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Abstract: This paper proposes a new design of a decentralized output-feedback tracking control for a class of switched large-scale systems with external bounded disturbances. The controller proposed herein is synthesized to satisfy the robust H_∞ tracking performance with local disturbance attenuation levels. Based on multiple switched Lyapunov functions, sufficient conditions proving the existence of the proposed controller are formulated in terms of Linear Matrix Inequalities (LMI). A deep simulation is proposed to illustrate the effectiveness of the obtained results.

Keywords: Output-feedback tracking control; large-scale switched systems; decentralized control; Lyapunov function.

1. INTRODUCTION

During the latter decades, switched interconnected large-scale systems have attracted considerable attention since they provide a convenient modelling approach for many physical systems. Thus, several studies dealing with the stability analysis and stabilization issues for both linear and nonlinear switched interconnected large-scale systems have been explored by (Chiou, 2006; Belkhiat et al., 2015; Jabri et al., 2010, 2011; Mahmoud and Al-Sunni, 2010; Sun et al., 2009; Thanh and Phat, 2014; Wang and Tong, 2015). Hence, the main challenge in treating such problems consists in determining the conditions ensuring the stability of the whole systems with consideration to the interconnections effects between its subsystems. For example, sufficient stability conditions for a class of switched large-scale time-delay systems have been delivered in (Chiou, 2006). Moreover, the problem of decentralized control for a class of switched interconnected large-scale systems with value-bounded uncertainties has been investigated in (Sun et al., 2009). By using Linear Matrix Inequalities (LMI) techniques, decentralized state-feedback controllers and decentralized switching laws have been designed to make this class of system asymptotically stable. In the same way, stabilization issue for discrete-time large-scale switched system has also been studied in (Jabri et al., 2010). Otherwise, the problem of a low-order H_∞ output-feedback controller design, with a decentralized switching rule, for a class of interconnected continuous-time switched systems subject to disturbances has been treated in (Mahmoud and Al-Sunni, 2010)'s work . The main objective of the work seeks to guarantee the asymptotic stability of the whole system with local disturbance attenuation. Furthermore, the problem of decentralized stabilization for a class of large-scale switched Takagi-Sugeno (T-S) systems has been investigated in (Jabri et al., 2011). To overcome the nonlinearity problem, the large-scale switched nonlinear system was divided into a set of low-order interconnected switched T-S Fuzzy subsystems. Then, in order to stabilize the overall system, a set of switched non-PDC (Parallel Distributed Compensation) controllers has been employed. In the same context, the problem of H_∞ control design, under asynchronous switching, for a class of switched discrete-time T-S Fuzzy large-scale systems has been explored by (Wang and Tong, 2015). Moreover, (Thanh and Phat, 2014) have studied the problem of decentralized stability for a class of switched nonlinear large-scale systems with time-varying delays in interconnections. By using a set of Lyapunov-Krasovskii functional, a delay-dependent sufficient condition for designing switching law has been established in terms of LMI. Recently, the design of an adaptive fuzzy output feedback control was developed by (Zhang and Yang, 2017) for a class of switched nonlinear large-scale systems with unknown dead zone. Based on the above, the main advantage of the decentralized control scheme is the tremendous reduction of computational load and the design easiness of locally feedback stabilization.

Likewise, tracking control is one of the most important issues currently under consideration by researchers in linear and nonlinear control theory in (Cabecinhas et al., 2014; Guan et al., 2014). This kind of control has a close relationship with the stability analysis and stabilization issues. Generally, tracking control deal with the stabilization and the minimization of the error between the system output (or state) and the reference signal via designing a controller. As regards the switched systems, few results have been reported on the tracking control problem (Belkhiat et al., 2014; Li et al., 2009; Lian and Ge, 2013; Liu and Xiang, 2014; Long and Zhoa, 2015; Tong et al., 2016). For example, the work carried out by (Liu et al., 2014) has focused on performing an exponential L_1 output tracking control for Switched Linear Systems (SLS) with time-varying delays. Similarly, the output tracking control has been studied for a class of switched systems containing stabilizable and unstabilizable subsystems in (Li et al., 2009). Moreover, in (Liu and Xiang, 2014), a controller design approach has been proposed for a class of switched systems with time-varying delay. This one takes into account the effect of the asynchronous switching phenomena and satisfies the robust H_∞ output-feedback tracking performance. In the same context, a robust H_∞ output tracking control for a class of SLS using switched Proportional-Derivative (PD) controller has been designed in the reference (Belkhiat et al., 2014). Based on the multiple Lyapunov functions and the adaptive fuzzy back-stepping technique, sufficient conditions for the adaptive fuzzy tracking control problem have been derived in (Long and Zhoa, 2015) for a class of switched uncertain nonlinear systems with unstable subsystems. Recently, an adaptive fuzzy decentralized tracking control for a class of switched nonlinear large-scale systems with unknown nonlinear functions has been performed in (Tong et al., 2016). The proposed approach has dealt with the problems related to dead zones and unmeasurable. In a nutshell, it is worth pointing out that the aforementioned results are mainly restricted to the tracking control of lower-dimensional switched systems. Moreover, although some progress have been made in several fields of interconnected switched systems such as the stability and stabilization issues, the tracking control problem of switched interconnected large-scale systems subject to external disturbances has rarely been explored so far, that which motivates the present study.

Hence, this paper presents the design of a decentralized tracking controller for a class of switched interconnected large-scale system under synchronous switching and with external bounded disturbances. The considered class consists of set of low-order interconnected subsystems. Each subsystem contains several switching modes which are described by using linear system. As regards the interconnections among different subsystems, two components, combining the state vectors and the external disturbances, are taken into account in the mathematical model representing the considered class. According to the existing literature, the contributions of this paper are the followings:

- provides a new synthesis approach to design a decentralized output-feedback tracking control for a class of switched interconnected large-scale systems subject to external disturbances. By using the descriptor redundancy formulation, the proposed approach resolves the crossing terms problem related to using the output-feedback control strategy.
- ensure with the designed controller, on the one hand, the stability of the overall switched interconnected large-scale systems and, on the other hand, the robust H_∞ output-feedback tracking performance with for each low-order subsystem a specific disturbance attenuation level.
- provides sufficient conditions to design the proposed controller, formulated in terms of LMI thanks to the multiple switched Lyapunov functions.

The remainder of the paper is organized as follows. Section 2 presents the considered class of switched interconnected large-scale system, followed by the problem statement. The design of the decentralized controller is presented in section 3. A numerical example is proposed to illustrate the efficiency of the proposed approach in section 4. The paper ends with conclusions, appendix and cited references.

2. SYSTEM'S DESCRIPTION AND PROBLEM STATEMENT

We consider the class of the switched interconnected large-scale system S. It is composed of N low-order switched subsystems denoted by S_i such that each subsystem S_i has its own mode number M_i. Thus, the switched interconnected large-scale system S can be represented as follows for $i = 1, \ldots, N$:

$$\dot{x}_i(t) = \sum_{j=1}^{M_i} \xi_{i,j}(t) \begin{bmatrix} A_{i,j} x_i(t) + B_{i,j} u_i(t) + B_{w_{i,j}} w_i(t) \\ + \sum_{\alpha=1,\alpha\neq i}^{N} \left[F_{i,\alpha,j} x_\alpha(t) + F_{w_{i,\alpha,j}} w_\alpha(t) \right] \end{bmatrix} \quad (1)$$

$$y_i(t) = \sum_{j=1}^{M_i} \xi_{i,j}(t) \left[C_{i,j} x_i(t) \right] \quad (2)$$

where $x_i(t) \in \Re^{n_i}$ is the i^{th} state vector, $u_i(t) \in \Re^{m_i}$ is the i^{th} input vector, $w_i(t) \in \Re^{m_i}$ is the L_2-norm bounded external disturbance associated to the i^{th} subsystem. $x_\alpha(t) \in \Re^{n_\alpha}$ and $w_\alpha(t) \in \Re^{m_\alpha}$ denote respectively the state vector and the L_2-norm bounded external disturbance of the α^{th} subsystem with $\alpha = 1, \ldots, N$ and $\alpha \neq i$. $y_i(t) \in \Re^{p_i}$ is the measurement (output) associated to the i^{th} subsystem. j denoted the mode of each i^{th} subsystem. $A_{i,j} \in \Re^{n_i \times n_i}$, $B_{i,j} \in \Re^{n_i \times m_i}$, $C_{i,j} \in \Re^{p_i \times n_i}$ are constant matrices, $F_{i,\alpha,j} \in \Re^{n_i \times n_\alpha}$ and $F_{w_{i,\alpha,j}} \in \Re^{n_i \times m_\alpha}$ are constant matrices which describe the influences of the α^{th} subsystem on the i^{th} one. Finally $\xi_{i,j}(t)$ are the switching rules. They are assumed to be real time available.

The switching rules $\xi_{i,j}(t)$ are defined that the i^{th} subsystem is active in the l^{th} mode, that is to say:

$$\begin{cases} \xi_{i,j}(t) = 1 & if \;\; j = l \\ \xi_{i,j}(t) = 0 & if \;\; j \neq l \end{cases} \quad (3)$$

Note that, the mode's evolution of each i^{th} subsystem is independent of the rest of the subsystems. Hence, the global system is represented by Fig. 1. In the sequel, we will deal with the output-feedback tracking problem for the considered class of large-scale system. The tracking control objective is to drive the outputs of the system (1)-(2) via static output-feedback controllers to track reference signals as close as possible. Hence, to specify the desired trajectory for the overall large-scale system S, we present the following reference model for $i=1,\dots,N$:

$$\dot{x}_{r_i}(t)=A_{r_i}x_{r_i}(t)+B_{r_i}r_i(t)+\sum_{\alpha=1,\alpha\neq i}^{N}F_{r_{i,\alpha}}x_{r_\alpha}(t) \tag{4}$$

$$y_{r_i}(t)=H_{r_i}x_{r_i}(t) \tag{5}$$

with $x_{r_i}(t)\in\Re^{n_i}$ and $r_i\in\Re^{m_i}$ are the i^{th} reference state vector and the i^{th} L_2-norm bounded reference input vector, respectively. $x_{r_\alpha}(t)\in\Re^{n_\alpha}$ and $y_{r_i}(t)\in\Re^{p_i}$ denote respectively the α^{th} state vector and the i^{th} reference measurement (output). $A_{r_i}\in\Re^{n_i\times n_i}$, $B_{r_i}\in\Re^{n_i\times m_i}$, $H_{r_i}\in\Re^{p_i\times n_i}$ are constant matrices, where $A_{r_i}\in\Re^{n_i\times n_i}$ are specified as asymptotically stable matrices.

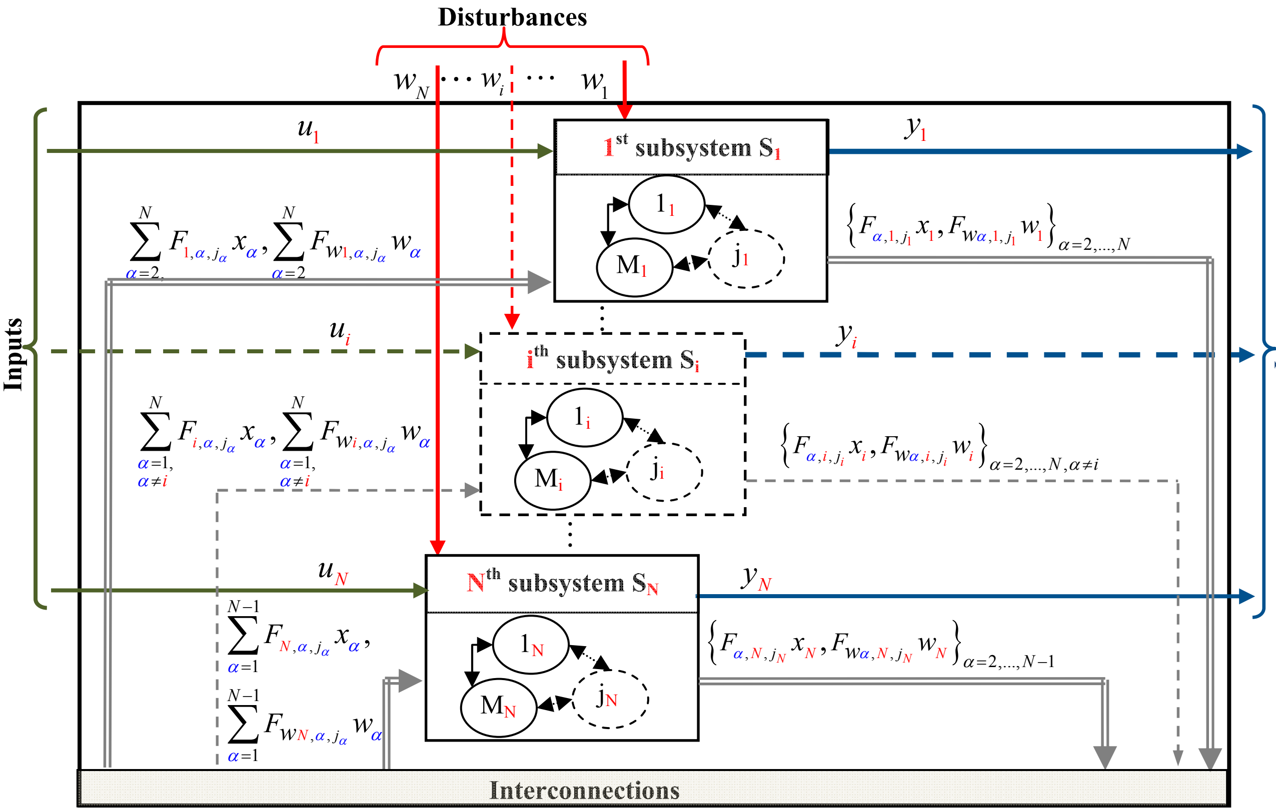

Fig. 1. System representation.

In order to drive our system S and to ensure the H_∞ tracking performance, decentralized switched controller is proposed in this work. The key idea is to synthesize a global controller composed of N local switched controllers. Each local controller is associated to a switched subsystem. The purpose of the local controllers is to ensure the stability and the output-feedback tracking performance of subsystems, while taking into account the problems related to the interconnections among the subsystems. In this work, the local switched controllers and the switched subsystems are synchronously orchestrated. Thus, the set of N local switched controllers are defined as follows for $i=1,\dots,N$:

$$u_i(t)=\sum_{j=1}^{M_i}\xi_{i,j}(t)\left[K_{i,j}e_i(t)\right] \tag{6}$$

where $e_i(t)=y_{r_i}(t)-y_i(t)\in\Re^{p_i}$ are the tracking errors, $K_{i,j}\in\Re^{m_i\times p_i}$ are the gain controllers[1].

In general, the classical way to write output-feedback dynamics consists on substituting the controller's equation (6) into the system's equation (1), this leads to, for $i=1,\dots,N$:

$$\dot{x}_i=\sum_{j=1}^{M_i}\xi_{i,j}\left[\begin{array}{l}A_{i,j}x_i+B_{i,j}K_{i,j}\left(H_{r_i}x_{r_i}-C_{i,j}x_i\right)+B_{w_{i,j}}w_i\\+\sum_{\alpha=1,\alpha\neq i}^{N}\left[F_{i,\alpha,j}x_\alpha+F_{w_{i,\alpha,j}}w_\alpha\right]\end{array}\right] \tag{7}$$

Thus, the problem considered in this study can be resumed as follows:

[1] The index time (t) will be omitted in the next when there is no ambiguity.

Problem 1**:** The objective is to design the controllers (6) such that the switched interconnected large-scale system (1)-(2) has a robust H_∞ output-feedback tracking performance.

Definition 1**:** The switched interconnected large-scale system (1)-(2) is said to have a robust H_∞ output-feedback tracking performance, if the following conditions are satisfied:

• ***Condition 1** (Stability condition)*: With zero disturbances input condition $w_i \equiv 0$, for $i = 1,\dots,N$, the closed-loop dynamics (7) is stable.

• ***Condition 2** (*Robustness *condition)*: For all non-zero $w_i \in L_2[0 \quad \infty)$, under zero initial condition $x_i(t_0) \equiv 0$, it holds that for $i = 1,\dots,N$,

$$J_i = \int_0^{+\infty} e_i^T e_i \, dt \le \gamma_i^2 \int_0^{+\infty} \left(w_i^T w_i + r_i^T r_i + \sum_{\alpha=1,\alpha\neq i}^{N} w_\alpha^T w_\alpha \right) dt \tag{8}$$

By observing the closed-loop dynamics (7), it can be seen that several crossing terms among the gain controllers $K_{i,j}$ and the system's matrices ($B_{i,j}K_{i,j}C_{i,j}$ and $B_{i,j}K_{i,j}H_{r_i}$, for $i = 1,\dots,N$, $j = 1,\dots,M_i$) are present. Hence, in view of the wealth of interconnections characterizing our system, these crossing terms lead surely to very conservative conditions for the design of the proposed controller. In order to decouple the crossing terms appearing in the equation (7) and to provide a LMI conditions, we use an interesting property called the descriptor redundancy (Tanaka et al., 2017). Thus, the equations (1)-(2), (4)-(5) and (6) are combined as follows for $i = 1,\dots,N$:

$$\dot{x}_i = \sum_{j=1}^{M_i} \xi_{i,j} \left[A_{i,j}x_i + B_{i,j}u_i + B_{w_{i,j}} w_i + \sum_{\alpha=1,\alpha\neq i}^{N} \left[F_{i,\alpha,j}x_\alpha + F_{w_{i,\alpha,j}} w_\alpha \right] \right]$$

$$\dot{x}_{r_i} = A_{r_i}x_{r_i} + B_{r_i}r_i + \sum_{\alpha=1,\alpha\neq i}^{N} F_{r_{i,\alpha}} x_{r_\alpha} \quad \text{and} \quad 0\,\dot{e}_i = -e_i + y_{r_i} - y_i$$

Then, we consider the following augmented variables:

$\tilde{x}_i^T = \begin{bmatrix} x_i^T & x_{r_i}^T & e_i^T \end{bmatrix}$, $\tilde{w}_{i,\alpha}^T = \begin{bmatrix} w_i^T & w_\alpha^T & r_i^T \end{bmatrix}$ and $\tilde{x}_\alpha^T = \begin{bmatrix} x_\alpha^T & x_{r_\alpha}^T & e_\alpha^T \end{bmatrix}$.

Hence, the large-scale system (1)-(2), the reference model (4) -(5) and the controllers (6) can be reformulated as follows for $i = 1,\dots,N$:

$$E_i\dot{\tilde{x}}_i = \sum_{j=1}^{M_i} \xi_{i,j} \sum_{\alpha=1,\alpha\neq i}^{N} \begin{bmatrix} \frac{1}{(N-1)}\tilde{A}_{i,j}\tilde{x}_i + \frac{1}{(N-1)}\tilde{B}_{i,j}u_i \\ +\tilde{F}_{i,\alpha,j}\tilde{x}_\alpha + \tilde{B}_{w_{i,\alpha,j}}\tilde{w}_{i,\alpha} \end{bmatrix} \tag{9}$$

The controllers are; for $i = 1,\dots,N$:

$$u_i = \sum_{j=1}^{M_i} \xi_{i,j}\tilde{K}_{i,j}\tilde{x}_i \tag{10}$$

The tracking errors are given by; for $i = 1,\dots,N$:

$$e_i = \sum_{j=1}^{M_i} \xi_{i,j}\tilde{C}_{i,j}\tilde{x}_i(t) \tag{11}$$

with $\tilde{K}_{i,j} = \begin{bmatrix} 0 & 0 & K_{i,j} \end{bmatrix}$, $\tilde{C}_{i,j} = \begin{bmatrix} -C_{i,j} & H_{r_i} & 0 \end{bmatrix}$,

$$E_i = \begin{bmatrix} I_{n_i} & 0 & 0 \\ 0 & I_{n_i} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{A}_{i,j} = \begin{bmatrix} A_{i,j} & 0 & 0 \\ 0 & A_{r_i} & 0 \\ -C_{i,j} & H_{r_i} & -I_p \end{bmatrix}, \quad \tilde{B}_{i,j} = \begin{bmatrix} B_{i,j} \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{F}_{i,\alpha,j} = \begin{bmatrix} F_{i,\alpha,j} & 0 & 0 \\ 0 & F_{r_{i,\alpha}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{B}_{w_{i,\alpha,j}} = \begin{bmatrix} \frac{1}{(N-1)}B_{w_{i,j}} & F_{w_{i,\alpha,j}} & 0 \\ 0 & 0 & \frac{1}{(N-1)}B_{r_i} \\ 0 & 0 & 0 \end{bmatrix}.$$

Note that the system (9) is called switched interconnected descriptor system ($rank(E) < \dim(E)$).

By substituting the equation (10) into the equation (9), the closed-loop dynamics can be written as follows for $i = 1,\dots,N$:

$$E_i\dot{\tilde{x}}_i = \sum_{j=1}^{M_i} \xi_{i,j} \sum_{\alpha=1,\alpha\neq i}^{N} \begin{bmatrix} \frac{1}{(N-1)}\left(\tilde{A}_{i,j} + \tilde{B}_{i,j}\tilde{K}_{i,j}\right)\tilde{x}_i \\ +\tilde{F}_{i,\alpha,j}\tilde{x}_\alpha + \tilde{B}_{w_{i,\alpha,j}}\tilde{w}_{i,\alpha} \end{bmatrix} \tag{12}$$

At this stage of the work, it is worth pointing out that the output-feedback tracking control problem of the system (1)-(2) can be converted into the stabilization problem of the augmented system (9). However, it is very complicated to work on the first problem due to the large number of crossing terms. Therefore, we will achieve our study by using the augmented system (9). Thus, the problem 1 can be reformulated as follows:

Problem 2**:** The objective is to design the controllers (10) such that the closed-loop switched interconnected descriptor systems (12) is stable with H_∞ disturbance attenuation performance.

At the end of this section, we introduce the following definition and some mathematical materials needed in the development of our results. Then, based on the works done within the context of the switched linear systems and large-scale systems in (Zhang and Feng, 2008; Belkhiat et al., 2014), we propose the following definition.

Definition 2**:** The closed-loop switched interconnected descriptor systems (12) is said stable with H_∞ disturbance attenuation performance, if the following conditions are satisfied:

• ***Condition 1** (Stability condition)*: With zero disturbances input condition $\tilde{w}_{i,\alpha} \equiv 0$, the closed-loop dynamics (12) is stable.

• ***Condition 2** (Robustness condition)*: For all non zero $\tilde{w}_{i,\alpha} \in L_2[0 \quad \infty)$, under zero initial condition $\tilde{x}_i(t_0) \equiv 0$, it holds that for $i = 1,\ldots,N$,

$$J_i \leq 0 \tag{13}$$

With $J_i = \int_0^{+\infty} \tilde{x}_i^T \, \Omega_i \, \tilde{x}_i \, dt - \gamma_i^2 \int_0^{+\infty} \left(\tilde{w}_{i,\alpha}^T \Xi_i \tilde{w}_{i,\alpha} + \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{w}_{i,\alpha}^T \Psi_i \tilde{w}_{i,\alpha} \right) dt$, γ_i are positive scalars which represents the disturbance attenuation level associated to the switched subsystem S_i [2],

$$\Omega_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{p_i} \end{bmatrix}, \Xi_i = \begin{bmatrix} I_{m_i} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{m_i} \end{bmatrix} \text{ and } \Psi_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_{m_i} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Indeed, the definition 2 is similar to the definition 1. It gives the conditions that should be satisfied to solve the problem 2, implicitly the problem 1. The first condition concerns the internal stability of the closed-loop dynamics, whereas the second one concerns the H_∞ disturbance attenuation performance. Furthermore, noting that the local criterion J_i given in definition 2 by the equation (13) is the same one given in definition 1 by equation (8).

Moreover, we provide the following lemma which is helpful, in the sequel, in formulating our results in terms of LMI.

Lemma 1: (Zhou and Khagonekar, 1988):
Let us consider A and B two matrices with appropriate dimensions, there exists a scalar $\tau > 0$ such that the following inequality holds:

$$A^T B + B^T A \leq \tau A^T A + \tau^{-1} B^T B \tag{14}$$

In the following, we describe the design of the proposed output-feedback tracking control.

3. ROBUST TRACKING CONTROL DESIGN

The main goal of this work is to propose sufficient LMI conditions in order to determine the gain matrices $K_{i,j}$, for $i = 1,\ldots,N$ and $j = 1,\ldots,M_i$, so that the robust H_∞ output-feedback tracking performance is satisfied. The main result is summarized in the following theorem [3].

[2] $I_{\bullet}$ denotes an identity matrix with appropriate dimension.

[3] As usual, in a matrix, $(*)$ indicates a symmetrical transpose quantity.

Theorem 1: Assuming that the current mode is present by j and the up-coming mode by j^+. Given positive scalars κ_i, $\mu_{i,j,j^+} \leq 1$, $\beta_{i,\alpha}$ for $i = 1,\ldots,N$, $\alpha = 1,\ldots,N$, $\alpha \neq i$, $j = 1,\ldots,M_i$ and $j^+ = 1,\ldots,M_i$, $j \neq j^+$, if there exist matrices $X_{i,j} = X_{i,j}^T$, $X_{i,j}^1 = \left(X_{i,j}^1\right)^T > 0$, $X_{i,j}^5 = \left(X_{i,j}^5\right)^T > 0$, $Y_{i,j}$ and non-singular matrices $X_{i,j}^9$, such that the LMIs (15) and (16) hold. Then, the closed-loop switched interconnected descriptor system (12) is stable with H_∞ disturbance attenuation levels $\sqrt[2]{\kappa_i}^{\,-\frac{1}{2}}$. Moreover, the controller gains are constructed by $K_{i,j} = Y_{i,j}\left(X_{i,j}^9\right)^{-1}$.

• $$\begin{bmatrix} -\mu_{i,j,j^+} X_{i,j} & X_{i,j} \\ (*) & -X_{i,j^+} \end{bmatrix} \leq 0 \tag{15}$$

• $$\left[\begin{array}{c|ccccccc} \theta_{i,\alpha,j} & X_{i,j}^T \tilde{C}_{i,j}^T \cdots & \cdots & \cdots & X_{i,j}^T \cdots & \cdots & X_{i,j}^T \cdots & X_{i,j}^T \\ \hline (*) & -I_{p_i} & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ \vdots & 0 & -\beta_{1,i} I_{2n_i+p_i} & 0 & \cdots & 0 & \cdots & \vdots \\ \vdots & \vdots & 0 & \ddots & \ddots & \cdots & \cdots & 0 \\ (*) & \vdots & \cdots & \ddots & -\beta_{i-1,i} I_{2n_i+p_i} & 0 & \cdots & \vdots \\ \vdots & 0 & 0 & \cdots & 0 & -\beta_{i+1,i} I_{2n_i+p_i} & \ddots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \cdots & \ddots & \ddots & 0 \\ (*) & 0 & \cdots & 0 & \cdots & \cdots & 0 & -\beta_{N,i} I_{2n_i+p_i} \end{array}\right] \leq 0 \tag{16}$$

with $\theta_{i,\alpha,j}$ and $X_{i,j}$ are described below.

Proof: Without loss of generality, we assume that the switched interconnected descriptor system (9) is regular and impulse free in (Sajja et al., 2013). Indeed, the present proof is divided in two parts corresponding to the conditions 1 and 2 are given in the definition 2.

• ***Condition 1**:* At this step of the study and according to the definition 2, we assume that we work under zero disturbance input condition $\tilde{w}_{i,\alpha} \equiv 0$. Then, our objective is to provide sufficient conditions ensuring that the closed-loop dynamics (12) is stable. For this purpose, we consider the following candidate multiple switched Lyapunov functions:

$$V\left(\{\tilde{x}_i(t)\}_{i=1,\ldots,N}\right) = \sum_{i=1}^{N} \vartheta_i\left(t, \tilde{x}_i(t)\right) \tag{17}$$

with $\vartheta_i\left(t, \tilde{x}_i(t)\right) = \sum_{i=1}^{M_i} \xi_{i,j} \nu_{i,j}\left(t, \tilde{x}_i(t)\right)$

and $\nu_{i,j}\left(t, \tilde{x}_i(t)\right) = \tilde{x}_i^T(t) E_i^T \left(X_{i,j}\right)^{-1} \tilde{x}_i(t)$

Thus, the closed-loop dynamics (12) is stable under arbitrary switching signal if the following conditions (18), (19) and (20) are verified:

- $V\left(\{\tilde{x}_i(t)\}_{i=1,\dots,N}\right) \geq 0$ (18)
- $\dot{V}\left(\{\tilde{x}_i(t)\}_{i=1,\dots,N}\right) < 0$ (19)
- There exists, for $i=1,\dots,N$, $j=1,\dots,M_i$, $j^+=1,\dots,M_i$, $j \neq j^+$, $\mu_{i,j,j^+} \leq 1$ such that:

$$\nu_{i,j^+}\left(t_{j\to j^+},\tilde{x}_i(t)\right) \leq \mu_{i,j,j^+}\nu_{i,j}\left(t_{j\to j^+},\tilde{x}_i(t)\right) \quad (20)$$

where the decreasing rates $\mu_{i,j,j^+} \leq 1$ are positive scalars describing the Lyapunov-like evolution at the switching time $t_{j\to j^+}$ from the current mode j to the up-coming j^+ and

$$\dot{V}\left(\{\tilde{x}_i(t)\}_{i=1,\dots,N}\right) = \sum_{i=1}^{N}\sum_{j=1}^{M_i}\xi_{i,j}(t)\begin{pmatrix}\dot{\tilde{x}}_i(t)E_i^T\left(X_{i,j}\right)^{-1}\tilde{x}_i(t)\\ +\tilde{x}_i^T(t)\left(X_{i,j}\right)^{-T}E_i\dot{\tilde{x}}_i(t)\end{pmatrix}.$$

In the sequel, we deal with the stability under arbitrary switching signal of the closed-loop switched interconnected descriptor system in three steps. In the first one, our purpose is to provide the conditions ensuring that the equality (18) is satisfied. Thus, we develop the inequality (18) as follows:

$$V\left(\{\tilde{x}_i(t)\}_{i=1,\dots,N}\right) = \sum_{i=1}^{N}\sum_{j=1}^{M_i}\xi_{i,j}(t)\left(\tilde{x}_i^T(t)E_i^T\left(X_{i,j}\right)^{-1}\tilde{x}_i(t)\right) \geq 0 \quad (21)$$

We can remark that the inequality (21) is verified if, for $i=1,\dots,N$, $j=1,\dots,M_i$, the following conditions are verified:

$$E_i^T\left(X_{i,j}\right)^{-1} = \left(X_{i,j}\right)^{-T}E_i \geq 0 \quad (22)$$

$$\theta_{i,\alpha,j} = \left[\begin{array}{c:c:c} \left(X_{i,j}^1\right)^T A_{i,j}^T + A_{i,j}X_{i,j}^1 + \sum_{\alpha=1,\alpha\neq i}^{N}\beta_{i,\alpha}F_{i,\alpha,j}F_{i,\alpha,j}^T + \kappa_i B_{w_{i,j}}B_{w_{i,j}}^T + \frac{\kappa_i}{N-1}\sum_{\alpha=1,\alpha\neq i}^{N}F_{w_{i,\alpha,j}}\sum_{\alpha=1,\alpha\neq i}^{N}F_{w_{i,\alpha,j}}^T & 0 & -\left(X_{i,j}^1\right)^T C_{i,j}^T + B_{i,j}Y_{i,j} \\ \hdashline (*) & \left(X_{i,j}^5\right)^T A_{r_i}^T + A_{r_i}X_{i,j}^5 + \sum_{\alpha=1,\alpha\neq i}^{N}\beta_{i,\alpha}F_{r_{i,\alpha}}F_{r_{i,\alpha}}^T & \left(X_{i,j}^5\right)^T H_{r_i}^T \\ \hdashline (*) & (*) & -\left(X_{i,j}^9\right)^T - X_{i,j}^9 \end{array}\right]$$

$X_{i,j} = diag\left(X_{i,j}^1 \quad X_{i,j}^5 \quad X_{i,j}^9\right)$.

with $E_i = \begin{bmatrix} I_{n_i} & 0 & 0 \\ 0 & I_{n_i} & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $X_{i,j}$ is considered as

$$X_{i,j} = \begin{bmatrix} X_{i,j}^1 & X_{i,j}^2 & X_{i,j}^3 \\ X_{i,j}^4 & X_{i,j}^5 & X_{i,j}^6 \\ X_{i,j}^7 & X_{i,j}^8 & X_{i,j}^9 \end{bmatrix}.$$

Multiplying (22), left by $X_{i,j}^T$ and right by its transpose, the inequalities (22) are equivalent to: for $i=1,\dots,N$, $j=1,\dots,M_i$,

$$X_{i,j}^T E_i^T = E_i X_{i,j} \geq 0 \quad (23)$$

The latter inequalities (23), implicitly the conditions (18), are verified if the following two conditions, (24) and (25), are satisfied for $i=1,\dots,N$, $j=1,\dots,M_i$,

$$X_{i,j}^1 = \left(X_{i,j}^1\right)^T > 0 \text{ and } X_{i,j}^5 = \left(X_{i,j}^5\right)^T > 0 \quad (24)$$

and $X_{i,j}^l = 0 \times I_\bullet$, for $l \in \{2,3,4,6\}$. (25)

Remark 1: The matrices $X_{i,j}^k$, for $k \in \{7,8,9\}$, are called decision matrices and they can be chosen freely. Moreover, these matrices are necessary to obtain in the sequel the LMI conditions ensuring the robustness of the tracking control.

In the second step of the stability study of the closed-loop switched interconnected descriptor system, we aim to develop the inequality (19).

$$\dot{V}\left(\{\tilde{x}_i(t)\}_{i=1,\dots,N}\right) = \sum_{i=1}^{N}\sum_{j=1}^{M_i}\xi_{i,j}\sum_{\alpha=1,\alpha\neq i}^{N}\begin{pmatrix}\tilde{x}_i^T\frac{1}{(N-1)}\left(\bar{\bar{A}}_{i,j}^T\left(X_{i,j}\right)^{-1}+\left(X_{i,j}\right)^{-T}\bar{\bar{A}}_{i,j}\right)\tilde{x}_i \\ +\tilde{x}_\alpha^T\tilde{F}_{i,\alpha,j}^T\left(X_{i,j}\right)^{-1}\tilde{x}_i+\tilde{x}_i^T\left(X_{i,j}\right)^{-T}\tilde{F}_{i,\alpha,j}\tilde{x}_\alpha\end{pmatrix} < 0 \quad (26)$$

with $\bar{\bar{A}}_{i,j} = \tilde{A}_{i,j} + \tilde{B}_{i,j}\tilde{K}_{i,j}$.

Using lemma 1 and assuming that, for $i=1,\dots,N$, $\tau_{i,i}=0$, the inequality (26) becomes:

$$\sum_{i=1}^{N}\sum_{j=1}^{M_i}\xi_{i,j}\left[\tilde{x}_i^T\begin{pmatrix}\bar{\bar{A}}_{i,j}^T\left(X_{i,j}\right)^{-1}+\left(X_{i,j}\right)^{-T}\bar{\bar{A}}_{i,j}+\sum_{p=1}^{N}\tau_{p,i}I_{2n_i+p_i} \\ +\left(X_{i,j}\right)^{-T}\sum_{\alpha=1,\alpha\neq i}^{N}\left(\tau_{i,\alpha}^{-1}\tilde{F}_{i,\alpha,j}\tilde{F}_{i,\alpha,j}^T\right)\left(X_{i,j}\right)^{-1}\end{pmatrix}\tilde{x}_i\right] < 0 \quad (27)$$

Inequality (27) is verified, $\forall \tilde{x}_i(t)$, for $i=1,\dots,N$, $\alpha=1,\dots,N$, $\alpha \neq i$, $j_i=1,\dots,M_i$, if:

$$\begin{aligned} &\bar{\bar{A}}_{i,j}^T \left(X_{i,j}\right)^{-1} + \left(X_{i,j}\right)^{-T} \bar{\bar{A}}_{i,j} + \sum_{p=1}^{N} \tau_{p,i} I_{2n_i+p_i} \\ &+ \left(X_{i,j}\right)^{-T} \sum_{\alpha=1,\alpha\neq i}^{N} \left(\tau_{i,\alpha}^{-1} \tilde{F}_{i,\alpha,j} \tilde{F}_{i,\alpha,j}^T\right) \left(X_{i,j}\right)^{-1} < 0 \end{aligned} \quad (28)$$

Then, multiplying the inequality (28), left by $\left(X_{i,j}\right)^T$ and right by $X_{i,j}$, it yields:

$$\begin{aligned} &\left(X_{i,j}\right)^T \bar{\bar{A}}_{i,j}^T + \bar{\bar{A}}_{i,j} X_{i,j} + \sum_{\alpha=1,\alpha\neq i}^{N} \left(\tau_{i,\alpha}^{-1} \tilde{F}_{i,\alpha,j} \tilde{F}_{i,\alpha,j}^T\right) \\ &+ \left(X_{i,j}\right)^T \sum_{p=1}^{N} \left(\tau_{p,i} I_{2n_i+p_i}\right) X_{i,j} < 0 \end{aligned} \quad (29)$$

Remark 2: At this step, it should be stressed that the conditions (29) are not LMI and they cannot be resolved with the LMI solvers. To skip to LMI formulation, some mathematical developments should be performed. However, we end the second step of the stability study at this development level. Indeed, this matter will be discussed in greater detail in the sequel, where we will show that the conditions (29), implicitly the conditions (19), will be omitted from the Theorem 1. In the third step of the stability study, we focus on the stability conditions (20). Their aim is to ensure the global behavior of the like-Lyapunov function at the switching time $t_{j\to j^+}$.

According to the condition (20), we can write:

$$\left(X_{i,j^+}\right)^{-1} \leq \mu_{i,j,j^+} \left(X_{i,j}\right)^{-1} \quad (30)$$

for $i=1,\dots,N$, $j=1,\dots,M_i$, $j^+=1,\dots,M_i$, $j\neq j^+$, $\mu_{i,j,j^+} \leq 1$.

Left and right multiplying the latter condition (30) by $X_{i,j}$, we can obtain:

$$X_{i,j} \left(X_{i,j^+}\right)^{-1} X_{i,j} - \mu_{i,j,j^+} X_{i,j} \leq 0 \quad (31)$$

Applying Schur's complement, the LMIs (15) presented in the Theorem 1 are provided.

In the next part of the proof, we are focused on dealing with the robustness of the proposed stabilization control according to the condition 2 of the definition 2.

• ***Condition 2****:* In this subsection, we provide some sufficient conditions concerning the robustness of the proposed tracking control with local disturbance attenuation levels. Under zero initial condition $\tilde{x}_i(t_0) \equiv 0$ and for any non-zero $\tilde{w}_{i,\alpha} \in L_2[0 \;\; \infty)$, our objective is to formulate the conditions (13) of the definition 2 in terms of LMI.

We can define, based on the condition (13), the global criterion $J = \sum_{i=1}^{N} J_i$. Such as $\tilde{x}_i^T \Omega_i \tilde{x}_i = e_i^T e_i$, the global criterion can be written as:

$$J = \sum_{i=1}^{N} \int_0^{+\infty} \left(e_i^T e_i - \gamma_i^2 \left(\tilde{w}_{i,\alpha}^T \Xi_i \tilde{w}_{i,\alpha} + \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{w}_{i,\alpha}^T \Psi_i \tilde{w}_{i,\alpha} \right) \right) dt \quad (32)$$

Thus, our objective now is to provide the sufficient conditions ensuring that both the global criterion J and the local criterions J_i (for $i=1,\dots,N$) are negative.

Let us consider the candidate multiple switched Lyapunov function (17). Hence, the equation (32) can be reformulated as follow:

$$\begin{aligned} J = &\sum_{i=1}^{N} \int_0^{+\infty} \left(e_i^T e_i - \gamma_i^2 \left(\tilde{w}_{i,\alpha}^T \Xi_i \tilde{w}_{i,\alpha} + \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{w}_{i,\alpha}^T \Psi_i \tilde{w}_{i,\alpha} \right) + \frac{d\vartheta_i(t,\tilde{x}_i)}{dt} \right) dt \\ &- \sum_{i=1}^{N} \vartheta_i(t,\tilde{x}_i) \end{aligned} \quad (33)$$

Such that $\vartheta_i(t,\tilde{x}_i)$ are positive for $i=1,\dots,N$, the criterion J is negative, if the following condition is verified:

$$\sum_{i=1}^{N} \left(e_i^T e_i - \gamma_i^2 \left(\tilde{w}_{i,\alpha}^T \Xi_i \tilde{w}_{i,\alpha} + \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{w}_{i,\alpha}^T \Psi_i \tilde{w}_{i,\alpha} \right) + \frac{d\vartheta_i(t,\tilde{x}_i)}{dt} \right) \leq 0 \quad (34)$$

By using the equation (12), the latter condition (34) can be reformulated such us:

$$\sum_{i=1}^{N} \left(\begin{aligned} &\sum_{j=1}^{M_i} \xi_{i,j} \left(\tilde{x}_i^T \tilde{C}_{i,j}^T \tilde{C}_{i,j} \tilde{x}_i \right) - \gamma_i^2 \left(\tilde{w}_{i,\alpha}^T \Xi_i \tilde{w}_{i,\alpha} + \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{w}_{i,\alpha}^T \Psi_i \tilde{w}_{i,\alpha} \right) \\ &+ \sum_{j=1}^{M_i} \sum_{\alpha=1,\alpha\neq i}^{N} \xi_{i,j} \left(\begin{aligned} &\tilde{x}_i^T \frac{1}{(N-1)} \left(\bar{\bar{A}}_{i,j}^T \left(X_{i,j}\right)^{-1} + \left(X_{i,j}\right)^{-T} \bar{\bar{A}}_{i,j} \right) \tilde{x}_i \\ &+ \tilde{x}_\alpha^T \tilde{F}_{i,\alpha,j}^T \left(X_{i,j}\right)^{-1} \tilde{x}_i + \tilde{x}_i^T \left(X_{i,j}\right)^{-T} \tilde{F}_{i,\alpha,j} \tilde{x}_\alpha \\ &+ \tilde{w}_{i,\alpha}^T \tilde{B}_{w_{i,\alpha,j}}^T \left(X_{i,j}\right)^{-1} \tilde{x}_i + \tilde{x}_i^T \left(X_{i,j}\right)^{-T} \tilde{B}_{w_{i,\alpha,j}} \tilde{w}_{i,\alpha} \end{aligned} \right) \end{aligned} \right) \leq 0 \quad (35)$$

As in the previous parts, one used lemma 1and assuming that, for $i=1,\dots,N$, $\tau_{i,i}=0$, the inequality (35) becomes:

$$\sum_{i=1}^{N} \sum_{j=1}^{M_i} \xi_{i,j} \left(\begin{aligned} &\tilde{x}_i^T \left(\begin{aligned} &\bar{\bar{A}}_{i,j}^T \left(X_{i,j}\right)^{-1} + \left(X_{i,j}\right)^{-T} \bar{\bar{A}}_{i,j} + \sum_{p=1}^{N} \tau_{p,i} I_{2n_i \times p_i} \\ &+ \left(X_{i,j}\right)^{-T} \sum_{\alpha=1,\alpha\neq i}^{N} \left(\tau_{i,\alpha}^{-1} \tilde{F}_{i,\alpha} \tilde{F}_{i,\alpha}^T\right) \left(X_{i,j}\right)^{-1} + \tilde{C}_{i,j}^T \tilde{C}_{i,j} \end{aligned} \right) \tilde{x}_i \\ &- \tilde{w}_{i,\alpha}^T \gamma_i^2 \left(\Xi_i + \sum_{\alpha=1,\alpha\neq i}^{N} \Psi_i \right) \tilde{w}_{i,\alpha} \\ &+ \tilde{w}_{i,\alpha}^T \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{B}_{w_{i,\alpha,j}}^T \left(X_{i,j}\right)^{-1} \tilde{x}_i + \tilde{x}_i^T \left(X_{i,j}\right)^{-T} \sum_{\alpha=1,\alpha\neq i}^{N} \tilde{B}_{w_{i,\alpha,j}} \tilde{w}_{i,\alpha} \end{aligned} \right) \leq 0 \quad (36)$$

The latter equation (36) can be reformulated such as:

$$\sum_{i=1}^{N}\sum_{j=1}^{M_i}\xi_{i,j}\begin{bmatrix}\tilde{x}_i\\ \tilde{w}_{i,\alpha}\end{bmatrix}^T \underbrace{\left[\begin{array}{c|c}\Omega_{11} & \sum_{\alpha=1,\alpha\neq i}^{N}\left(X_{i,j}\right)^{-T}\tilde{B}_{w_{i,\alpha,j}}\\ \hline (*) & -\gamma_i^2\left(\Xi_i+\sum_{\alpha=1,\alpha\neq i}^{N}\Psi_i\right)\end{array}\right]}_{\mathrm{T}_{i,\alpha,j}}\begin{bmatrix}\tilde{x}_i\\ \tilde{w}_{i,\alpha}\end{bmatrix}\leq 0 \tag{37}$$

with

$$\Omega_{11}=\bar{\bar{A}}_{i,j}^T\left(X_{i,j}\right)^{-1}+\left(X_{i,j}\right)^{-T}\bar{\bar{A}}_{i,j}+\sum_{p=1}^{N}\tau_{p,i}I_{2n_i\times p_i}$$
$$+\left(X_{i,j}\right)^{-T}\sum_{\alpha=1,\alpha\neq i}^{N}\left(\tau_{i,\alpha}^{-1}\tilde{F}_{i,\alpha}\tilde{F}_{i,\alpha}^T\right)\left(X_{i,j}\right)^{-1}+\tilde{C}_{i,j}^T\tilde{C}_{i,j}$$

Based on the inequality (37), the global criterion J, given by equation (33), is negative if the following sufficient condition is verified for $i=1,\ldots,N$, $\alpha=1,\ldots,N$, $\alpha\neq i$, $j=1,\ldots,M_i$;

$$T_{i,\alpha,j}\leq 0 \tag{38}$$

In other words, the verification of the inequality $\mathrm{T}_{i,\alpha,j}\leq 0$, for $i=1,\ldots,N$, $\alpha=1,\ldots,N$, $\alpha\neq i$, $j=1,\ldots,M_i$, implies that each local criterion J_i, for $i=1,\ldots,N$, is satisfied separately as required in the definition 2.

Applying the inverse of Schur's complement, we can write $\mathrm{T}_{i,\alpha,j}\leq 0$ as follows; for $i=1,\ldots,N$, $\alpha=1,\ldots,N$, $\alpha\neq i$, $j=1,\ldots,M_i$:

$$\bar{\bar{A}}_{i,j}^T\left(X_{i,j}\right)^{-1}+\left(X_{i,j}\right)^{-T}\bar{\bar{A}}_{i,j}+\tilde{C}_{i,j}^T\tilde{C}_{i,j}$$
$$+\sum_{\alpha=1,\alpha\neq i}^{N}\left(X_{i,j}\right)^{-T}\tilde{B}_{w_{i,\alpha,j}}\left(\kappa_i\left(\Xi_i+\sum_{\alpha=1,\alpha\neq i}^{N}\Psi_i\right)^{-1}\right)\sum_{\alpha=1,\alpha\neq i}^{N}\tilde{B}_{w_{i,\alpha,j}}^T\left(X_{i,j}\right)^{-1}$$
$$+\left(X_{i,j}\right)^{-T}\sum_{\alpha=1,\alpha\neq i}^{N}\left(\tau_{i,\alpha}^{-1}\tilde{F}_{i,\alpha}\tilde{F}_{i,\alpha}^T\right)\left(X_{i,j}\right)^{-1}+\sum_{p=1}^{N}\tau_{p,i}I_{2n_i\times p_i}\leq 0 \tag{39}$$

with $\kappa_i=\left(\gamma_i^2\right)^{-1}$.

Multiplying the latter inequality (39), left by $\left(X_{i,j}\right)^T$ and right by $X_{i,j}$, we obtain the following inequalities for $i=1,\ldots,N$, $\alpha=1,\ldots,N$, $\alpha\neq i$, $j=1,\ldots,M_i$:

$$\left(X_{i,j}\right)^T\bar{\bar{A}}_{i,j}^T+\bar{\bar{A}}_{i,j}X_{i,j}+\left(X_{i,j}\right)^T\tilde{C}_{i,j}^T\tilde{C}_{i,j}X_{i,j}+\sum_{\alpha=1,\alpha\neq i}^{N}\left(\tau_{i,\alpha}^{-1}\tilde{F}_{i,\alpha}\tilde{F}_{i,\alpha}^T\right)$$
$$+\sum_{p=1}^{N}\tau_{p,i}\left(X_{i,j}\right)^TX_{i,j}+\sum_{\alpha=1,\alpha\neq i}^{N}\tilde{B}_{w_{i,\alpha,j}}\left(\kappa_i\left(\Xi_i+\sum_{\alpha=1,\alpha\neq i}^{N}\Psi_i\right)^{-1}\right)\sum_{\alpha=1,\alpha\neq i}^{N}\tilde{B}_{w_{i,\alpha,j}}^T\leq 0 \tag{40}$$

Remark 3: At this step of study, a significant simplification, that relates both inequalities (29) and (40), can be considered. Indeed, regarding the inequality (29), we can remark that it is bounded by the inequality (40). Then, the inequality (29) will be systematically verified when the inequality (40) is satisfied. For this reason, the inequality (29) is omitted from the Theorem 1 as mentioned above in the remark 2. Hence, the Theorem 1 contains only the condition (40) formulated in terms of LMI (readers are referred to the appendix section).

Remark 4: Based on the study above, the design procedure of the switched decentralized tracking control for interconnected switched systems is summarized as follows:

Step.1. Construct the mathematical model of the real system according to equations (1) and (2)

Step.2. Choose stable interconnected systems as a reference model according to equations (4) and (5).

Step.3. Solve the LMI conditions (15) and (16) of the theorem 1, using LMI solver, in order to obtain the matrices $X_{i,j}^1$, $X_{i,j}^5$, $X_{i,j}^9$ and $Y_{i,j}$ such that the positive scalars κ_i, $\beta_{i,\alpha}$ and $\mu_{i,j,j^+}\leq 1$ are chosen freely.

Step.4. Construct the controller gains $K_{i,j}=Y_{i,j}\left(X_{i,j}^9\right)^{-1}$.

Step.5. Implement the obtained switched decentralized controller in the control scheme.

4. SIMULATION AND RESULTS

In this section, we illustrate the effectiveness of the proposed decentralized tracking control via a numerical simulation. For that, we consider the following switched interconnected large-scale system S composed of three continuous-time switched subsystems S_1, S_2 and S_3:

Subsystem S_1 is:

$$\dot{x}_1(t)=\sum_{j=1}^{2}\xi_{1,j}(t)\begin{bmatrix}A_{1,j}x_1(t)+B_{1,j}u_1(t)+B_{w_{1,j}}w_1(t)+F_{1,2,j}x_2(t)\\ +F_{1,3,j}x_3(t)+F_{w_{1,2,j}}w_2(t)+F_{w_{1,3,j}}w_3(t)\end{bmatrix}$$

$$y_1=\sum_{j=1}^{2}\xi_{1,j}(t)\left[C_{1,j}x_1(t)\right]$$

with, for ***mode 1***, $A_{1,1}=\begin{bmatrix}-0.1 & 1\\ 0.5 & -1.1\end{bmatrix}$, $B_{1,1}=\begin{bmatrix}2\\ 0\end{bmatrix}$, $B_{w_{1,1}}=\begin{bmatrix}-0.1\\ 0.1\end{bmatrix}$, $F_{1,2,1}=\begin{bmatrix}0.01 & 0.2\\ 0 & 0.1\end{bmatrix}$, $F_{1,3,1}=\begin{bmatrix}0.1 & 0\\ 0.1 & 0.1\end{bmatrix}$, $F_{w_{1,2,1}}=\begin{bmatrix}0.01\\ 0.1\end{bmatrix}$, $F_{w_{1,3,1}}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}$ and $C_{1,1}=\begin{bmatrix}4 & 0\end{bmatrix}$

and for ***mode 2*** , $A_{1,2}=\begin{bmatrix}-1 & 0.25\\ 0.39 & -1.5\end{bmatrix}$, $B_{1,2}=\begin{bmatrix}0\\ 0.3\end{bmatrix}$, $B_{w_{1,2}}=\begin{bmatrix}0.1\\ -0.1\end{bmatrix}$, $F_{1,2,2}=\begin{bmatrix}0.1 & 0\\ 0.2 & 0.1\end{bmatrix}$, $F_{1,3,2}=\begin{bmatrix}0.1 & 0\\ 0.02 & 0.1\end{bmatrix}$, $F_{w_{1,2,2}}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}$, $F_{w_{1,3,2}}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}$ and $C_{1,2}=\begin{bmatrix}0 & 3\end{bmatrix}$.

Subsystem S_2 is:

$$\dot{x}_2(t)=\sum_{j=1}^{2}\xi_{2,j}(t)\begin{bmatrix}A_{2,j}x_2(t)+B_{2,j}u_2(t)+B_{w_{2,j}}w_2(t)\\ +F_{2,1,j}x_1(t)+F_{2,3,j}x_3(t)\\ +F_{w_{2,3,j}}w_3(t)+F_{w_{2,1,j}}w_1(t)\end{bmatrix}$$

$$y_2=\sum_{j=1}^{2}\xi_{2,j}(t)\left[C_{2,j}x_2(t)\right]$$

with, for ***mode 1***, $A_{2,1}=\begin{bmatrix}-4.6 & 2\\ 1.5 & -0.1\end{bmatrix}$, $B_{2,1}=\begin{bmatrix}2.5\\ 0\end{bmatrix}$, $B_{w_{2,1}}=\begin{bmatrix}-0.2\\ 0.1\end{bmatrix}$, $F_{2,1,1}=\begin{bmatrix}1 & 0.2\\ 0.01 & 0.1\end{bmatrix}$, $F_{2,3,1}=\begin{bmatrix}1 & 0.2\\ 0.01 & 0\end{bmatrix}$, $F_{w2,1,1}=\begin{bmatrix}1\\ 0.1\end{bmatrix}$, $F_{w_{2,3,1}}=\begin{bmatrix}1\\ 0\end{bmatrix}$ and $C_{2,1}=\begin{bmatrix}3.5 & 0\end{bmatrix}$,

and for ***mode 2***, $A_{2,2}=\begin{bmatrix}-4 & 0.2\\ 1.2 & -3\end{bmatrix}$, $B_{2,2}=\begin{bmatrix}0\\ 0.6\end{bmatrix}$, $B_{w_{2,2}}=\begin{bmatrix}0.2\\ -0.1\end{bmatrix}$, $F_{2,1,2}=\begin{bmatrix}0.1 & 1\\ 0.5 & 0.1\end{bmatrix}$, $F_{2,3,2}=\begin{bmatrix}0.4 & 1\\ 0.02 & 0.1\end{bmatrix}$, $F_{w_{2,1,2}}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}$, $F_{w_{2,3,2}}=\begin{bmatrix}0.4\\ 0.1\end{bmatrix}$ and $C_{2,2}=\begin{bmatrix}0 & 3.2\end{bmatrix}$

Subsystem S_3 is:

$$\dot{x}_3(t)=\sum_{j=1}^{2}\xi_{3,j}(t)\begin{bmatrix}A_{3,j}x_3(t)+B_{3,j}u_3(t)+B_{w_{3,j}}w_3(t)\\ +F_{3,1,j}x_1(t)+F_{3,2,j}x_2(t)\\ +F_{w_{3,1,j}}w_1(t)+F_{w_{3,2,j}}w_2(t)\end{bmatrix}$$

$$y_3=\sum_{j=1}^{2}\xi_{3,j}(t)\left[C_{3,j}x_3(t)\right]$$

with, for ***mode 1***, $A_{3,1}=\begin{bmatrix}-14.6 & 11\\ 10.5 & -12.1\end{bmatrix}$, $B_{3,1}=\begin{bmatrix}2.75\\ 0.1\end{bmatrix}$, $B_{w_{3,1}}=\begin{bmatrix}-1.2\\ 0.1\end{bmatrix}$, $F_{3,1,1}=\begin{bmatrix}0.01 & 0.7\\ 0 & -0.1\end{bmatrix}$, $F_{3,2,1}=\begin{bmatrix}0.1 & 0\\ 1 & -0.1\end{bmatrix}$, $F_{w_{3,1,1}}=\begin{bmatrix}0.01\\ -0.1\end{bmatrix}$, $F_{w_{3,2,1}}=\begin{bmatrix}0.1\\ -0.1\end{bmatrix}$ and $C_{3,1}=\begin{bmatrix}2.65 & 0\end{bmatrix}$

and for ***mode 2***, $A_{3,2}=\begin{bmatrix}-15.9 & 9.2\\ 10.2 & -13\end{bmatrix}$, $B_{3,2}=\begin{bmatrix}0\\ 0.76\end{bmatrix}$, $B_{w_{3,2}}=\begin{bmatrix}0.45\\ -0.1\end{bmatrix}$, $F_{3,1,2}=\begin{bmatrix}0.01 & -0.1\\ 0.8 & 0.1\end{bmatrix}$, $F_{3,2,2}=\begin{bmatrix}0.1 & 0.01\\ 2 & 0.1\end{bmatrix}$, $F_{w_{3,1,2}}=\begin{bmatrix}0.01\\ 0.1\end{bmatrix}$, $F_{w_{3,2,2}}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}$ and $C_{3,2}=\begin{bmatrix}0 & 3.45\end{bmatrix}$.

The initial states are given as follow: $x_1(0)=[10 \quad 20]^T$, $x_2(0)=[-10 \quad 20]^T$, $x_3(0)=[-15 \quad 23]^T$.

As stated above, each subsystem has its own mode's evolution independently from other subsystems. Noting that the large-scale system S is unstable.

As mentioned above, we consider the following reference large-scale model to specify the desired trajectory for the overall large-scale system S. This reference model is composed of three subsystems:

1st Subsystem:

$$\dot{x}_{r_1}(t)=A_{r_1}x_{r_1}(t)+B_{r_1}r_1(t)+F_{r_{1,2}}x_{r_2}(t)+F_{r_{1,3}}x_{r_3}(t)$$

$$y_{r_1}=H_{r_1}x_{r_1}(t)$$

with $A_{r_1}=\begin{bmatrix}-1.5 & 0.25\\ 0.125 & -0.125\end{bmatrix}$, $B_{r_1}=\begin{bmatrix}0.1\\ 2\end{bmatrix}$, $F_{r_{1,2}}=\begin{bmatrix}0.1 & 0.2\\ 0 & 0.1\end{bmatrix}$, $F_{r_{1,3}}=\begin{bmatrix}0.1 & 0\\ 0.1 & 0.1\end{bmatrix}$ and $H_{r_1}=\begin{bmatrix}1 & -0.3\end{bmatrix}$.

2nd Subsystem:

$$\dot{x}_{r_2}(t)=A_{r_2}x_{r_2}(t)+B_{r_2}r_2(t)+F_{r_{2,1}}x_{r_2}(t)+F_{r_{2,3}}x_{r_3}(t)$$

$$y_{r_2}=H_{r_2}x_{r_2}(t)$$

with $A_{r_2}=\begin{bmatrix}-3 & 0.5\\ 0.3 & -0.3\end{bmatrix}$, $B_{r_2}=\begin{bmatrix}-1\\ 1\end{bmatrix}$, $F_{r_{2,1}}=\begin{bmatrix}1 & 0.2\\ 0.01 & 0.1\end{bmatrix}$, $F_{r_{2,3}}=\begin{bmatrix}1 & 0.2\\ 0.01 & 0\end{bmatrix}$ and $H_{r_2}=\begin{bmatrix}2 & -0.6\end{bmatrix}$.

3rd Subsystem:

$$\dot{x}_{r_3}(t)=A_{r_3}x_{r_3}(t)+B_{r_3}r_3(t)+F_{r_{3,1}}x_{r_1}(t)+F_{r_{3,2}}x_{r_2}(t)$$

$$y_{r_3}=H_{r_3}x_{r_3}(t)$$

with $A_{r_3} = \begin{bmatrix} -13 & 6.5 \\ 8 & -10 \end{bmatrix}$, $B_{r_3} = \begin{bmatrix} -1.01 \\ 1 \end{bmatrix}$, $F_{r_{3,1}} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0 \end{bmatrix}$, $F_{r_{3,2}} = \begin{bmatrix} 0.2 & 0.2 \\ 0.01 & 0 \end{bmatrix}$ and $H_{r_3} = \begin{bmatrix} 2.3 & -0.6 \end{bmatrix}$.

Now, by using the Matlab LMI toolbox, a solution of the Theorem 1 is obtained and leads to the design of the proposed control. This latter is composed of three local switched controllers of the form (6) and defined as follows:

1st controller: $u_1(t) = \sum_{j=1}^{2} \xi_{1,j}(t)\left[K_{1,j} e_1(t)\right]$ for the attenuation level $\tau_1 = 2.7$ and the decreasing rates $\mu_{1,j,j^+} = 0.99$, we obtain the following gains: $K_{1,1} = 39.54$ and $K_{1,2} = 54.03$.

2nd controller: $u_2(t) = \sum_{j=1}^{2} \xi_{2,j}(t)\left[K_{2,j} e_2(t)\right]$ for the attenuation level $\tau_2 = 2.3$ and the decreasing rates $\mu_{2,j,j^+} = 0.81$, we obtain the following gains: $K_{2,1} = 17.66$ and $K_{2,2} = 28.90$.

3rd controller: $u_3(t) = \sum_{j=1}^{2} \xi_{3,j}(t)\left[K_{3,j} e_3(t)\right]$ for the attenuation level $\tau_3 = 2.5$ and the decreasing rates $\mu_{3,j,j^+} = 0.87$, we obtain the following gains: $K_{3,1} = 18.35$ and $K_{3,2} = 29.27$.

The following reference signals are considered under this simulation:

$$r_1(t) = \begin{cases} 1 & 0 \le t \le 5s \\ 20 \times sin(0.02 * t) & 5 < t \le 10s \\ 7 \times square(0.01 * t) & 10 < t \le 15s \end{cases},$$

$$r_2(t) = \begin{cases} 20 \times cos(0.008 * t) & 0 \le t \le 7.5s \\ 0.5 & 7.5 < t \le 15s \end{cases},$$

$$r_3(t) = \begin{cases} 10 \times square(0.005 * t) & 0 \le t \le 5s \\ 1 & 5 < t \le 10s \\ 20 \times cos(0.008 * t) & 10 < t \le 15s \end{cases},$$

and the external disturbances signal $w_i(t)$, for $i = 1;2;3$, are considered as a white noise sequence.

The outputs $y_i(t)$ and $y_{r_i}(t)$, for $i = 1,\ldots,3$, are shown in Fig. 2. The tracking errors evolution, computed between the outputs of the large-scale system S and the reference model, are depicted in Fig. 3. As expected, the outputs $y_i(t)$ of the large-scale system S can track the desired trajectories $y_{r_i}(t)$ (for $i = 1,\ldots,3$) after a finite time interval. Thus, the obtained results demonstrate that the proposed controller can guarantee the H_∞ output-tracking performance.

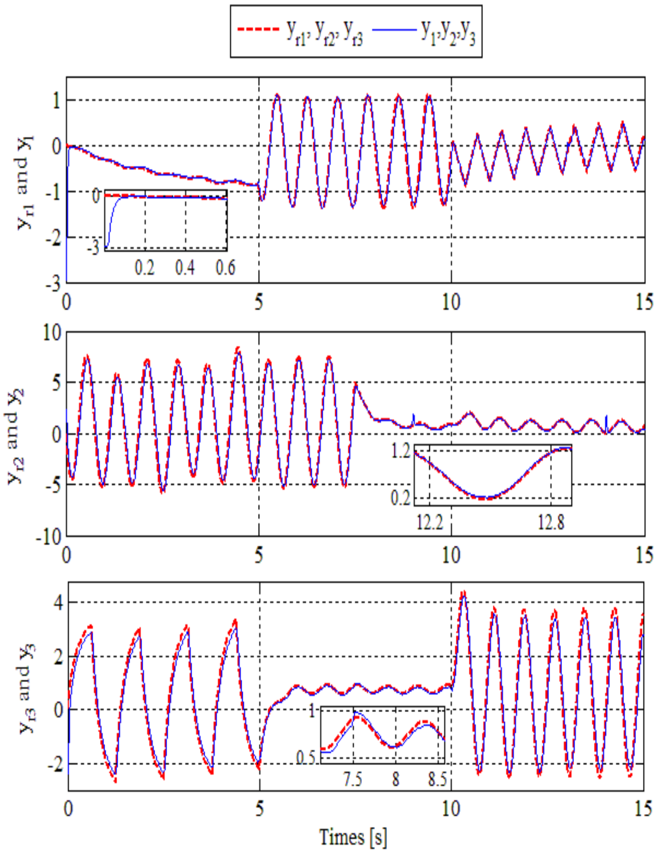

Fig. 2. Outputs trajectory (system and reference model).

5. CONCLUSION

In this paper, the problem of the decentralized output-feedback tracking control, intended to drive a class of switched interconnected large-scale systems subject to disturbances, has been studied. Two kinds of the interconnections between different subsystems have been considered: interconnections depending on state vector and those depending on unknown disturbances. By effectively using the descriptor redundancy formulation, some new sufficient conditions, which guarantee that the proposed controller had the H_∞ output-feedback tracking performance, have been derived. These conditions have been reformulated in terms of LMI thanks to the multiple switched Lyapunov functions. Some concrete simulations are achieved to illustrate the effectiveness of the proposed approach. Moreover, considering the performances obtained in this work in terms of output-feedback tracking control under synchronous switching, relaxation of this assumption and extension of the proposed approach to general switched systems under asynchronous switching will be the focus of future work.

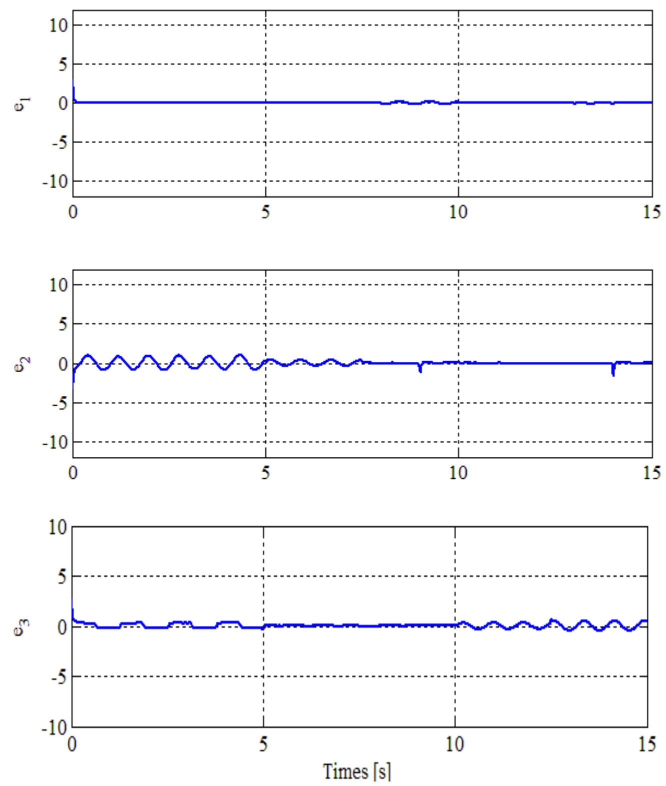

Fig. 3. Evolution of tracking errors.

APPENDIX A

In this section, we are focused on formulating the inequality (40) in terms of LMI. We recall that $\tau_{ii}=0$, the latter inequality (40) can be written as follows for $i=1,\dots,N$, $\alpha=1,\dots,N$, $\alpha\neq i$, $j=1,\dots,M_i$:

$$\left(X_{i,j}\right)^T \bar{\bar{A}}_{i,j}^T+\bar{\bar{A}}_{i,j}X_{i,j}+\sum_{\alpha=1,\alpha\neq i}^{N}\left(\tau_{i,\alpha}^{-1}\tilde{F}_{i,\alpha}\tilde{F}_{i,\alpha}^T\right)+\bar{\bar{X}}_{i,j}^T\bar{\bar{I}}_i\bar{\bar{X}}_{i,j}$$
$$+\sum_{\alpha=1,\alpha\neq i}^{N}\tilde{B}_{w_{i,\alpha,j}}\left(\kappa_i\left(\Xi_i+\sum_{\alpha=1,\alpha\neq i}^{N}\Psi_i\right)^{-1}\right)\sum_{\alpha=1,\alpha\neq i}^{N}\tilde{B}_{w_{i,\alpha,j}}^T\leq 0 \tag{41}$$

with $\bar{\bar{X}}_{i,j}^T=\left[X_{i,j}^T\tilde{C}_{i,j}^T \quad X_{i,j}^T \quad \cdots \quad \cdots \quad X_{i,j}^T \quad \cdots \quad X_{i,j}^T\right]$,

$$\bar{\bar{I}}_i=\begin{bmatrix} I_{p_i} & 0 & \cdots & \cdots & 0 & \cdots & 0\\ 0 & \tau_{1,i}I_{2n_i+p_i} & 0 & \cdots & \cdots & \cdots & 0\\ \vdots & 0 & \ddots & \ddots & \cdots & \cdots & \vdots\\ \vdots & \cdots & \ddots & \tau_{i-1,i}I_{2n_i+p_i} & 0 & \cdots & \vdots\\ 0 & \cdots & \cdots & 0 & \tau_{i+1,i}I_{2n_i+p_i} & \ddots & \vdots\\ \vdots & \cdots & \cdots & \cdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & \cdots & \cdots & 0 & \tau_{N,i}I_{2n_i+p_i} \end{bmatrix}$$

Then, by applying the Schur's complement, the latter equation (41) can be written for $i=1,\dots,N$, $\alpha=1,\dots,N$, $\alpha\neq i$, $j=1,\dots,M_i$:

$$\left[\begin{array}{c|c} \Lambda_{i,\alpha,j} & \bar{\bar{X}}_{i,j}^T\\ \hline (*) & -\left(\bar{\bar{I}}_i\right)^{-1} \end{array}\right]\leq 0 \tag{42}$$

with:

$$\Lambda_{i,\alpha,j}=\left(X_{i,j}\right)^T\bar{\bar{A}}_{i,j}^T+\bar{\bar{A}}_{i,j}X_{i,j}+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{N}\left(\tau_{i,\alpha}^{-1}\tilde{F}_{i,\alpha}\tilde{F}_{i,\alpha}^T\right)$$
$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{N}\tilde{B}_{w_{i,\alpha,j}}\left(\kappa_i\left(\Xi_i+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{N}\Psi_i\right)^{-1}\right)\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{N}\tilde{B}_{w_{i,\alpha,j}}^T$$

We substitute $\bar{\bar{A}}_{i,j}$, $\tilde{C}_{i,j}$, $\tilde{F}_{i,\alpha}$, $\tilde{B}_{w_{i,\alpha,j}}$, $\bar{\bar{X}}_{i,j}$, $\bar{\bar{I}}_i$ in the inequality (42), we can obtain the inequality (43).

Remark 5: It is important to note that the inequality (43) is not a LMI due to the presence of the quadratic terms ($K_{i,j}X_{i,j}^7$, $K_{i,j}X_{i,j}^8$, $K_{i,j}X_{i,j}^9$). A way to skip to LMI conditions is to act on decision matrices mentioned in the remark 1. Hence, we choose (for $i=1,\dots,N$, $j=1,\dots,M_i$) matrices $X_{i,j}^7=0$ and $X_{i,j}^8=0$. Therefore, using the following change of variable $Y_{i,j}=K_{i,j}X_{i,j}^9$, under condition that the matrices $X_{i,j}^9$ are non-singular, the LMI (16) given in the Theorem 1 is provided.

$$\left[\begin{array}{c|ccccccc} \bar{\Lambda}_{i,\alpha,j_i} & X_{i,j_i}^T\tilde{C}_{i,j_i}^T\cdots\cdots & \cdots\ \cdots\ \cdots & X_{i,j_i}^T & \cdots\ \cdots\ \cdots & X_{i,j_i}^T & \cdots & X_{i,j_i}^T\\ \hline (*) & -I_{p_i} & 0 & \cdots & \cdots & 0 & \cdots & 0\\ \vdots & 0 & -\tau_{1,i}^{-1}I_{2n_i+p_i} & 0 & \cdots & 0 & \cdots & \vdots\\ \vdots & \vdots & 0 & \ddots & \ddots & \cdots & \cdots & 0\\ (*) & \vdots & \cdots & \ddots & -\tau_{i-1,i}^{-1}I_{2n_i+p_i} & 0 & \cdots & \vdots\\ \vdots & 0 & 0 & \cdots & 0 & -\tau_{i+1,i}^{-1}I_{2n_i+p_i} & \ddots & \vdots\\ \vdots & \vdots & \cdots & \cdots & \cdots & \ddots & \ddots & 0\\ (*) & 0 & \cdots & 0 & \cdots & \cdots & 0 & -\tau_{N,i}^{-1}I_{2n_i+p_i} \end{array}\right]\leq 0 \tag{43}$$

with $\overline{\Lambda}_{i,\alpha,j} = \begin{bmatrix} \begin{array}{l} \left(X^1_{i,j}\right)^T A^T_{i,j} + \left(X^7_{i,j}\right)^T K^T_{i,j} B^T_{i,j} + A_{i,j} X^1_{i,j} \\ + B_{i,j} K_{i,j} X^7_{i,j} + \sum\limits_{\alpha=1,\alpha\neq i}^{N} \tau^{-1}_{i,\alpha} F_{i,\alpha,j} F^T_{i,\alpha,j} \\ + \kappa_i B_{w_{i,j}} B^T_{w_{i,j}} + \dfrac{\kappa_i}{N-1} \sum\limits_{\alpha=1,\alpha\neq i}^{N} F_{w_{i,\alpha,j}} \sum\limits_{\alpha=1,\alpha\neq i}^{N} F^T_{w_{i,\alpha,j}} \end{array} & B_{i,j} K_{i,j} X^8_{i,j} & \begin{array}{l} -\left(X^1_{i,j}\right)^T C^T_{i,j} - \left(X^7_{i,j}\right)^T \\ + B_{i,j} K_{i,j} X^9_{i,j} \end{array} \\ (*) & \left(X^5_{i,j}\right)^T A^T_{r_i} + A_{r_i} X^5_{i,j} + \sum\limits_{\alpha=1,\alpha\neq i}^{N} \tau^{-1}_{i,\alpha} F_{r_{i,\alpha}} F^T_{r_{i,\alpha}} & \left(X^5_{i,j}\right)^T H^T_{r_i} - \left(X^8_{i,j}\right)^T I_p \\ (*) & (*) & -\left(X^9_{i,j}\right)^T - X^9_{i,j} \end{bmatrix}$

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