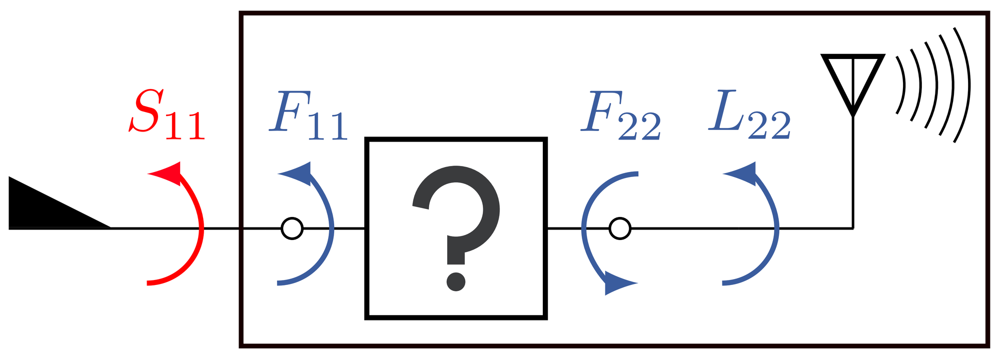


Belevitch model

$$F = \frac{1}{q}\begin{pmatrix} \epsilon p^* & -\epsilon r^* \\ r & p \end{pmatrix}$$

$$qq^* = pp^* + rr^*$$

CLASSICAL APPROACH

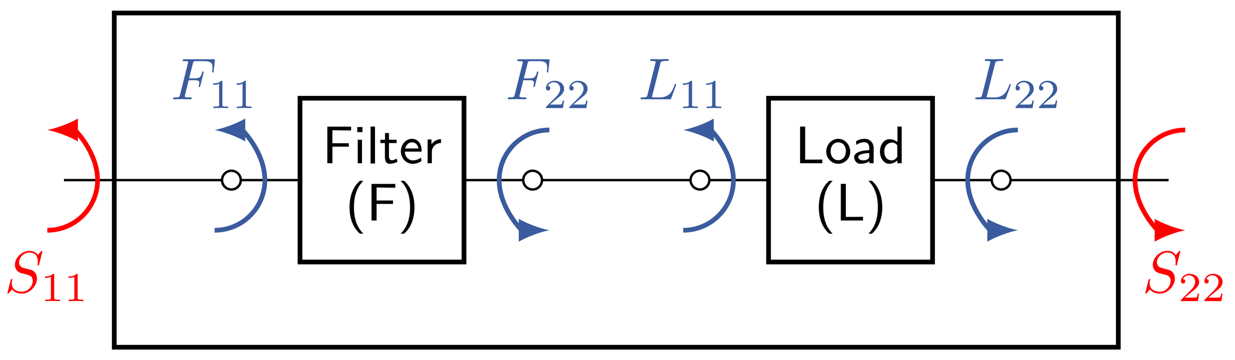


matching filter optimisation → **non convex** techniques → **no optimality** guaranteed

CONCEPT: OVERALL DESIGN + DE-EMBEDDING

YOULA'S DE-EMBEDDING CONDITIONS

Synthesize global system instead of matching filter



CONCEPT
Darlington equivalent
+
design: global system
+
load de-embbeding

Interpolation conditions at load transmission zeros allow **de-embbeding**

$$L_{21}(\alpha_i)L_{12}(\alpha_i) = 0 \quad 1 \le i \le M \qquad S_{22} = L_{22} + \frac{L_{12}F_{22}L_{21}}{1 - L_{11}F_{22}} \Rightarrow S_{22}(\alpha_i) = L_{22}(\alpha_i)$$

BRINGING OPTIMISATION INTO CONTEXT

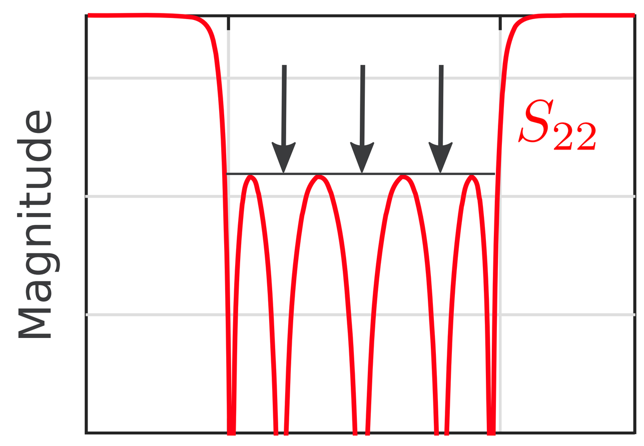
$S_{22} = UB$: U outer factor, B inner factor

Belevitch form of U:

$$|U_P(\omega)|^2 = \frac{P(\omega)}{P(\omega) + R(\omega)} \qquad P = pp^* \quad R = rr^*$$

$P, R \in \mathbb{P}^+_{2N}$ R fixed, $R(\alpha_i) = 0$

MINIMISE REFLECTION



Optimisation problem:

$$\min_{P \in \mathbb{P}^+_{2N}} \max_{\omega \in \mathbb{I}} |U_P(\omega)| \quad \text{s.t.}$$

$$\exists B : U_P(\alpha_i)B(\alpha_i) = L_{22}(\alpha_i)$$

relaxed set of S_{22} functions: **degree** can **increase** by deg(B)

PRACTICAL USE

Choice of global response: **Butterword / Tchebyshev**

Interpolation conditions on S_{22}:
Rigid approach: **Not optimisation** friendly

EXISTENCE OF FACTOR B

Given U_P, does B inner exist s.t $B(\alpha_i) = \frac{L_{22}(\alpha_i)}{U_P(\alpha_i)}$?

THEORETICAL RESULTS & NUMERICAL IMPLEMENTATION

NEVANLINNA-PICK INTERPOLATION

Existence of inner function *B* satisfying interpolation conditions
Practical implementation of Youla's characterisation

$$\exists B \text{ s.t. } B(\alpha_i) = \gamma_i \Leftrightarrow \Delta(P) \succeq 0$$

$$\Delta(P)_{i,k} = \frac{1 - \gamma_i \overline{\gamma_k}}{j(\alpha_i - \overline{\alpha_k})} \qquad \gamma_i = \frac{L_{22}(\alpha_i)}{U_P(\alpha_i)}$$

$$\min_{P \in \mathbb{P}^+_{2N}} \max_{\omega \in \mathbb{I}} \frac{P(\omega)}{R(\omega)} : \Delta(P) \succeq 0$$

CONVEX OPTIMISATION PROBLEM

Modulus of *Up* is obtained from the filtering function *P/R*
SDP with constrains on positive polynomials

define $\Gamma \ge \frac{P(\omega)}{R(\omega)}, \quad \omega \in \mathbb{I}$

$$\min_{P \in \mathbb{P}^+_{2N}} \Gamma : \begin{array}{l} \Delta(P) \succeq 0 \\ P(\omega) \le \Gamma R(\omega) \quad \omega \in \mathbb{I} \end{array}$$

$\Delta(P)$: CONCAVE OPERATOR $\Longrightarrow \{P : \Delta(P) \succeq 0\}$ CONVEX SET

$$\Delta((1-\lambda)P_1 + \lambda P_2) \succeq (1-\lambda)\Delta(P_1) + \lambda\Delta(P_2)$$

Allow handling of Pick matrix by augmented lagrangian techniques

POSITIVITY: GRAM MATRIX PARAMETRISATION

$$P = P(\Theta_P), \quad P \in \mathbb{P}^+_{2N} \Longleftrightarrow \Theta_P \succeq 0$$

$$Q(\omega)_{a \le \omega \le b} \ge 0 \Longleftrightarrow Q = F - (\omega - a)(\omega - b)G, \quad F, G \in \mathbb{P}^+_{2N}$$

AT THE OPTIMUM $\Delta(P)$ IS SINGULAR AND deg(B)<M

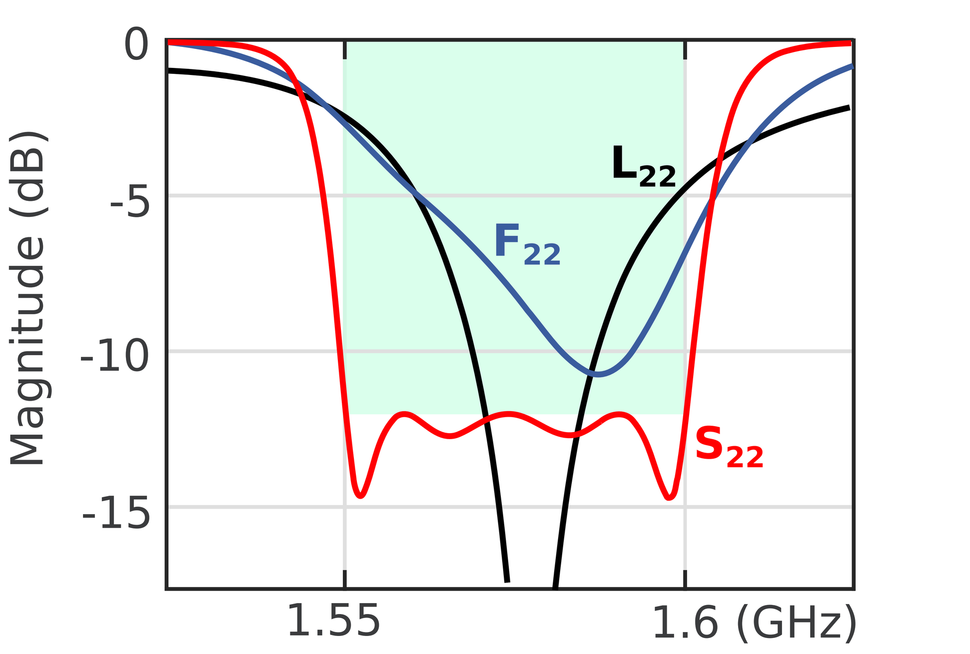
$$\deg(U_P^{opt}) \le \deg(S_{22}^{opt}) \le \deg(U_P^{opt}) + M - 1$$

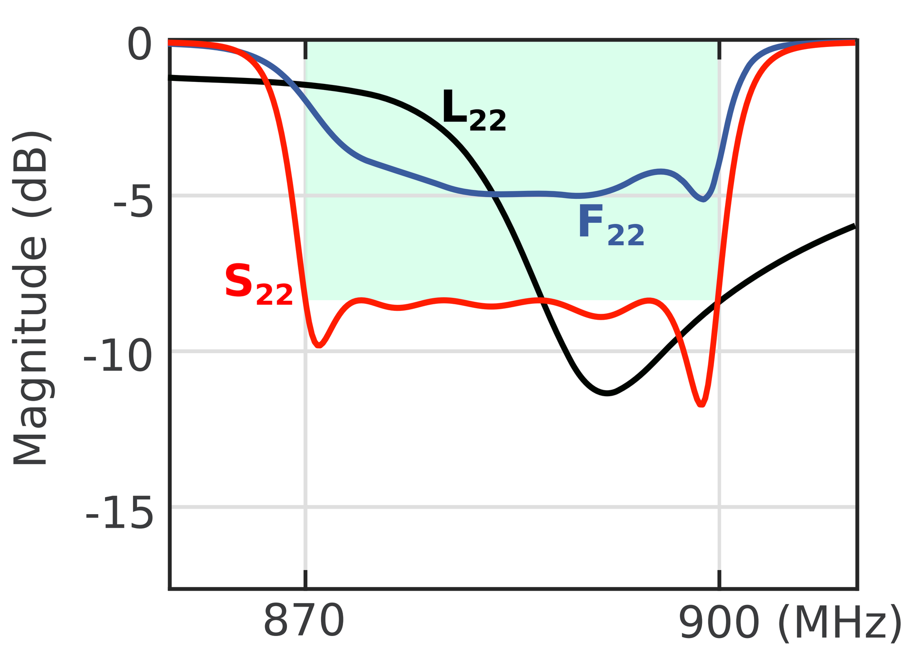
A NON-LINEAR SEMI-DEFINITE PROGRAM

$$\min_{\Theta_P, \Theta_F, \Theta_G, \Gamma} \Gamma \quad \text{s.t.} \quad \begin{array}{l} \Theta_P, \Theta_F, \Theta_G, \Delta[P(\Theta_P)] \succeq 0 \\ Q(\Theta_F, \Theta_G) = \Gamma R - P(\Theta_P) \end{array}$$

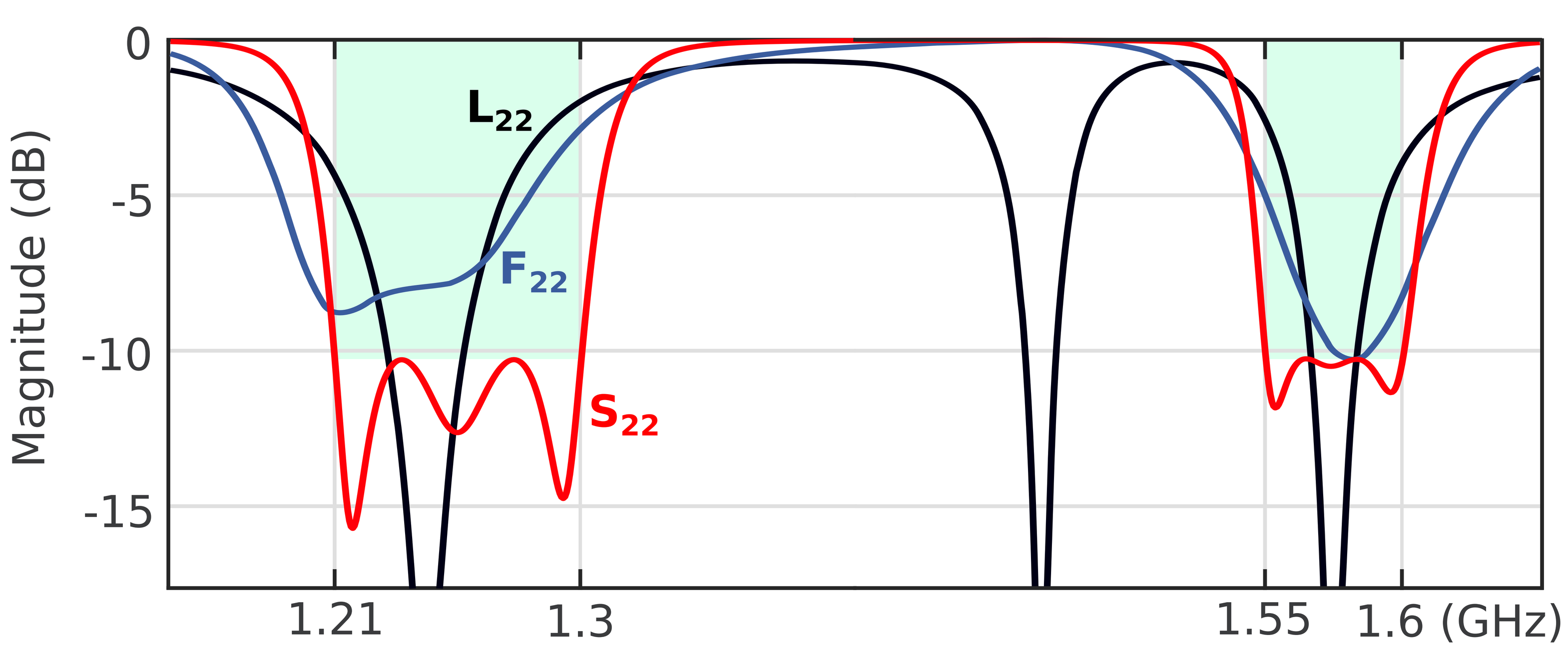
RESULTS

SINGLE-INTERVAL RESULTS





DUAL-INTERVAL RESULTS



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