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Principles for a Judgement Editor Based on BDD

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Abstract

We describe the theoretical principles that underlie the design of a software tool which could be used by judges for writing judgements and for making decisions about litigations. The tool is based on Binary Decision Diagrams (BDD), which are graphical representations of truth–valued functions associated to propositional formulas. Given a specific litigation, the tool asks questions to the judge; each question is represented by a propositional atom. Their answers, true or false, allow to evaluate the truth value of the formula which encodes the overall recommendation of the software about the litigation. Our approach combines some sort of 'theoretical' or 'legal' reasoning dealing with the core of the litigation itself together with some sort of 'procedural' reasoning dealing with the protocol that has to be followed by the judge during the trial: some questions or group of questions must necessarily be examined and sometimes in a specific order. That is why we consider extensions of BDD called Multi-BDD. They are BDD with multiple entries corresponding to the different specific issues that must necessarily be addressed by the judge during the trial. We illustrate our ideas on a case study dealing with French union trade elections, an example that has been used throughout a project with the French Cour de cassation.

1 Introduction

Law has so much impact on our daily lives that any mistake related to its application must be avoided. Law must be transparent, accountable and understandable by anybody it can affect. Clearly, computer science can be instrumental to reach these goals, in particular because French law (stemming from Roman law) is organized in a systematic manner and lends itself easily to a formalization under the form of rules and principles which are very common in the models used

in computer science. Logic, sometimes viewed as the "calculus of computer science" [17, 29], is a natural theoretical background to address these issues. As it turns out, there is already an important body of work in the field of logic and law (see [9, 24, 16] for details and pointers).

1.1 A Status Quo in Logic and Law

Originally, logic was intended to be used for the representation of law in a clear and unambiguous manner. On top of this representation, some kind of reasoning could then take place to infer some information. Sergot and Kowalski were pionneers in the use of logic programming in that field, which they applied to the formalization of the British Nationality Act [25]. However, they encountered difficulties with the Prolog treatment of negation as failure and with the lack of deontic operators [18]. In Prolog, something is said to be false if it is not known or cannot be inferred to be true. Hence, if a program cannot show that an infant was born in the UK, it assumes that it was not. But more generally, researchers realized that several aspects of the law which can not be dealt with standard (Fregean) logic had to be taken into account, such as the need to handle exceptions, conflicting rules, vagueness, open texture (i.e. the failure of natural languages to determine future usage of terms), counterfactual conditionals and the possibility of rational disagreement. This led to the development of a broader conception of logic, which is in fact a special case of a more general and relatively recent development of logic where the central focus is the study of rational agency and intelligent interaction [21].

Representation of law. Among the logical formalisms used to represent law, deontic logic provides formal tools for the clarification of the meaning of normative terms such as 'may', 'must' and 'shall', which play a central role in specifying the legal relations between agents (which can be human beings or machines). Therefore, it has been used in the analysis of law [18] and in the area of automated contract management. Other notions which play an important role in law and legal reasoning were also analysed logically, such as the notion of power, as involved in sentences such as "the president has the power to declare a state of emergency". Normative systems propounded by Alchourron and Bulygin were another influential approach to represent legal systems [1]. A normative system is a set of norms, which are pairs of the form $\langle condition, consequence \rangle$: if the condition holds then the consequent *must* hold. Unlike formulas of deontic logic, they do not bear truth values.

Several structural features of legal regulations, such as the use of exception, the use of hierarchies of legislation to resolve conflicts, cross references to other parts of the legislation, deeming provisions, conditions under which the legislation is applicable, and conditions for the validity of particular norms, led to the use of non-monotonic logics [23]. These are logics where the consequence relation is not monotonic, meaning that adding a formula to a theory does not necessarily produce an increase of its set of consequences. However, these formalisms proved inadequate to provide a general means of conflict resolution. These conflicts are sometimes due to conflicting interpretations of the law and get even more difficult to handle in the context of *stare decisis* (a legal principle by which judges are obliged to respect the precedents established by prior decisions). These difficulties led to a shift of focus from the modes of representation to the modes of reasoning.

Reasoning about law. Law and its practice are subject to different kinds of reasoning:

- *Case-based reasoning* uses considerations about precedent legal cases to show how they justify particular outcomes in a new case (following the *stare decisis* principle). The problem is then to map appropriately the precedent cases to the new case. Several models have

been developped and logically formalized, notably by McCarty [20]. This problem involves the classification of the facts of a case under legal concepts and the interpretation of these legal concepts.

- *Practical and teleological reasoning* deals with the reasoning involved in the justification of choices that the arbiter has to make in some legal decisions. These justifications should be in line with the underlying purpose of the law. This involves to be able to derive normative consequences from the classification of facts and the interpretation of legal concepts.

- *Evidential reasoning* is the kind of reasoning that occurs when judges strive to establish the facts on the basis of evidences.

Different kinds of logical formalisms were developped to address these different kinds of reasonings, and in particular numerous works resorted to argumentation theory. However, legal reasoning mostly arises in the conduct of a dispute which is regulated by a particular procedure. The outcome of this dispute does not only depend on facts and a body of law, but also on the procedure itself: whether it is a criminal proceedings or a civil proceedings, to which party is assigned the burden of proof in this procedure, etc. A number of dialogue games models of legal procedure have been produced in the last 20 years [22].

All this said, a striking particularity of most of the works which have been pursued at the interface of logic and law in the last decades is that they were mostly driven by theoretical considerations and without much interaction with jurists and lawyers. Arguably, this work did not really catch the attention of the lawyers and jurists. In particular, they did not change the way they work or their actual practice of the law, except maybe for the adoption of large and online databases such as LexisNexis or Legifrance[1] (based on standards for legal documents such as LegalRuleXML) and the use of knowledge management systems [10]. This theoretical work did not seem for jurists to answer an actual need and it was somehow remote from their daily preoccupations, although the researchers could sense the potential and the important applicability of their work in the practice of the law.

1.2 Current Problems in the Application of Law in France

The work that we are going to describe in this article stems from actual problems expressed to us by jurists of the Cour de cassation, the highest legal institution in France.[2] These problems are in fact not specific to France. The application of law is plagued with a series of problems which are difficult to overcome with the standard and present methods employed by jurists [19, 5].[3] First, the increasing diversity and complexity of legal texts and jurisprudence makes the work of jurists (and lawyers) very difficult to pursue nowadays. This complexity appears not only at the local or national level but is sometimes worsened by its interaction with the european level, and sometimes even the international level. Second, legal texts and jurisprudence are changing at a high pace in some areas and it is difficult for jurists (and lawyers) to cope and keep up-to-date with the current legislation and regulations. Third, there is a lack of uniformity in the application of law, depending on the geographical part in which trials take place, on the local customs and

[1] See `www.lexisnexis.com` and `https://www.legifrance.gouv.fr/`.

[2] The main role of the "Cour de cassation" is to check that the law is applied correctly and uniformly in France, mainly from a formal point of view. The 'Cour de cassation' is also the last legal institution to which French citizens can resort to in case they want to 'break' ('casser' in French) a legal decision that concerns them. In that case, the decision of the Cour de cassation cannot be revoked.

[3] In some countries subject to common law, like the United Kingdom, such problems are worsen by the fact that some legal decisions are not made by professional jurists [5].

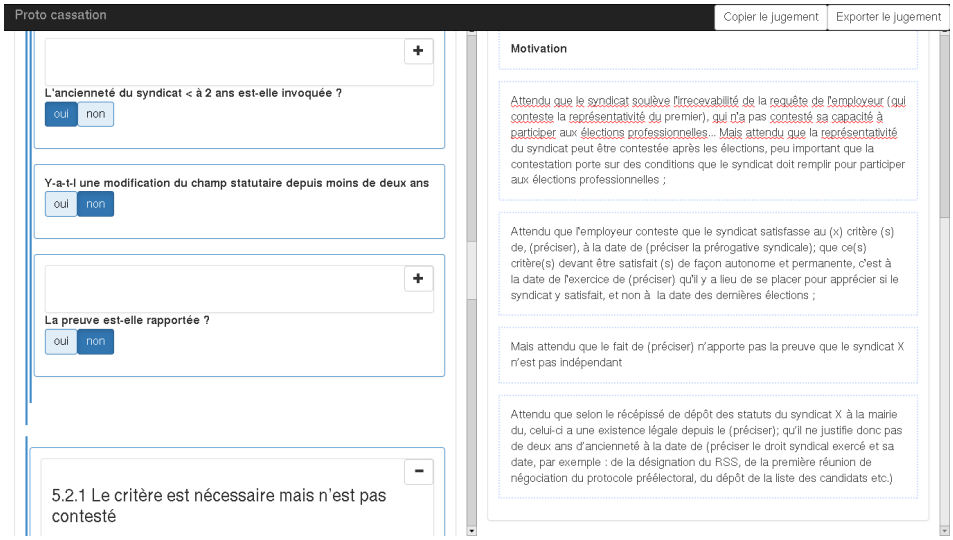


Figure 1: Screenshot of the graphical user interface

sometimes the personnality of the judges, and more generally on the specific political or social context in which a legal decision is taken. Altogether, these three problems call for a new kind of solution.

1.3 A New Kind of Solution: a Software Assistant

The solution propounded by the Cour de cassation is to make use of a software, whose ultimate role is to help judges write a judgement and take better and well-informed decisions thanks to a series of questions to which she/he has to answer. These questions would be backed up by the corresponding legal texts and jurisprudence.

This software assistant would indeed be a solution to the problems mentionned above. First, it would unify and uniformize the application of law in France: the kind of reasoning proposed by the software to sort out a given (type of) litigation could be controlled by the Cour de cassation and it could also be the same in every jurisdiction of France. Second, like any software, it could take into account the evolution of legal texts and jurisprudence to update the different kinds of reasoning and therefore cope with the increasing complexity of law. Third, its easy access to a large and up-to-date knowledge base comprising the current legal texts and jurisprudence would increase the chance for the judge to make well–informed decisions.

The graphical user interface (GUI) of this software is depicted in Figure 1. On the left hand side of the GUI, a guide for reasoning consisting of a series of questions is displayed. These questions and their underlying reasoning are backed up by legal texts and jursiprudence to which the user can have access whenever she/he wants. On the right hand side of the GUI, a judgement is produced automatically as the user answers the questions. The user can modify at any time the judgement produced and can also have an alternative graphical representation of the web of questions to which she/he has to answer on the left hand side.

1.4 Structure of the Article

The article is organized as follows. In Section 2, we recall the basics of propositional logic and BDD. In Section 3, we extend propositional logic with the examination operator $!\varphi$ and we provide a semantics to this operator by means of Multi-BDD. In Section 4, we consider as case study the problems of determining whether an association of employees in a firm can indeed be considered ('qualified') as a trade union. In Section 5, we show how the various algorithms that have been designed for BDD can be used to solve and address specific kinds of legal issues that arise in the practice of the law. In Section 6, we describe the prototype that we have implemented during our project with the Cour de cassation. Finally, we conclude in Section 7.

2 Propositional logic and BDD

In this section, we recall the basics of propositional logic and Binary Decision Diagrams (BDD for short, [11]) and we show how they are related to each other. BDD can be viewed as an operational semantics of propositional logic and it is this *operational* feature that will play a role in the legal context. Indeed, it will allow us to represent the *procedural* aspect of the practice of law (during a trial especially).

2.1 Propositional Logic

In the sequel, $\mathbb{P}$ is a set of *atoms* (propositional letters) denoted $p, q, r, \ldots$ and T and F are two symbols called *truth values* standing for True and False.

Definition 1 (Propositional language $\mathcal{L}$)**.** The language $\mathcal{L}$ is the set that contains $\mathbb{P}$ and such that

- if $\varphi, \psi \in \mathcal{L}$, then $\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi) \in \mathcal{L}$;
- $\mathcal{L}$ contains no more formulas.

We introduce the following abbreviations: $\varphi \leftrightarrow \psi \triangleq (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$, $\top \triangleq p \vee \neg p$, $\bot \triangleq p \wedge \neg p$ (for some $p \in \mathbb{P}$). The formula $\varphi[p \backslash \psi]$ denotes the formula φ where the atom p is uniformly substituted with ψ.

The intuitive reading of the formulas is as follows: $\neg\varphi$: "φ does not hold"; $\varphi \wedge \psi$: "φ holds and ψ holds"; $\varphi \vee \psi$: "φ holds or ψ holds"; $\varphi \rightarrow \psi$: "If φ holds then ψ holds".

Definition 2 (Interpretation)**.** A *total (partial) interpretation* is a total (resp. partial) function $\mathcal{I} : \mathbb{P} \mapsto \{T, F\}$ that assigns one of the *truth values* T or F to *every* (resp. *some* of the) atoms in $\mathbb{P}$. The set of total interpretations is denoted $\mathcal{C}$ and the set of partial interpretations is denoted $\mathcal{C}^p$. Note that $\mathcal{C} \subseteq \mathcal{C}^p$. If $\mathcal{I} \in \mathcal{C}^p$, then $Ext(\mathcal{I})$ is the set of *total* interpretations extending the interpretation $\mathcal{I}$, that is, for all $\mathcal{I}' \in Ext(\mathcal{I})$, for all $p \in \mathbb{P}$ such that $\mathcal{I}(p)$ is defined, we have that $\mathcal{I}(p) = \mathcal{I}'(p)$.

We can extend the domain of an interpretation function from the set of atoms to the set of all formulas of $\mathcal{L}$. This extension is inductively defined by the truth table given in Figure 2. If E is a set of interpretations, we say that a formula φ of $\mathcal{L}$ is *valid on* E when for all $\mathcal{I} \in E$, we have that $\mathcal{I}(\varphi) = T$. When $E = \mathcal{C}$, we simply say that φ is *valid*.

$\mathcal{I}(\varphi)$	$\mathcal{I}(\psi)$	$\mathcal{I}(\neg\varphi)$	$\mathcal{I}(\varphi \wedge \psi)$	$\mathcal{I}(\varphi \rightarrow \psi)$	$\mathcal{I}(\varphi \vee \psi)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	F

Figure 2: Semantics of Propositional Connectives

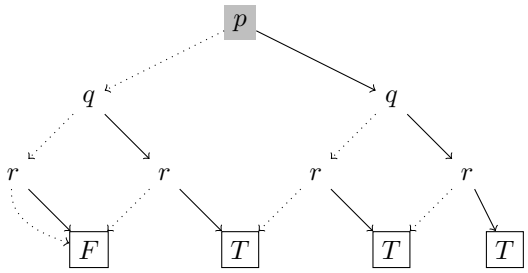


Figure 3: Example of a binary decision diagram

2.2 Ordered Binary Decision Diagrams (OBDD)

Definition 3 (Binary Decision Diagram, BDD, [8])**.** A *binary decision diagram (BDD)* is a directed acyclic graph with a unique root, which is also called the *entry point*. Each leaf is labeled with a constant T or F. Each interior node is labeled with an atom and has two outgoing edges: one, the *false edge*, is denoted by a dotted line, while the other, the *true edge*, is denoted by a solid line. No atom appears more than once in a branch from the root to an edge.

Example 1. Figure 3 shows a binary decision diagram.

During a trial, some questions have to be examined in a certain temporal order. This temporal order does not play a role from a logical point of view, in the sense that the truth value of a given statement will not depend on the order in which its arguments are examined. However, this temporal order plays a role from a procedural point of view when the judge constructs its judgment while answering the different questions.

This ordering is made explicit in *ordered* binary decision diagrams (OBDD): we can canonically associate to each OBDD an ordering corresponding to the order in which the different questions should be examined by the judge.

Definition 4 (Compatible set of orderings)**.** Let $\mathcal{P}$ be a finite set of atoms or propositional constants. Let $\mathcal{O} = \{(\mathcal{O}^1, <^1), \ldots, (\mathcal{O}^n, <^n)\}$, where for each i, $\mathcal{O}^i$ is a set of elements of $\mathcal{P}$ ordered by a total relation $<^i$. $\mathcal{O}$ is a *compatible set of orderings for* $\mathcal{P}$ iff for all $i \neq j$, there are no atoms $p, p' \in \mathcal{O}^i \cap \mathcal{O}^j$ such that $p <^i p'$ while $p' <^j p$.

Definition 5 (Ordered Binary Decision Diagrams, OBDD)**.**

- Let *bdd* be a BDD. The *set of orderings associated to bdd* is the set $\{(\mathcal{O}^1, <^1), \ldots, (\mathcal{O}^n, <^n)\}$ where each set corresponds to the atoms appearing in the i^{th} branch of the *bdd* and the ordering $<^i$ is defined by the order of appearance of each atom on the i^{th} branch (starting from the root of the branch).

Input: An OBDD *bdd* for a formula φ; an interpretation $\mathcal{I} \in \mathcal{C}^p$ (partial or total).
Output: An OBDD `Restrict`$(\mathcal{I}, bdd)$ for a formula ψ such that $\varphi \leftrightarrow \psi$ is valid on the set of interpretations $Ext(\mathcal{I})$.

Perform a recursive traversal of the OBDD:

- If the root of *bdd* is a leaf, return the leaf.
- If the root of *bdd* is labeled p and $\mathcal{I}(p)$ is defined, return the sub-BDD reached by its true edge if $\mathcal{I}(p) = T$ and the sub-BDD reached by its false edge if $\mathcal{I}(p) = F$.
- Otherwise (the root of *bdd* is labeled p and $\mathcal{I}(p)$ is *not* defined), apply the algorithm to the left and right sub-BDD, and return the BDD whose root is p and whose left and right sub-BDD are those returned by the recursive calls.

Figure 4: Schematic algorithm `Restrict`.

- An *ordered binary decision diagram (OBDD)* is a BDD such that the set of orderings associated to this BDD is compatible.

Note that the set of orderings associated to an OBDD induces an ordering on *all* the atoms appearing in the OBDD.

Definition 6 (Operations on OBDD, [8])**.** The operations `Apply`, `Restrict` and `Reduce` are defined in Figure 5 4. Two BDD *bdd* and bdd' are said to be *equivalent*, written $bdd \equiv bdd'$, when `Reduce`(bdd)=`Reduce`(bdd').

Example 2. When we apply `Reduce` step-by-step on the BDD of Figure 3, we obtain the BDD shown in Figure 6. First, we merge all F-leaves in one single leaf and we merge all T-leaves in one single leaf. Then we bypass the r-node and the two r-nodes on the right because all outgoing edges lead to the same node. In the last step, we bypass the q-node on the right.

Example 3. Figure 7 shows the application of `Restrict` with $\mathcal{I}(r) = T$ and $\mathcal{I}(\alpha)$ undefined for $\alpha \neq r$.

Theorem 1 ([11])**.** *For all BDD bdd,* `Reduce`$(bdd) \equiv bdd$.

Instead of the set-theoretical semantics of propositional logic based on the notion of interpretation, we can provide a semantics to propositional logic in terms of OBDD. The meaning of a propositional formula is completely determined by the OBDD associated to that formula, which is itself built inductively from the `Apply` Algorithm. The soundness of `Apply` is ensured by Shannon's expansion Theorem:

Theorem 2 (Shannon expansion, [8])**.** *For all formulas $\varphi, \psi \in \mathcal{L}_M$, for all $\star \in \{\wedge, \vee, \rightarrow\}$, the following formula is valid:*

$$\varphi \star \psi \leftrightarrow (p \wedge (\varphi[p\backslash\top] \star \psi[p\backslash\top])) \vee (\neg p \wedge (\varphi[p\backslash\bot] \star \psi[p\backslash\bot]))$$

For example, $\varphi \wedge \psi \leftrightarrow (p \wedge (\varphi[p\backslash\top] \wedge \psi[p\backslash\top])) \vee (\neg p \wedge (\varphi[p\backslash\bot] \wedge \psi[p\backslash\bot]))$.

Definition 7 (OBDD associated to a formula)**.** Let $\varphi \in \mathcal{L}$. The OBDD associated to φ, written bdd_φ, is defined inductively on φ as follows:

- $bdd_\top$ is the OBDD consisting of a single node labeled T;

1. Algorithm `Apply`:

   Input: OBDD bdd_φ for formula φ and bdd_ψ for formula ψ such that the set of orderings $\{(\mathcal{O}_\varphi, <_\varphi), (\mathcal{O}_\psi, <_\psi)\}$ is compatible (where $(\mathcal{O}_\varphi, <_\varphi)$ and $(\mathcal{O}_\psi, <_\psi)$ are the orderings associated to bdd_φ and bdd_ψ respectively). A connective $\star \in \{\neg, \wedge, \vee, \rightarrow\}$.

   Output: An OBDD for the formula $\varphi \star \psi$, denoted $bdd_\varphi \star bdd_\psi$, if $\star \in \{\wedge, \vee, \rightarrow\ \}$, or for the formula $\neg\varphi$, denoted $bdd_{\neg\varphi}$.

   Perform a recursive traversal of both OBDD as follows.

   - If $\star = \neg$, then return bdd_φ where the leafs labeled T are replaced by leafs labeled F, and vice versa.
   - Otherwise ($\star$ is then a binary connective)
     - If the root of bdd_φ or bdd_ψ is a leaf, then return either T, F, the other BDD, or `Apply`(the other BDD, $\neg$), according to the truth table of $\star$.
     - Otherwise select the bdd_φ or bdd_ψ that have the root p of lowest rank in the atom ordering, apply recursively the algorithm to its left and right sub-BDD, with the other BDD as other parameter, and return the BDD whose root is p and whose left and right sub-BDD are those returned by the recursive calls.

2. Algorithm `Reduce`:

   Input: An OBDD bdd.

   Output: A reduced OBDD bdd'.

   Perform a recursive traversal of the OBDD:

   - If bdd has more than two distinct leaves (one labeled with T and one labeled with F), remove duplicate leaves. Direct all edges that pointed to leaves to the remaining two leaves.
   - Perform the following steps as long as possible:
     (a) If all outgoing edges of a node labeled p_i point at the same node labeled p_j, delete this node for p_i and direct p_i's incoming edges to p_j.
     (b) If two nodes labeled p_i are the roots of identical sub-BDD, delete one sub-BDD and direct its incoming edges to the other node.

Figure 5: Schematic algorithms `Apply` and `Reduce`.

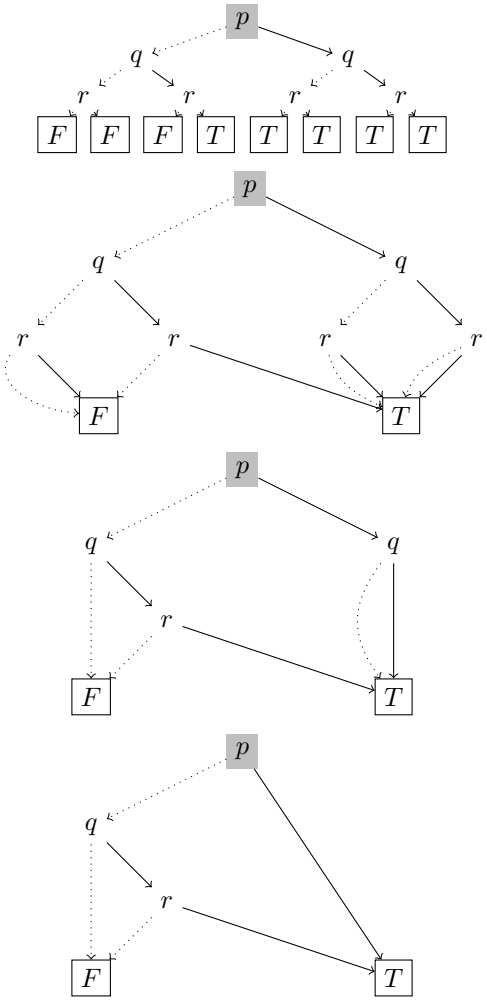


Figure 6: Step-by-step of `Reduce`

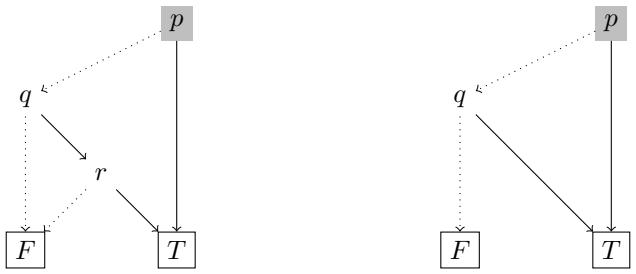
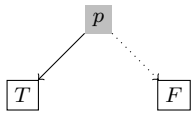


Figure 7: `Restrict` with $\mathcal{I}(r) = T$ and $\mathcal{I}(\alpha)$ undefined for all $\alpha \neq r$

- $bdd_\bot$ is the OBDD consisting of a single node labeled F;
- bdd_p is the following OBDD:



- $bdd_{\varphi \star \psi}$ is the OBDD $bdd_\varphi \star bdd_\psi$ obtained from Algorithm `Apply` of Figure 5, for all $\star \in \{\wedge, \vee, \rightarrow\}$.

Example 4. Let us consider the following set of atoms: $\mathbb{P} \triangleq \{p, r, s, t\}$. Then, we consider the following formula: $\varphi \triangleq p \rightarrow ((r \wedge \neg s) \vee (\neg r \wedge \neg t))$. The syntactic decomposition of formula φ is represented at the top of Figure 8. From this decomposition, we can apply the induction process of Definition 7 to obtain the OBDD bdd_φ. This process is represented in Figure 8, it consists in applying successively the Algorithm `Apply`.

All operations on BDD, `Apply`, `Reduce`, and `Restrict`, have a polynomial algorithmic complexity with the size of the BDD they operate on. The size of the result of `Reduce` strongly depends on the atom ordering. It is exponential in the worst case. Some formulas even have an exponential size BDD for all orderings. However, BDD have shown to be of great pratical value, and have become a standard solution for dealing with industrial size propositional formulas, e.g. for the verification of complex digital circuits [4].

3 The Examination Operator

Sometimes during a trial, the judge must necessarily examine or raise a specific issue. For example, the complainant can attack the legitimity of a syndicate that presents a candidate to the professionnal elections of a firm. The complainant could argue that this association of employees cannot really be qualified as a syndicate because it is not senior enough. In that case, even if the syndicate turns out to be senior enough (older than 2 years of existence), the judge *must* nevertheless examine *all* the other criteria (different from seniority) that make the association of employees qualify as a syndicate (even if she/he does not decide to raise the issue of the other criteria during the trial).

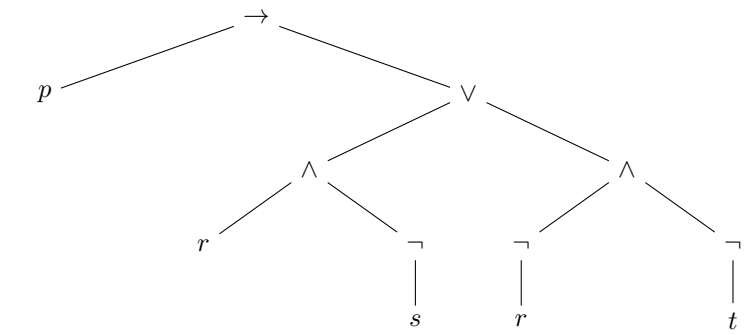
Definition 8 (Language $\mathcal{L}_M$)**.** The language $\mathcal{L}_M$ is the set that contains $\mathbb{P} \cup \{\top, \bot\}$ and such that

- if $\varphi, \psi \in \mathcal{L}_M$, then $!\varphi, \neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi) \in \mathcal{L}_M$;
- $\mathcal{L}_M$ contains no more formulas.

We use the same abbreviation and notation as in Definition 1.

The intuitive reading of the formula $!\varphi$ is as follows: "φ is examined and it holds". For example, the formula $q \wedge !p$ holds if, and only if, p is examined and p and q both hold.

We must provide a semantics to our extended language $\mathcal{L}_M$, in particular to the examination operator $!\varphi$. The Algorithm `Apply` provides already a semantics to every formula of the propositional language in terms of OBDD. Indeed, it suffices to apply it inductively to every subformula of a given formula φ (from the atoms up to the formula φ) to obtain an OBDD that captures the meaning of φ. Thus, we extend this algorithm to cover the examination operator. This leads us to introduce an extended form of BDD with several entry points. This extension is called a *Multi-BDD* and, contrary to BDD, it can also have several roots.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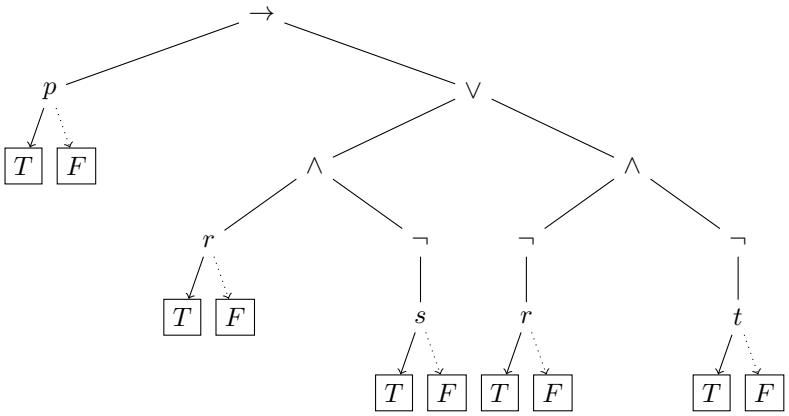
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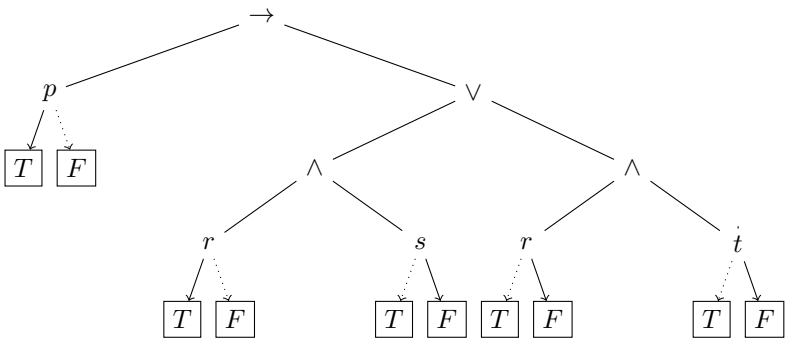
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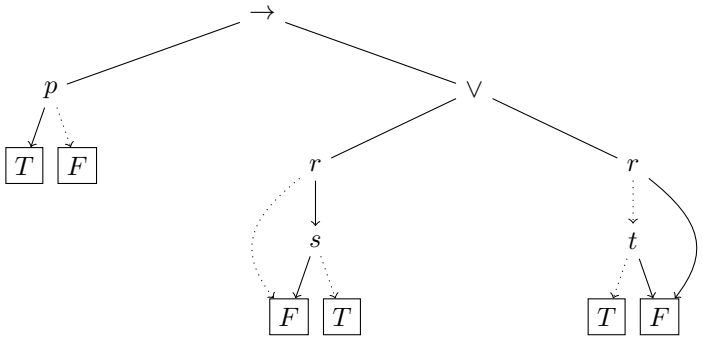
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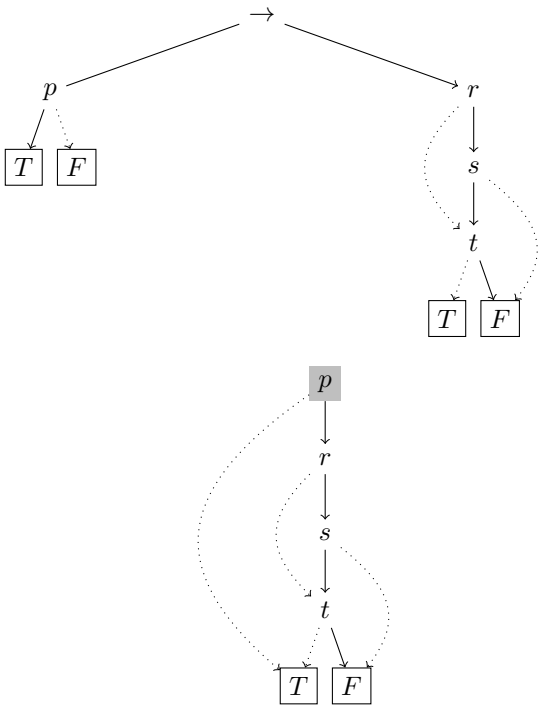

Figure 8: Successive applications of `Apply` with the ordering $p < q < r < s$: from the syntactic tree of $\varphi \triangleq p \rightarrow ((r \wedge \neg s) \vee (\neg r \wedge \neg t))$ (*top*) to bdd_φ (*bottom*).

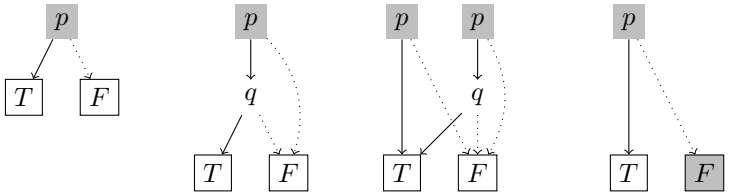

Figure 9: bdd_p (*first*), $bdd_{q\wedge p}$ (*second*), $mbdd_{q\wedge !p}$ reduced (*third*), $\texttt{Restrict}(\mathcal{I}, mbdd_{q\wedge !p})$ reduced (*fourth*) where $\mathcal{I}(q) = F$ and $\mathcal{I}(p)$ is undefined.

Definition 9 (Multi-BDD)**.** A *Multi-BDD* (MBDD for short) is a directed acyclic graph with the same property as a BDD except that it can have multiple roots. The set of *entry points* of a MBDD is a set of nodes comprising the roots of the MBDD and possibly others.

Example 5. Figure 12 shows a MBDD. The construction of this MBDD is detailed in the next section.

Definition 10 (MBDD associated to a formula)**.** Let $\varphi \in \mathcal{L}_M$. The MBDD associated to φ, written $mbdd_\varphi$, is defined inductively on φ as follows. Let $\star \in \{\wedge, \vee, \rightarrow\}$.

- $mbdd_\top$, $mbdd_\bot$, $mbdd_p$ are respectively $bdd_\top$, $bdd_\bot$, bdd_p of Definition 7.
- if $\varphi \neq !\varphi'$ and $\psi \neq !\psi'$ (for any $\varphi', \psi' \in \mathcal{L}_M$), then $mbdd_{\varphi\star\psi}$, is the MBDD $mbdd_\varphi \star mbdd_\psi$ obtained from Algorithm `Apply` of Figure 5;
- if $\varphi = !\varphi'$ and $\psi \neq !\psi'$ (for some $\varphi' \in \mathcal{L}_M$ and any $\psi' \in \mathcal{L}_M$) then $mbdd_{\varphi\star\psi}$ is the disjoint union of the MBDD $mbdd_\varphi$ and $mbdd_{\varphi\star\psi}$. Hence, the set of entry points of $mbdd_{!\varphi\star\psi}$ is the union of the entry points of $mbdd_\varphi$ and $mbdd_{\varphi\star\psi}$;
- if $\varphi = !\varphi'$ and $\psi = !\psi'$ (for some $\varphi' \in \mathcal{L}_M$ and some $\psi' \in \mathcal{L}_M$) then $mbdd_{\varphi\star\psi}$ is the disjoint union of the MBDD $mbdd_\varphi$, $mbdd_\psi$ and $mbdd_{\varphi\star\psi}$. Hence, the set of entry points of $mbdd_{!\varphi\star\psi}$ is the union of the entry points of $mbdd_\varphi$, $mbdd_\psi$ and $mbdd_{\varphi\star\psi}$.

In the same spirit, we could also define the notion of *Ordered* Multi-BDD, which might be even more suited to deal with the procedural reasoning of legal practice. It is important to notice that in order to reduce a MBDD to T or F, the user/judge must evaluate *every* formula that corresponds to the sub-BBD generated by an entry point of the MBDD. The theorem below shows that our notion of MBDD does capture that: if $\texttt{Restrict}(\mathcal{I}, mbdd_\varphi) \in \{T, F\}$, then every occurrence of a subformula $!\psi$ in φ is such that $\texttt{Restrict}(\mathcal{I}, mbdd_\psi) \in \{T, F\}$.

Theorem 3. *Let φ be a formula of $\mathcal{L}_M$ containing a subformula $!\psi$ and let $\mathcal{I} \in \mathcal{C}^p$ be a partial interpretation. If $\mathcal{I}(\psi) \notin \{T, F\}$, then $\texttt{Restrict}(\mathcal{I}, mbdd_\varphi) \notin \{T, F\}$.*

Example 6. In Figure 9, we represent the MBDD for the formula $q\wedge!p$ (third graph). This formula is true if, and only if, p and q are both true and p is examined. Hence, if q is given the value F (by the judge), then the truth value of the formula $q\wedge!p$ is still not determined. Indeed, the MBDD is still not completely reduced and we must *examine* the 'question' p and give a truth value to p.

4 Case Study: French Trade Unions

Our case study deals with French professional election in a firm [26]. Among the problems to be decided upon, one is to determine whether an association of employees is really qualified as a trade union so that this association can indeed propose candidates to professional elections in the firm. Law introduces four criteria that have to be fulfilled so that an association can indeed be qualified as a trade union. These four criteria have to hold altogether, and during a trial, the judge must necessarily examine all of them. They are the following:

1. the association of employees should respect the 'Republican values';
2. the association of employees should be 'Independent' (from the directorate of the firm for example);
3. the association of employees should be 'Senior' enough (minimum 2 years of existence);
4. the association of employees should be within the appropriate 'Geographical and professional' range.

Hence, we introduce the formula R (resp. I, S and G) which stands respectively for "the criteria of the *Republican values* (resp. *Independence*, *Seniority*, *Geographical and professional range*) is fulfilled". The four criteria must hold for an association of employees to be legally qualified as trade union in a firm, and they must all be examined by the judge, even if they are not contested. Therefore, the following formula must be true: $!R \wedge !I \wedge !G \wedge !S$.

4.1 The Criterias of 'Republican values' and 'Independence'

To determine whether the criteria of 'Republican values' holds, the judge has to answer a number of questions. To formulate these questions, we introduce the following set of atoms:

$$\mathbb{P}_R \triangleq \{Lit_R, OldJug_R, NoNewElt_R, Proof_{\neg R}\}$$

These atoms stand for the following propositions:

- Lit_R:"The criteria of republican values is contested";
- $OldJug_R$:"An old judgement dealing with 'Republican values' has already established that the association of employees fulfills the criteria of Republican Values";
- $NoNewElt_R$:"No new elements have been brought to the fore that oblige to reconsider the old judgement";
- $Proof_{\neg R}$:"The plaintiff presents the proof that the criteria of 'Republican values' is not fulfilled".

Then, we can give the formula in the language $\mathcal{L}_M$ that determine in which case the criteria of 'Republican values' holds. It is the following:

$$R \triangleq Lit_R \rightarrow ((OldJug_R \wedge NoNewElt_R) \vee (\neg OldJug_R \wedge \neg Proof_{\neg R}))$$

The above formula reads as follows: "if the plaintiff contests that the criteria of republican value is satisfied, then either there is an old judgement that already established that this criteria was fulfilled and no new element has been brought to the fore that oblige to reconsider this old

judgement, or there is no old judgement and the plaintiff has not been able to prove that the criteria is not fulfilled".

Dealing with the criteria of 'Independence' is completely similar. Hence, we introduce the following set of atoms:

$$\mathbb{P}_I \triangleq \{Lit_I, OldJug_I, NoNewElt_I, Proof_{\neg I}\}$$

Their interpretation is the same as for the criteria of Republican values, except that the term "Republican values" has to be replaced by "Independence" everywhere. So, the formula in the language $\mathcal{L}_M$ that determines in which case I holds is the following:

$$I \triangleq Lit_I \rightarrow ((OldJug_I \wedge NoNewElt_I) \vee (\neg OldJug_I \wedge \neg Proof_{\neg I}))$$

Its intuitive interpretation is the same as for the previous criteria.

Three equivalent representations. In Figure 10, we give two equivalent and alternative representations of the semantics of the formula R (in propositional logic): the truth table of R and the MBDD associated to R.

4.2 The Criteria of 'Geographical and Professional Range'

To determine whether the criteria of "Geographical and professional range" holds, the judge has to answer a number of questions. To formulate these questions, we introduce the following set of atoms:

$$\mathbb{P}_G \triangleq \{Lit_G, Decide_G, Proof_{\neg G}\}$$

These atoms stand for the following propositions:

- Lit_G:"The criteria of Geographical and professional range is contested";
- $Decide_G$:"The judge decides to examine the criteria of Geographical and professional range";
- $Proof_{\neg G}$:"The plaintiff presents the proof that the criteria of 'Geographical and professional range' is not fulfilled".

Then, we can give the formula in the language $\mathcal{L}_M$ that determine in which case the criteria of 'Geographical and professional range' holds. It is the following:

$$G \triangleq (Lit_G \vee Decide_G) \rightarrow Proof_{\neg G}$$

The above formula reads as follows: "if the plaintiff contests that the criteria of 'Geographical and professional range' is fulfilled or if the judge decides to consider this criteria, then the plaintiff presents the proof that the criteria is not fulfilled". The MBDD associated to the formula G is depicted in Figure 11, it is the third OBDD from the left.

4.3 A BDD for Qualifying as a Trade Union

We have not considered the criteria of 'Seniority' so far and we will not deal with it in order to ease the presentation and because it is more complex to represent than the other criteria, in the sense that many more atoms (questions) are needed to deal with it.

$\mathcal{I}(Lit_R)$	$\mathcal{I}(OldJug_R)$	$\mathcal{I}(NoNewElt_R)$	$\mathcal{I}(Proof_{\neg R})$	$\mathcal{I}(R)$
T	T	T	T	F
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	F
T	F	T	F	T
T	F	F	T	F
T	F	F	F	T
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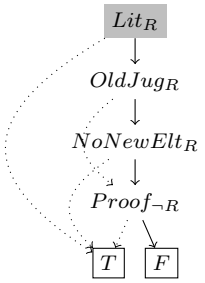

Figure 10: Two equivalent and alternative semantics for the formula $Lit_R \rightarrow ((OldJug_R \wedge NoNewElt_R) \vee (\neg OldJug_R \wedge \neg Proof_{\neg R}))$.

The MBDD for $!R \wedge !I \wedge !G$ is given in Figure 11. It is simply the disjoint union of the three OBDD R, I, G and $R \wedge I \wedge G$. Then, it can be reduced equivalently thanks to the Algorithm `Reduce` of Figure 5. This yields the MBDD of Figure 12. Note that in this MBDD, all the leaf nodes have been merged into two nodes ($bdd_\top$ and $bdd_\bot$) and the OBDD for the formula G has been merged with the OBDD for the formula $R \wedge I \wedge G$. That is why we have an entry node Lit_G which is not a root of the MBDD.

5 Applications of BDD Algorithms to Legal Reasoning

Because our solution is based on BDD, we inherit from the vast amount of work for BDD a number of algorithms and software applications that can play an important role in legal reasoning. Even if they were not initially intended to be used in the legal domain, these algorithms and software applications turn out to be really relevant for solving specific problems or answer specific queries of the judge. We list below some of these algorithms (some of them have already been considered above) and show how they can be used by a judge during, before or after a trial. We start with the algorithms of this article:

- Algorithm `Restrict`. This algorithm is used when the judge answer questions: each question answered corresponds in fact to an application of the algorithm `Restrict`. After each answer, the MBDD is instanciated and the node corresponding to the question disappears.
- Algorithm `Reduce`. This algorithm can be used to determine whether two kinds of legal reasoning represented by two different MBDD are in fact equivalent: in case the MBDD returned by this algorithm is the same in both cases, then they are indeed equivalent (see Theorem 1).
- Algorithm `Apply`. This algorithm can be used to construct a MBDD corresponding to a formula expressed in our language $\mathcal{L}_M$. It can also be used when we want to combine

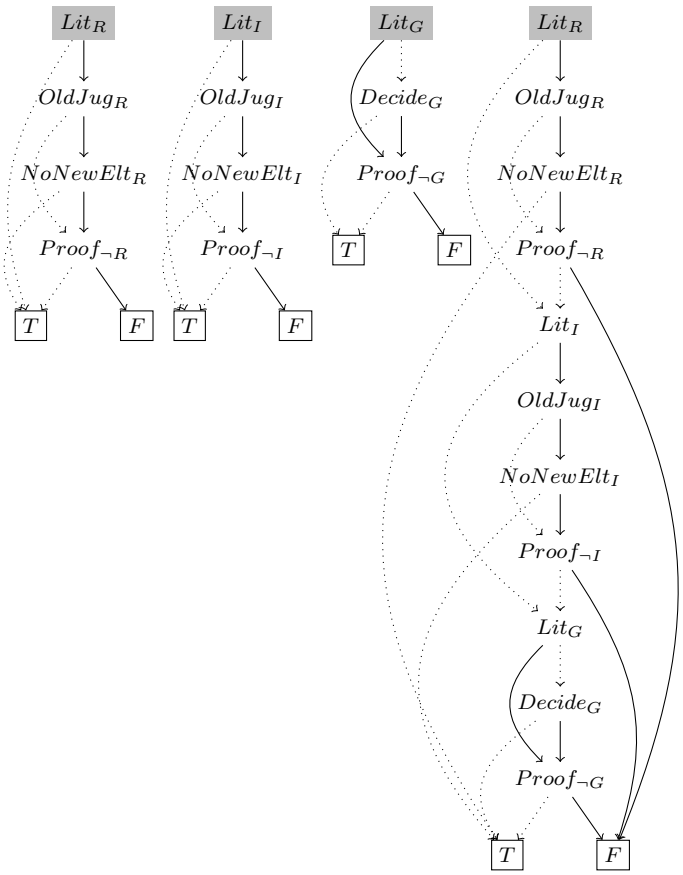

Figure 11: MBDD for $!R \wedge !I \wedge !G$.

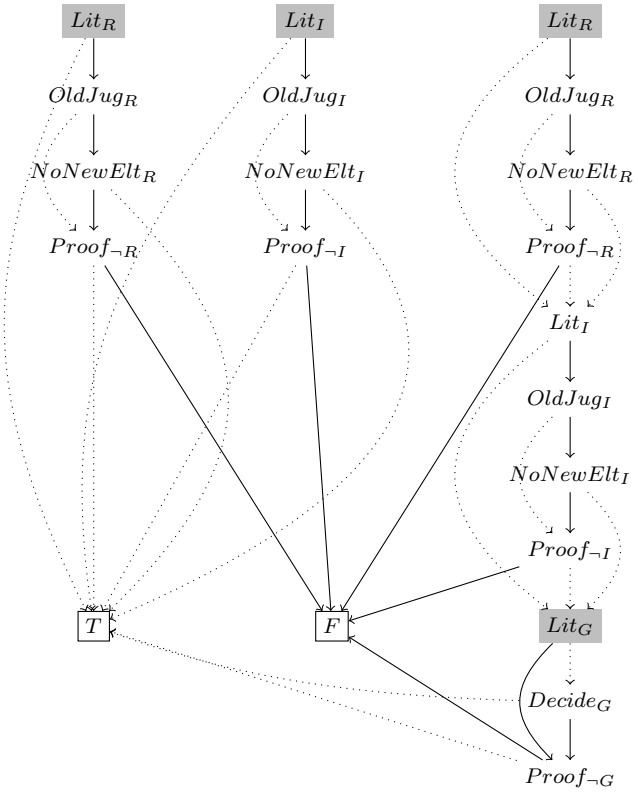

Figure 12: MBDD for $!R \wedge !I \wedge !G$ reduced. It has four entry points (in gray) corresponding to R, I, G and $R \wedge I \wedge G$.

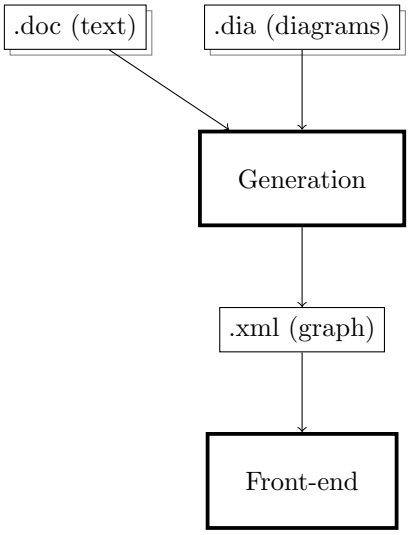

Figure 13: Global architecture

two kinds of legal reasoning that deal with different but complementary issues that have already been represented by two MBDD.

Other algorithms based on BDD dealing with quantification over propositional atoms can be used to solve the following tasks:

- Determine whether the answer to a specific question will allow the judge to conclude about a litigation or a specific subproblem without having to examine all the other questions exhaustively.
- Determine whether a question is redondant and can thus be removed from the MBDD.

These algorithms are only a few among the large amount of algorithms for BDD that are available. Many other algorithms can be used and even designed to solve specific legal reasoning tasks.

6 Implementation

In this section, we describe the prototype that we have developed during our project with the 'Cour de cassation'. In a first subsection, we describe the front-end graphical user interface. The development process was iterative (agile method). The second subsection explains our choices for being able to design it interactively, namely the use of Microsoft Word and Dia (`http://dia-installer.de/`) for creating the input. But our front-end takes a graph as an input and the third subsection describes the architecture of the graph generation from .doc and .dia files. Finally, we describe the architecture of the front-end. The global architecture of the prototype is given in Figure 13.

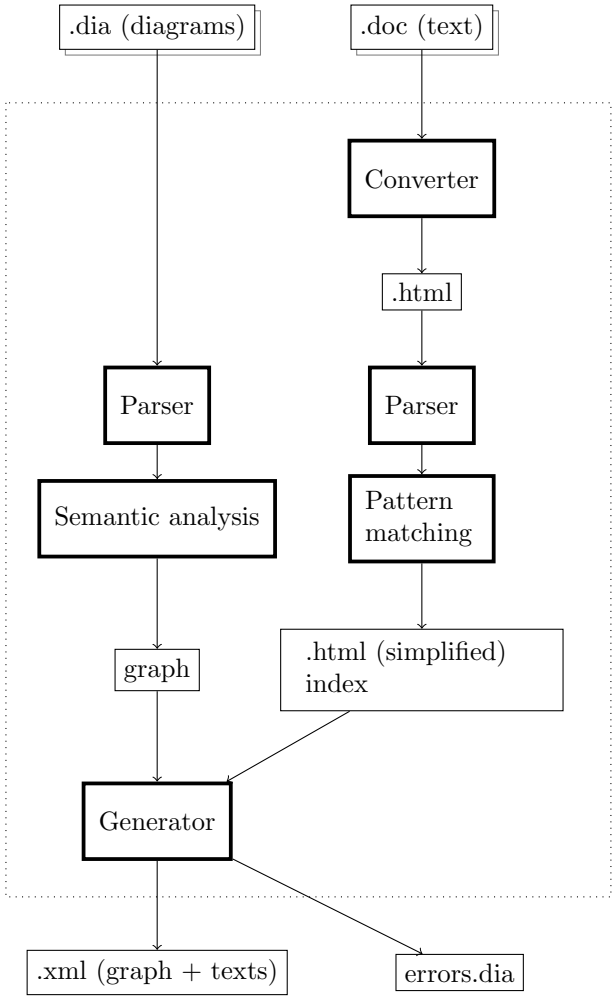

Figure 14: Graph generation architecture

6.1 Graphical user interface

The graphical user interface (see Figure 1) front-end is divided in two parts. The left part proposes questions to be answered and the documentation for helping the judge to take decisions. While the judge answers to the questions, the right part shows the produced judgment text.

6.2 Graph generation architecture

Figure 14 shows the logical graph generation architecture. The input is divided in two parts: diagrams in .dia and the documentation in .doc files. The logic of a case is represented by diagrams saved in .dia files. We used some graphical conventions to encode all logical elements (questions, answers, connectives). The diagrams are parsed and a semantic analysis generates a logical graph from diagrams by using the graphical conventions. On the other hand, .doc files are converted in .html files, that are then parsed. We then identify the different paragraphs in

.html files and we produce a single simplified .html file and an index that identifies what are the sections and what are the judgment paragraphs.

The last step generates a .xml file that merges the logical graph and the textual documentation (sections and judgment paragraphs). Errors are reported graphically in a .dia file.

6.3 Front-end architecture

The front-end is implemented in Javascript and is fully executed in the web browser. The front-end is based on a model-view-controller architecture. The model is dynamically generated from the .xml file given as an input. The generated .xml file contains all the data for displaying questions and judgment paragraphs in the front-end. We can export the produced judgment output as a text file.

7 Conclusion

One could argue that our approach is not suitable for legal reasoning because it is merely based on propositional logic and does not integrate non-classical reasoning such as deontic, causal or defeasible reasoning, which has often been claimed to be more appropriate to deal with legal reasoning (see the various references in Section 1.1 or [18, 9, 16]). Criticisms regarding the adequacy of propositional logic for legal reasoning do not apply to our work because propositional logic is intended to serve as a theoretical basis for the *rewriting* of most of legal texts and regulations. It is not intended to model the current state of legal texts and regulations. In the course of our project, we had to resort to BDD because this kind of representation suited better the actual practice of the jurists of our project. In particular, the procedural and temporal aspect of BDD (in a BDD, one has to answer one after the other questions corresponding to the nodes of the BDD) was in fact very close to their actual practice as lawyers or jurists. This procedural and temporal aspect of the legal practice cannot be captured by the usual syntactic representation of propositional formulas.

This rewriting would then lead to the development of software tools that could be used by jurists and lawyers, and probably change their actual practice of the law. If our proposal is adopted, the current legal texts and regulations would have to be all rewritten and adapted in order to fit the format based on BDD propounded in this article, as we did during our project with the Cour de Cassation with the case study of the "French trade union" (see Section 4). This case study shows that our approach is feasible and can be extended to any kind of regulations, even if a tremendous amount of work would need to be carried out by the jurists (in collaboration with computer scientists and logicians) to rewrite and adapt all the existing texts and regulations in order to define corresponding BDD. This said, the rewriting and adaptation phase could start with small fragments of the law and it could then be expanded step by step to all regulations.

As such, our work is only a first step and does not preclude to be extended by other kinds of reasoning hardly amenable to propositional reasoning. In fact, even if we have not addressed in this article standard problems in AI and law such as those related to "contrary to duties" or exceptions, we did encounter such kinds of exceptional reasoning in the course of our project with the Cour de cassation. As it turns out, this problem of exceptions could also be overcome with BDD and we actually introduced to that aim in the course of our project an operator called "sinon" ("otherwise" in English) with a semantics based on BDD. Our solution for handling exceptions based on BDD turned out to be also very appealing to jurists. Another solution could consist in extending our approach with some sort of defeasible reasoning, like in the work of Beierle & Al. [7].

This research was carried out from the outset hand in hand by both jurists and computer scientists, with regular and numerous communications between both parties. This explains why we had to resort to BDD: this type of representation turned out to fit better the actual practice of the jurists. Our case study based on the "French union trade" (Section 4) has been implemented in a prototype in the course of our project with the French Cour de cassation (see Section 6). This prototype was tried out by three judges of 'cour d'instances' in France. They were all impressed and satisfied by the prototype tool that they tested on our case study. Moreover, we also made an experiment with law students of the Ecole Normale Supérieure of Rennes to determine whether they prefer to write the kind of meta-regulations that we use in the software tool with formulas of (propositional) logic or with a graph-based representation like BDD. We designed a kind of experimental protocol to figure this out. Even if the results were hard to interpret because they had no prior teaching in logic or graph theory, it turns out that they had somehow more facility to use the graph-based representation than the formula-based representation.

From a theoretical point of view, we believe that our solution is the most promising and realistic approach to answer the needs expressed to us by the jurists of the Cour de cassation. Indeed, we believe that it is the most rigorous, accurate and solid approach: it is based on methods and techniques of logic that are very well understood, worked out and applied. Our approach allows an important control over our representation of the legal reasoning and over the changes that we may want to make to this reasoning. Moreover, BDD are very well–studied and their associated algorithms are able to scale–up to a large number of nodes. Thus, using BDD is clearly a realistic solution to deal with the complexity of the law and the large amount of texts and jurisprudences. Finally, our approach is very flexible and it allows to take into account the evolubility and dynamicity of the law. Indeed, because it is based on propositional logic, the various changes in the law, such as abrogation and annulment, can be modeled naturally as update operations in propositional logic (Governatori & Al. [15, 14] attempt to model abrogation and annulment in a more realistic fashion). As it turns out, dealing with dynamism and change has been at the core of most of the recent development in logic and artificial intelligence in the last decades (see for instance [12, 27, 28]) and many extensions of propositional logic with dynamic operators have been introduced. Hence, the dynamic character of the law could be dealt with in the software by importing, adapting and implementing the various methods and techniques that have been developed in logic for dealing with dynamism and change.

The software tool proposed is only the first part of a bigger project since this software tool would only *use* the BDD representing the legal reasoning underlying specific litigations. But these BDD would first need to be defined and created by a legal expert or a legislator. The second tool that complements the software described in this article still has to be specified precisely. It should be able to create and edit these BDD that represent other and all case studies. In particular, this second software tool should provide mechanisms for modifying the graphs that underly the BDD. From a theoretical point of view, graph modifications and change is also an area of research that is currently very active in logic [3, 13, 2, 6]. These theoretical works could provide algorithms and associated software tools for checking and verifying that a particular change or modification of the BDD has indeed been made and that these changes do correspond to the idea and the intention of the user/legislator who triggered them.

Finally, if such kinds of softwares would ever be developed and used by judges, this would entail as a prerequisite that the jurists be trained and taught some rudiments of logic, especially the law experts who would have to rewrite and adapt the legal texts and regulations to create and edit the corresponding BDD. This would also call for the development of regulations to determine in which kind of legal context and litigations these softwares can and should be used. Indeed, the novelty of such softwares and their impact on society would raise a number of ethical issues that would need to be harnessed by the law. This said, we want to stress that this work will

not replace by any means judges by 'machines', nor will suppress the responsibility that judges endorse when they make decisions. In this article, we only propose some theoretical foundations that could lead to the development of a software tool that would help judges to make fair and maybe sometimes better or well-informed decisions. This software tool would mainly provide and recall judges some organized and well–structured information about the relevant legal texts and regulations that they should not omit, together with some crucial information about the legal procedure that they should follow in order to deal with a specific litigation. If such a software tool were available some day to judges, they should in any case remain fully responsible of their decisions and they should not have the possibility to discharge the responsibility of their decisions to that software tool.

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