Security Enforcement in IoT Systems using Attack Trees

Delphine Beaulaton, Najah Ben Said, Ioana Cristescu, Axel Legay, Jean Quilbeuf

Security Enforcement in IoT Systems using Attack Trees

Delphine Beaulaton[1], Najah Ben Said[1], Ioana Cristescu[1], Régis Fleurquin[2], Axel Legay[1,3], Jean Quilbeuf[1], and Salah Sadou[2]

[1] INRIA Rennes, France
[2] Univ. South Brittany, Irisa, Vannes, France
[3] UC Louvain, Belgium

Abstract. Attack trees are graphical representations of the different scenarios that can lead to a security failure. In this paper we propose a security-based framework for modeling IoT systems where attack trees are defined alongside the model to detect and prevent security risks in the system. A successful attack can be a *rare event* in the execution of a well designed system. When rare, such attacks are hard to detect with usual model checking techniques. Hence, we use *importance splitting* as a statistical model checking technique for rare events.

1 Introduction

IoT is a rapidly emerging paradigm that provides a way to the user to instrument and control a large variety of objects interacting between each other over the Internet. In IoT systems, the security risks are multiplied as they involve heterogeneous devices that are connected to a shared network and that carry critical tasks, and hence, are targets for malicious users [2]. Vulnerabilities are discovered in opportunistic manner since security concerns have mostly an ad-hoc treatment. Therefore, developing a systematic mechanism that considers security aspects at an early stage of system design helps detecting and preventing attacks against IoT systems [18, 11].

Attack trees [15, 9] are intuitive and practical formal methods to identify and analyze attacks on the security of a system. As their name suggests, attacks are modeled as trees, where the leaves represent elementary steps needed for the attack, and the root represents a successful attack. The internal nodes are of two types, indicating whether all the sub-goals (an AND node) or one of the sub-goals (an OR node) must be achieved in order to accomplish the main goal.

In this paper we present a framework to analyze security in IoT systems consisting of a formal languages for modeling IoT systems and of attack trees for modeling the possible attacks on the system. In our approach a malicious entity is present in the system, called the *Attacker*. The other IoT entities can inadvertently help the Attacker, by *leaking* their sensitive data. Equipped with the acquired knowledge the Attacker can then communicate with the IoT entities undetected. The attack tree provided with the model acts as a *monitor*: It

observes the interactions the Attacker has with the system and detects when an attack is successful.

An IoT system is then analyzed using statistical model checking (SMC) [5]. The first method we use is Monte Carlo, which consists of sampling the executions of an IoT system and computing the probability of a successful attack based on the number of executions for which the attack was successful. However, the evaluation may be difficult if a successful attack is *rare*. We therefore use a second SMC method, developed for *rare events*, called *importance splitting* [12]. Importance splitting assumes that an execution leading to a rare event can be decomposed into several intermediate steps. Instead of executing a system until the rare event occurs, the execution is stopped after one of the intermediate steps is reached. The execution is restarted then from that step onwards. The method of importance splitting is well suited for attack trees, as the intermediate steps leading to a rare event are the nodes in the tree leading to a successful attack.

An IoT system and its attack tree are transformed into a $\mathcal{S}BIP$ system [3, 6], a stochastic heterogeneous component-based model for which an execution engine is developed and maintained [9]. We use Plasma [7] for the SMC analysis that infers the probability of a successful attack. Such analysis can help in the design phase of a system to decide how and where to modify the system in order to meet some security specifications. The main contributions of the paper are:

- a formal *probabilistic* language for modeling security in IoT systems;
- a formal analysis framework which consists of: (i) translating an IoT system in $\mathcal{S}$BIP; (ii) translating an attack tree into a $\mathcal{S}$BIP *monitor* and (iii) using SMC to estimate the probability of success;
- the implementation of a tool chain to automate the analysis presented above;
- the modelisation of an example involving cyber-attacks on a Smart Hospital. The example is based on existing attacks as reported by TrapX [1] and ENISA [8].

The paper is structured as follows. Section 2 presents the IoT modeling language, Section 3 introduces attack trees and Section 4 presents $\mathcal{S}$BIP. The transformation from IoT to $\mathcal{S}$BIP is shown in Section 5. In Section 6 we use SMC (based on Monte Carlo and importance splitting) and attack trees to analyze an IoT system. Section 7 presents our tool chain and a validation of the approach using our running example. We conclude in Section 8.

2 Probabilistic IoT Models

The formal language we propose for specifying IoT systems is a continuation the one presented in [4] targeting probabilistic systems. The components of an IoT system, called *entities*, have each a *knowledge*, used to allow (or disallow) its interaction with the rest of the system. For instance, an entity can send an email to another entity only if it *knows* its email address. Or an user needs to *know* the `url` of a website in order to access it; we say that the `url` is part of the user's knowledge. For simplicity, we represent knowledge as a finite set of *values*.

Protocols are used at each interaction to verify the knowledge of the interacting entities. Each value is associated to a protocol. Two entities can communicate through a protocol if they have a common value for that protocol.

Each entity has a unique identifier, denoted by $e_1, \cdots e_n$ and a running process. The grammar of processes is defined in Figure 1; we use CCS-like processes [16] augmented with probabilities [19]. We write C for a set of protocols, ranged over by c and *Val* for a set of values ranged over by v.

The actions of a process consists of sending and receiving values under an agreed upon protocol. We distinguish between "safe" interactions and the ones that can potentially lead to security issues, called *leak* and *collect*. A *leak* is a send action where there is no protocol governing the interaction and *collect* is its receive counterpart. Processes can also do silent moves, denoted by τ. Processes are composed of *threads*, which can only do sequential computations. We write 0 for the inactive process and A for the (recursive) definitions of threads. Actions are equipped with a probability, denoted by $[n]a$, for an action a and a probability $n \in [0, 1]$. Threads can therefore do a probabilistic choice between actions, with the restriction that the sum of the probabilities of all available actions is 1. If there is only one available action, its probability is 1 and can be omitted.

$$
\begin{array}{rrl}
\textit{Process} & P, Q & ::= T \mid\mid P \mid Q \\
\textit{Thread} & T, U & ::= 0 \mid\mid A \mid\mid \sum_{i \in I} [n_i] a_i.T_i \text{ where } n_i \in (0,1], \sum_{i \in I} n_i = 1 \\
\textit{Action} & a, b & ::= e \xrightarrow[v]{c} e' \quad (\textsc{Send}) \mid\mid e \xleftarrow{c} e' \quad (\textsc{Receive}) \mid\mid e \xrightarrow[v]{} \!\!\!\!\!\rightarrow e' \quad (\textsc{Leak}) \\
 & & \quad \mid\mid e \twoheadleftarrow e' \quad (\textsc{Collect}) \mid\mid \tau \quad (\textsc{Internal}) \\
\textit{Definition} & A & \stackrel{\text{def}}{=} T \\
\textit{State} & s & ::= \emptyset \mid\mid \langle P, k \rangle \mid\mid s \mid s.
\end{array}
$$

Fig. 1: Syntax of the probabilistic IoT-calculus

A *knowledge* function $K : E \times C \to \mathcal{P}(\mathit{Val})$ associates a set of values to each entity and protocol. For simplicity we write k_i^c for the knowledge of entity i under protocol c. The function $\mathsf{protocol} : \mathit{Val} \to C$ associates each value to a protocol.

Each entity, has at any state of its computation, a running process P and a knowledge k. The global state of the system consists of the parallel composition of all entities states $s_1 \mid \cdots \mid s_n$, where s_i is the current state of the entity e_i.

We define a *congruence* relation on processes $\equiv_P \subseteq P \times P$ as the smallest equivalence relation which includes the associativity and the commutativity for $+$ and $|$; the identity element 0 for $|$ and the unfolding law for definitions: $A \equiv_P T$ if $A \stackrel{\text{def}}{=} T$. We also introduce a congruence relation on states $\equiv_s \subseteq s \times s$, as the smallest equivalence relations which includes the associativity and the commutativity for $|$, the identity element $\emptyset$ and generalizes the congruence on processes: if $P \equiv_P Q$ then $\langle P, k \rangle \equiv_s \langle Q, k \rangle$.

$$\textsc{Choice}\qquad \langle \sum_{i\in I}[n_i]a_i.T_i, k\rangle \xrightarrow[\tau]{[n_i]} \langle a_i.T_i, k\rangle$$

$$\textsc{Internal}\qquad \langle \tau.P, k\rangle \xrightarrow[\tau]{[1]} \langle P, k\rangle$$

$$\textsc{SendReceive}\quad \frac{\exists v \in k_1^c \text{ s.t. } v \in k_2^c \qquad c' = \mathsf{protocol}(v')}{\langle e_1 \xrightarrow[\mathrm{v'}]{\mathrm{c}} e_2.P_1, k_1\rangle | \langle e_2 \xleftarrow{\mathrm{c}} e_1.P_2, k_2\rangle \xrightarrow[\mathrm{SR}:v']{[1]} \langle P_1, k_1\rangle | \langle P_2, k_2^{c'} \uplus \{v'\}\rangle}$$

$$\textsc{LeakCollect}\quad \frac{c' = \mathsf{protocol}(v')}{\langle e_1 \xrightarrow[\mathrm{v'}]{} e_2.P_1, k_1\rangle | \langle e_2 \xleftarrow{} e_1.P_2, k_2\rangle \xrightarrow[\mathrm{LC}:v']{[1]} \langle P_1, k_1\rangle | \langle P_2, k_2^{c'} \uplus \{v'\}\rangle}$$

$$\textsc{ParProc}\quad \frac{\langle P_i, k_i\rangle | \langle P_j, k_j\rangle \xrightarrow[l]{[n]} \langle P_i', k_i'\rangle | \langle P_j', k_j'\rangle}{\langle P_i \mid Q_i, k_i\rangle | \langle P_j \mid Q_j, k_j\rangle \xrightarrow[l]{[n]} \langle P_i' \mid Q_i, k_i'\rangle | \langle P_j' \mid Q_j, k_j'\rangle}$$

$$\textsc{Congruence}\quad \frac{s \equiv_s t \xrightarrow[l]{[n]} s' \equiv_s t'}{s \xrightarrow[l]{[n]} s'}$$

$$\textsc{ParState\_Tau}\quad \frac{\langle P, k\rangle \xrightarrow[\tau]{[n]} \langle P', k'\rangle \qquad \mathsf{count}_\tau(\langle P, k\rangle | s) = m}{\langle P, k\rangle | s \xrightarrow[\tau]{[1/m \cdot n]} \langle P', k'\rangle | s}$$

$$\textsc{ParState\_Interaction}\quad \frac{\langle P_i, k_i\rangle | \langle P_j, k_j\rangle \xrightarrow[l]{[1]} \langle P_i', k_i'\rangle | \langle P_j', k_j'\rangle \qquad \mathsf{count}_{\mathtt{SR},\mathtt{LC}}(\langle P_i, k_i\rangle | \langle P_j, k_j\rangle | s) = m \qquad \mathsf{count}_\tau(\langle P_i, k_i\rangle | \langle P_j, k_j\rangle | s) = 0}{\langle P_i, k_i\rangle | \langle P_j, k_j\rangle | s \xrightarrow[l]{[1/m]} \langle P_i', k_i'\rangle | \langle P_j', k_j'\rangle | s}$$

Fig. 2: The operational semantics of an IoT system

The operational semantics of Figure 2, defines a transition system (S, T, L, s_0) where we write S for a set of states, ranged over by s with s_0 the initial state, $L \subseteq \{\tau\} \cup (\{\text{SR, LC}\} \times \mathit{Val})$ for a set of labels, ranged over by l, and $T \subseteq S \times [0,1] \times L \times S$ for a set of transitions, where each transition is decorated by a probability and by a label.

In our semantics, a probabilistic choice is always resolved locally, using the Choice rule. A transition derived by the Choice rule is considered internal and is labeled τ. A process can also do internal transitions using rule Internal. Rule SendReceive defines the interaction between two components e_1 and e_2. The interaction is allowed if the sender and the receiver share some common values under the protocol c. After the interaction, the receiver's knowledge is updated by adding the received value under the corresponding protocol. A LeakCollect interaction proceeds similarly, except that there are no checks on the knowledge of the two components. Rules Congruence and ParProc allows one to use congruence and parallel composition on states to derive transitions.

The rules for the global states, ParState_Tau and ParState_Interaction, give priority to the internal transitions over the binary interactions. Moreover, in

each case, we choose a global transition from several local ones using an uniform distribution. We rely on two auxiliary functions, count_τ and $\mathsf{count}_{\mathtt{SR},\mathtt{LC}}$ that count the number of local transitions with labels τ and labels SR, LC, respectively.

The actions of the IoT system are :

$$\text{AH} = A \xrightarrow[\text{getSensitiveData}]{\text{url}} H \qquad \text{leakEmail} = H \xrightarrow[\text{emailEmployee}]{} \!\!\!\!\rightarrow A$$

$$\text{HA} = H \xleftarrow{\text{url}} A \qquad \text{leakPhone} = H \xrightarrow[\text{phoneEmployee}]{} \!\!\!\!\rightarrow A$$

$$\text{AE\_mail} = A \xrightarrow[\text{getCredential}]{\text{mail}} E \qquad \text{leakIssue} = H \xrightarrow[\text{issueEmployee}]{} \!\!\!\!\rightarrow A$$

$$\text{EA\_mail} = E \xleftarrow{\text{mail}} A \qquad \text{leakCredentials} = E \xrightarrow[\text{credEmployee}]{} \!\!\!\!\rightarrow A$$

$$\text{AE\_phone} = A \xrightarrow[\text{getCredential}]{\text{phone}} E$$

$$\text{EA\_phone} = E \xleftarrow{\text{phone}} A$$

and the following process definitions:

$$\begin{aligned}
\text{Attacker} &= \text{AChoice} \mid \text{collectH} \mid \text{collectE}\\
\text{AChoice} &= [0.4]\text{AH.AChoice} + [0.3]\text{AE_mail.AChoice} + [0.3]\text{AE_phone.AChoice}\\
\text{collectH} &= A \twoheadleftarrow H.\text{collectH}\\
\text{collectE} &= A \twoheadleftarrow E.\text{collectE}\\
\text{Hospital} &= \text{HA.}([n_1]\text{leakPhone.Hospital} + [n_2]\text{leakIssue.Hospital} +\\
&\qquad [n_3]\text{leakEmail.Hospital} + [n_4]\tau.\text{Hospital})\\
\text{Employee} &= [m_1]\text{EA_mail.EChoice} + [m_2]\text{EA_phone.EChoice}\\
\text{EChoice} &= [m_3]\text{leakCredentials.Employee} + [m_4]\tau.\text{Employee}
\end{aligned}$$

A has Attacker as initial process and similarly for H and E. Their initial knowledge is:

$$\begin{aligned}
k_A &= \{url = \{\text{urlHospital}\}, message = \{\text{getSensitiveData, getCredentials}\}, mail = phone = \emptyset\}\\
k_H &= \{url = \{\text{urlHospital}\}, message = \emptyset, mail = \{\text{emailEmployee}\}, phone = \{\text{phoneEmployee}\}\}\\
k_E &= \{url = \emptyset, message = \emptyset, mail = \{\text{emailEmployee}\}, phone = \{\text{phoneEmployee}\}\}
\end{aligned}$$

Fig. 3: The Smart Hospital in the IoT language

Given an IoT model (S, T, L, s_0) with an initial state s_0, an *execution* is a sequence of transitions in T, $\sigma = \{s_i \xrightarrow[l_i]{[n_i]} s'_i\}_{0 \le i \le k}$, such that $s_0 = s$ and $\forall i \ge 1$, $s'_{i-1} = s_i$. The probability of a transition $s \xrightarrow[l]{[n]} s'$ is n and the probability of an execution σ is $\prod_{0 \le i \le k} n_i$.

Example 1. Let us now introduce our running example, the Smart Hospital system. Let $E = \{A(ttacker), H(ospital), E(mployee)\}$ be three entities which communicate with each other using the protocols $C = \{url, message, mail, phone\}$.

The IoT system is defined in Figure 3. Let us consider the following transitions:

$$\langle \text{Attacker}, k_A \rangle \mid \langle \text{Hospital}, k_H \rangle \xrightarrow[\tau]{[0.4]} \tag{1}$$

$$\langle \text{AH.AChoice} \mid \text{collectH} \mid \text{collectE}, k_A \rangle \mid \langle \text{Hospital}, k_H \rangle \xrightarrow[\text{SR:getSensitiveData}]{[1]} \tag{2}$$

$$\langle \text{Attacker}, k_A \rangle \mid \langle [n_3]\text{leakEmail.Hospital} + \cdots, k'_H \rangle \xrightarrow[\tau]{[n_3]} \tag{3}$$

$$\langle \text{Attacker}, k_A \rangle \mid \langle \text{leakEmail.Hospital}, k'_H \rangle \xrightarrow[\text{LC:emailEmployee}]{[1]} \tag{4}$$

$$\langle \text{Attacker}, k'_A \rangle \mid \langle \text{Hospital}, k'_H \rangle$$

The Attacker starts by choosing to contact the Hospital using the internal transition (1), after which the two entities can communicate in transition (2). At this point the hospital can either leak some sensitive information (*emailEmployee*, *phoneEmployee* or *issueEmployee*) or it can do an internal transition. Transitions $(3) - (4)$ represent the scenario where *emailEmployee* is leaked. The knowledge of the attacker is augmented with the leaked data: $k_A^{mail} \cup \{\text{emailEmployee}\}$ and the attacker can now communicate with the employee using the email protocol. Suppose that after transition (2), the Hospital does not leak any information:

$$\langle \text{Attacker}, k_A \rangle \mid \langle [n_3]\text{leakEmail.Hospital} + \cdots, k'_H \rangle \xrightarrow[\tau]{[n_4]} \langle \text{Attacker}, k_A \rangle \mid \langle \text{Hospital}, k'_H \rangle.$$

Then the Attacker cannot communicate with the Employee. If the Attacker tries to communicate without knowing the *emailEmployee*, the system deadlocks. A sequence of transitions ending with a deadlock represents an unsuccessful attack.

3 Attack Trees

In this section we formally introduce attack trees. Figure 4 shows an attack tree for the Smart Hospital in Example 1. The root of the tree is the main goal of the Attacker, which is getting the Employee's credentials. The nodes represent the possible attacks. The *qui pro quo attack* consists of contacting the Employee and posing as a technician. For this attack to succeed the Attacker needs the Employee's phone number and technical issue, which both are leaked by the Hospital. The second possibility is a *phishing attack*. First the Attacker contacts the Hospital, then the Hospital has to leak the Employee's email. If either of the two attacks succeed, the Attacker can then try to get the Employee's credentials.

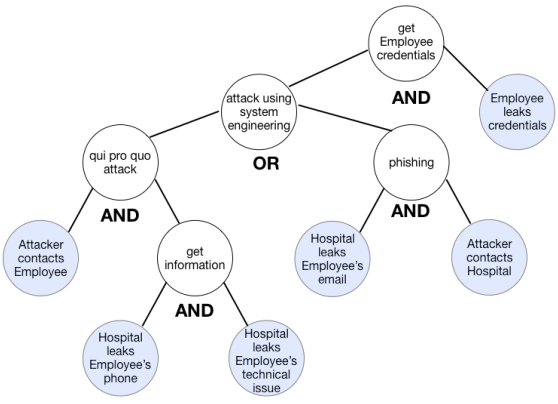


Fig. 4: An attack tree for the Smart Hospital

The leaves of the tree corresponds to events happening in the system. An event consists of the exchange of a value between some entities. For example, the node *Employee leaks credentials* stands for the pair *(LC, "credEmployee")*, meaning that the value *credEmployee* has been leaked.

Definition 1 (Attack Tree). *Let $\Delta \subseteq \{SR,LC\} \times Val$ be a set of events. An* attack tree $\mathtt{t}$ *is a term constructed recursively from the set Δ using the operators $\vee$ and $\wedge$.*

4 SBIP: A Stochastic Component Based Model

$\mathcal{S}$BIP [6, 3] is a stochastic, component based framework that allows modeling hierarchical systems from atomic components. In this section we introduce $\mathcal{S}$BIP in four steps: first we introduce variables and valuation domains; then we introduce the syntax of the stochastic atomic components; next its semantics and lastly, we define the composition of the atomic components.

4.1 Preliminaries

Let $\{D_j\}_{j\in\mathcal{J}}$ be a set of data domains including the Boolean domain $D_{\text{Bool}} = \{true, false\}$. Let $\mathbb{E}\text{xpr}$ be a set of operators. We denote by $\mathbb{E}\text{xpr}[V]$ the set of expressions constructed from a set of variables V and operators in $\mathbb{E}\text{xpr}$. We denote by $\mathbb{A}\text{sgn}[V]$ a set of assignments to variables in V, that is, any subset $\{(v_i, e_i)\}_{i\in\mathcal{I}} \subseteq V \times \mathbb{E}\text{xpr}[V]$ where $(v_i)_{i\in\mathcal{I}}$ are all distinct. An assignment (v, e) is denoted by $v := e$.

A *valuation* for the variables in V is a function $\mathbf{X} : V \to \cup_{j\in J} D_j$ which assigns values to variables. We denote $\mathbf{X}(v)$ the valuation of the variable $v \in V$ and $\mathbf{X}(e)$ the valuation of the expression $e \in \mathbb{E}\text{xpr}[V]$ according to values of V.

We distinguish between two types of variables: the deterministic variables and the *random* variables, used for encoding the stochastic behavior. A random variable v is associated with a probability distribution μ over its valuation domain D, denoted as $v \sim \mu$, where $\mu : D \to [0, 1]$ and $\sum_{x\in D} \mu(x) = 1$.

4.2 Stochastic Atomic Components

$\mathcal{S}$BIP components are 1-safe Petri-Nets equipped with (i) ports that allow the component to communicate with other components; and (ii) variables that can be read and updated during communications.

Definition 2 (Stochastic Atomic Components). *A stochastic atomic component consists of the tuple $\mathcal{B} = (P, V, N)$, where*

- *P is a set of communication ports;*
- *$V = V^d \uplus V^p$, with $V^d = \{v_1, \ldots, v_n\}$ a set of deterministic variables and $V^p = \{v_1^p, \ldots, v_m^p\}$ a set of random variables with an associated probability distribution $v_i^p \sim \mu_i$.*

– $N = (L, L_0, T)$ *is a Petri-Net*[4] *where*
  - L *is a set of places and* $L_0 \subseteq L$ *is a set of initial places;*
  - T *is a finite set of transitions* $t = ({}^\bullet t, \langle p, g, f\rangle, t^\bullet)$ *where* ${}^\bullet t$ *(resp.* $t^\bullet$*) is the set of input (resp. output) places of* t*. Transitions are labeled by the triple* $\langle p, g, f\rangle$ *where* $p \in P$ *is the port triggered by* t*,* $g \in \mathbb{E}xpr[V]$ *is the* guard *of* t *and* $f = (f^d, R^p)$ *is the* update *function of* t*, such that* $f^d = \{v := f(V) \mid v \in V^d\} \in \mathbb{A}sgn[V]$ *is a set of functions that update the deterministic variables and* $R^p \subseteq V^p$ *is a subset of random variables to be updated.*

We sometimes write p_t, g_t and f_t^d, R_t^p for the label of a transition t. We define *(1-safe) markings* as the set of functions $m : L \to \{0, 1\}$. Given two markings m_1, m_2 we define inclusion $m_1 \leq m_2$ iff for all $l \in L$, $m_1(l) \leq m_2(l)$. Also, we define addition $m_1 + m_2$ as the marking m_{12} such that, for all $l \in L$, $m_{12}(l) = m_1(l) + m_2(l)$.

A priority order on a set of ports is a partial order, where each element $p < p'$ of the order is called a *priority*. Whenever the system has a choice between the two interactions on two ports p or p', the interaction on p' is chosen. Due to lack of space, we refer the reader to [10] for a formal definition of priorities.

4.3 Semantics of Stochastic Atomic Components

The semantics of a $\mathcal{S}$BIP component $\mathcal{B} = (P, V, N)$ uses a transition system $\mathcal{M}$ obtained from its Petri-Net N. The states in $\mathcal{M}$ are of the form $(m, \mathbf{X})$ where m is a marking of the Petri-Net N and where $\mathbf{X}$ is a valuation of V.

The random variables engender a probabilistic behavior over transitions of $\mathcal{M}$. Let us consider an atomic component $\mathcal{B}$ that has a transition going from place l_1 to place l_2 using port p, with a guard that is always true, and which updates a random variable v according to a distribution μ. Assuming the initial value of v is x_0, when executing $\mathcal{B}$, there will be several possible transitions, from state $(\{l_1\}, x_0)$ to states $(\{l_2\}, x_i)$ for all $x_i \in D$, where D is the valuation domain of v. The probabilities of these transitions is given by μ. Since the random variables are independent, when several random variables are updated, the resulting distribution on transitions will be the product of the distributions associated to each variable. We recall that these distributions are fixed during variables declaration.

Atomic components with random variables lead to behaviors that combine both stochastic and non-deterministic aspects. A transition is possible is a communication is ready on its associated port. At any given state, several ports can be ready for a communication, and thus several transitions can be enabled, regardless of whether they are associated or not with random variables.

Non-determinism is always resolved in $\mathcal{S}$BIP to a probabilistic choice on an uniform distribution. To formally state this, consider a stochastic component

[4] N is equivalent to the extended 1-safe Petri-Net (L, L_0, T, F) where $F = \{(l, t) \mid l \in {}^\bullet t\} \cup \{(t, l) \mid l \in t^\bullet\}$ is the token flow relation and can be deduced from T.

$\mathcal{B} = (P, V, N)$ with $N = (L, L_0, T)$ its Petri-Net and let m be a marking. We denote with $\mathsf{Enabled}(m; \mathbf{X})$ the set of transitions in T that are enabled by m for a valuation $\mathbf{X}$: $\mathsf{Enabled}(m; \mathbf{X}) = \{t \in T \mid {}^\bullet t \leq m \text{ and } \mathbf{X}(g_t) \text{ is true }\}$. Remark that $|\mathsf{Enabled}(m; X)|$ can be greater than one[5].

In the associated semantics of $\mathcal{B}$, first a transition is selected with a uniform distribution from the set of enabled transitions. Then, the next state is selected according to the distributions attached to the random variables.

Definition 3 (Semantics of a stochastic atomic components). *The semantics of a stochastic atomic component $\mathcal{B} = (P, V, N)$ with $N = (L, L_0, T)$ its Petri-Net and with $\mathbf{X_{init}}$ an initial valuation, is defined as a probabilistic transition system $\mathcal{M} = \langle Q, \pi, P, q_0 \rangle$, where:*

- *Q is the set of states, where each state $(m, \mathbf{X})$ consists of m, a marking, and of $\mathbf{X}$, a valuation of the variables in V. $q_0 = (m_0, \mathbf{X_{init}})$ is the initial state where m_0 is the marking associated to L_0, i.e. $m_0(l) = 1 \iff l \in L_0$ and 0 otherwise;*
- *$\rightarrow \ \subseteq Q \times P \times Q$ is a set of transitions defined by the following rule:*

$$\frac{\begin{array}{c} t \in T \quad {}^\bullet t \leq m \quad m' = m - {}^\bullet t + t^\bullet \quad \mathbf{X}(g_t) = \mathit{true} \\ \mathbf{X}' = [v^d := \mathbf{X}(f_t^d), v^p := \mathit{random}(\mu)] \quad v^d \in V^d \quad v^p \in R_t^p, v^p \sim \mu \end{array}}{(m, \mathbf{X}) \xrightarrow{p_t} (m', \mathbf{X}')}$$

Lastly, we defined the probability of a transition as follows:

$$\mathbb{P}\big(q \xrightarrow{p} q'\big) = \frac{1}{|\mathsf{Enabled}(m; \mathbf{X})|} \cdot \prod_{v_i \in R^p, v_i \sim \mu_i} \mu_i(X'(v_i)).$$

In the definition above we say that the state $(m', \mathbf{X}')$ is a successor of state $(m, \mathbf{X})$, if t is a transition of T enabled by the marking m, the guard g_t evaluates to *true* and the new valuation $\mathbf{X}'$ on the variables $V^d \cup V^p$ is obtained by applying f_t^d on the deterministic variables V^d and updating the random variables in R_t^p. The probability of a transition, as explained above, is given by an uniform distribution on the enabled interactions and by the probabilistic distribution of the random variables updated during the interaction.

4.4 Composition of Stochastic Components

Let us now define the semantics of the parallel composition of stochastic atomic components.

Definition 4 (Interaction between components). *An* interaction $\gamma = (P_\gamma, G_\gamma, F_\gamma)$ *on a set of components $\mathcal{B}_i = (P_i, V_i, N_i)$, for $i \in I$, where I is set of indexes consists of:*

[5] $|\cdot|$ denotes the cardinality of a set.

- $P_\gamma = \{p_i \mid p_i \in P_i, i \in I\}$ *is a disjoint set of ports containing exactly one port from each component* $\mathcal{B}_i$, $i \in I$;
- G_γ *is a global guard defined on* $V_\gamma = \cup_{i \in I} V_i$;
- $F_\gamma = \{v := F(V_\gamma) \mid v \in \cup_{i \in I} V_i^d\}$ *is a global update function used to exchange values between components.*

Definition 5 (Composition of Stochastic Components). *Let* Γ *be a set of interactions defined on* n *components* $\mathcal{B}_i = (P_i, V_i, N_i)$, *with* $N_i = (L_i, L_{0,i}, T_i)$ *for* $i \leq n$. *The composition of the* n *components, denoted as* $\Gamma(\mathcal{B}_1, \ldots, \mathcal{B}_n)$, *is a stochastic component* $\mathcal{B} = (\Gamma, V, N)$, *with* $N = (L, L_0, T)$, *defined as follows:*

- $V = \cup_{i \leq n} V_i$;
- $L = \cup_{i \leq n} L_i$ *with* $L_0 = \cup_{i \leq n} L_{0,i}$;
- $T = \left\{({}^\bullet T_\gamma, \langle \gamma, g, f \rangle, T_\gamma^\bullet) \mid \gamma \in \Gamma\right\}$, *where* $T_\gamma = \{t_i \mid p_i \in P_\gamma\}$ *is the set of transitions that synchronize on the interaction* $\gamma \in \Gamma$. *Then* ${}^\bullet T_\gamma = \{l \mid l \in {}^\bullet t_i, t_i \in T_\gamma\}$ *and* $T_\gamma^\bullet = \{l \mid l \in t_i^\bullet, t_i \in T_\gamma\}$. *Each transition is labeled by the triple* $\langle \gamma, g, f \rangle$ *where* $g = G_\gamma \wedge (\bigwedge_{t_i \in T_\gamma} g_{t_i})$ *and* $f = (\sqcup_{t_i \in T_\gamma} f_{t_i}) \circ F_\gamma$[6] *consists of the composition of all* f_{t_i} *with* F_γ.

Assembling stochastic atomic components produces a stochastic atomic component, and thus its semantics is given by Definition 3. We use a priority order, denoted $\ll$, which gives priority to the internal transitions over the binary interactions. We write then $\langle \ll \rangle(\Gamma(\mathcal{B}_1, \ldots, \mathcal{B}_n))$ for a $\mathcal{S}$BIP system.

5 Transformation from IoT to $\mathcal{S}$BIP

In this section we transform an IoT system to $\mathcal{S}$BIP. Entities of an IoT model become atomic components in $\mathcal{S}$BIP and the communication between them are represented with interactions.

An entity can have several threads running, all sharing the same knowledge. To model this we define an atomic component as the union of several Petri Nets, which all have a common set of variables, guards and update functions. The deterministic variables are used to model the entity's knowledge. The random variables, similarly to the transformation of DTMC into $\mathcal{S}$BIP of [6], encode the probabilities associated to actions in a summation process. In our transformation we use labeling functions on places and on the random variables, denoted by ℓ. The labels are the threads of the original IoT system. Moreover we identify places that have congruent labels, i.e. $l_1 \equiv_L l_2 \iff \ell(l_1) \equiv_P \ell(l_2)$. We write l_T and v_T when $\ell(l) = T$ and $\ell(v) = T$, respectively.

Definition 6 (Transformation of an IoT Thread into an Atomic Component). *For a thread* T, *let* Definitions *and* Actions *be the sets of thread definitions and of actions, respectively, used recursively in* T. *We define the transformation of* T *to be the atomic component* $(\mathsf{Actions}, V^d \uplus V^p, (\mathcal{L}, \mathcal{L}_0, \mathcal{T}))$ *with:*

[6] We denote $(f \circ g)(x) = f(g(x))$ the composition of functions. If the two functions have disjoint domains, we write $f \sqcup g$ for their disjoint composition.

– $V^d = \{v_c \mid c \text{ is a protocol used in } T\}$;
– $V^p = [\![T]\!]_v \cup \{[\![U]\!]_v \mid A \stackrel{def}{=} U \text{ and } A \in \mathsf{Definitions}\}$;
– $\mathcal{L} = \big([\![T]\!]_s \cup \{[\![U]\!]_s \mid A \stackrel{def}{=} U \text{ and } A \in \mathsf{Definitions}\}\big)_{\equiv_L}$ *is a set of places partitioned in equivalence classes by the* $\equiv_L$ *relation, with* $\mathcal{L}_0 = \{l_T\}$;
– $\mathcal{T} = [\![T]\!]_t \cup \{[\![U]\!]_t \mid A \stackrel{def}{=} U \text{ and } A \in \mathsf{Definitions}\}$.

and where $[\![\cdot]\!]_v$, $[\![\cdot]\!]_s$ *and* $[\![\cdot]\!]_t$ *are defined in Figure 5.*

The random variables are defined by the function $[\![T]\!]_v$, for a thread T. Whenever a thread T is of the form $\sum_{i\in I}[n_i]a_i.T_i$ we introduce a new random variable v_T. The valuation domain D for v_T is the set of states associated to the possible continuations i.e. $D = \{a_i.T_i\}_{i\in I}$. The probability distribution of v_T is defined by the probabilities n_i i.e. $\mu(a_i.T_i) = n_i$. In the transformation above, $l_T^\star$ denotes the state in which a random variable v_T is updated. The guards are only used when making a probabilistic choice: Suppose we are currently running thread T and we wish to go from state $l_T^\star$ to a state l_{T_i}. The guard then checks that the value of the random variable v_T is updated to $a_i.T_i$. For the rest of transitions, the guard is the constant *true*.

$$
\begin{aligned}
[\![\sum_{i\in I}[n_i]a_i.T_i]\!]_v &= \bigcup_{i\in I}[\![a_i.T_i]\!]_v \cup \{v_T \mid v_T \sim \mu \text{ s.t. } \mu(a_i.T_i) = n_i, \forall i \in I\} \\
&\qquad \text{where } T = \sum_{i\in I}[n_i]a_i.T_i \text{ and } |I| > 1 \\
[\![a.T]\!]_v &= [\![T]\!]_v \\
[\![A]\!]_v = [\![0]\!]_v &= \emptyset \\
[\![\sum_{i\in I}[n_i]a_i.T_i]\!]_s &= \bigcup_{i\in I}[\![T_i]\!]_s \cup \{l_T, l_T^\star\}, \text{ where } T = \sum_{i\in I}[n_i]a_i.T_i \text{ and } |I| > 1 \\
[\![a.T]\!]_s &= [\![T]\!]_s \cup \{l_{a.T}\} \\
[\![A]\!]_s &= \{l_A\} \\
[\![0]\!]_s &= \{l_0\} \\
[\![\sum_{i\in I}[n_i]a_i.T_i]\!]_t &= \bigcup_{i\in I}\Big((\{l_T^\star\}, \langle a_i, g = (v_T == a_i.T_i), f\rangle, \{l_{T_i}\}) \cup [\![T_i]\!]_t\Big) \\
&\qquad \cup (\{l_T\}, \langle \tau, \text{true}, f^\star\rangle, \{l_T^\star\}) \text{ where } T = \sum_{i\in I}[n_i]a_i.T_i \text{ and } |I| > 1 \\
[\![a.T]\!]_t &= (\{l_{a.T}\}, \langle a, \text{true}, f\rangle, \{l_T\}) \cup [\![T]\!]_t \\
[\![0]\!]_t = [\![A]\!]_t &= \emptyset
\end{aligned}
$$

where $f = \{v := v \mid v \in V^d\}$ and $R^p = \emptyset$ and $f^\star$ defined as f but with $R^p = \{v_T\}$.

Fig. 5: The functions used in the transformation in Definition 6

Definition 7 (Transformation of an IoT Entity into an Atomic Components). *Let* e *be an entity in an IoT system with the initial state* $s_0 = \langle P, k\rangle$ *and* $P = T_1 \mid \cdots \mid T_m$. *Let* $(P^j, V^j, (\mathcal{L}^j, \mathcal{L}_0^j, \mathcal{T}^j))$ *be the atomic components*

obtained from each T_j, $j \leq m$. We define the transformation of e as the atomic component $\mathcal{B}_e = (P, V, N)$ with $P = \cup_{j \leq m} P_j$, $V^d = \cup_{j \leq m} V_j^d$, $V^p = \uplus_{j \leq m} V_j^p$ and with $N = (\uplus_{j \leq m} \mathcal{L}^j, \uplus_{j \leq m} \mathcal{L}_0^j, \uplus_{j \leq m} \mathcal{T}^j)$. We also define the initial valuation $\mathbf{X_{init}}(v_c) = k(c)$ where for each protocol c we initialize the variable v_c to the set of values $k(c)$.

The transformation of an IoT entity into an atomic component proceeds in two steps: first encode each thread of the entity's process (Definition 6) and then define the atomic component associated with an entity (Definition 7). Note that we are using set union for ports and variables as the different threads of an entity share their ports and knowledge. However, in order to clearly separate the behaviour of the different threads, we use disjoint union for combining the Petri Nets of the different threads. This is allowed because the different threads in an entity cannot interact with each other, but only with other entities.

Communications between two entities e_1 and e_2 in the IoT language are transformed into a set of guarded interactions between components $\mathcal{B}_{e_1}$ and $\mathcal{B}_{e_2}$.

Definition 8 (Interactions for an IoT state). *Let $\mathcal{B}_{e_i} = (P_i, V_i, N_i)$ be the transformation of an IoT system with n entities e_i, $i \leq n$ and with the initial state s_0. For all actions $a \in \mathsf{Actions}$, if*

- *$a = e_1 \xrightarrow[v]{c} e_2$ and there exists $a' \in \mathsf{Actions}$ such that $a' = e_2 \xleftarrow{c} e_1$,*
- *or $a = e_1 \xrightarrow[v]{} e_2$ and there exists $a' \in \mathsf{Actions}$ such that $a' = e_2 \twoheadleftarrow e_1$*

then we define an interaction γ=(P, G, F) where

- *$P = \{a, a'\}$;*
- *if $a = e_1 \xrightarrow[v]{c} e_2$ then $G = (\exists x \in v_c^1 \text{ s.t. } x \in v_c^2)$ for $v_c^1 \in V_1^d$, $v_c^2 \in V_2^d$; otherwise $G = true$;*
- *$F = \{v_{c'}^2 := v_{c'}^2 \cup \{v\} \mid \mathsf{protocol}(v) = c', v_{c'}^2 \in V_2^d\}$*

and where V_1^d, V_2^d are the deterministic variables of $\mathcal{B}_{e_1}$ and $\mathcal{B}_{e_2}$, respectively.

We also define the interaction $(\{\tau\}, true, F)$, where $F = \{v := v \mid v \in V^d\}$, for every component $\mathcal{B}_e = (P, V, N)$.

Note that interactions are only defined for consistent pairs of *SendReceive* and *LeakCollect* reflecting the rules from the operational semantics of Figure 2.

Given an IoT system with n entities and an initial state s_0 let us write Γ for the set of interactions of Definition 8. Recall that $\ll$ is the priority order of Section 4.4 and that $\Gamma(\mathcal{B}_1, \ldots, \mathcal{B}_n)$ is the composition of the n entities, as in Definition 5. Then $\langle\ll\rangle(\Gamma(\mathcal{B}_1, \ldots, \mathcal{B}_n))$ is the resulting $\mathcal{S}$BIP system. We have everything in place to show a correspondence between the *semantics* of the IoT system and of its $\mathcal{S}$BIP transformation.

Theorem 1. *Let (S, L, T, s_0) be an IoT system where $s_0 = \langle P_1, k_1\rangle \mid \cdots \langle P_n, k_n\rangle$ and where $\langle P_i, k_i\rangle$ is the initial state of each enity e_i, $i \leq n$. Let $\mathcal{B}_{e_i}$ be the $\mathcal{S}$BIP transformation and $\mathbf{X_{init}^i}$ be the initial valuation of the entity e_i and let Γ be*

the corresponding set of interactions. For $\mathcal{M} = (Q, \pi, P, q_0)$ *the semantics of* $\langle\ll\rangle(\Gamma(\mathcal{B}_1, \ldots, \mathcal{B}_n))$ *with the valuation* $\mathbf{X^1_{init}} \sqcup \cdots \mathbf{X^n_{init}}$*, there exists* $\mathcal{R} \subseteq S \times Q$ *a symmetric relation such that*

- $(s_0, q_0) \in \mathcal{R}$*;*
- *if* $(s, q) \in \mathcal{R}$ *then for all* $s' \in S$ *and* $s \xrightarrow[l]{[n]} s' \in T$ *there exists* $q' \in Q$ *and* $q \xrightarrow{p} q' \in \pi$ *with* $\mathbb{P}(q \xrightarrow{p} q') = n$ *such that* $(s', q') \in \mathcal{R}$*.*

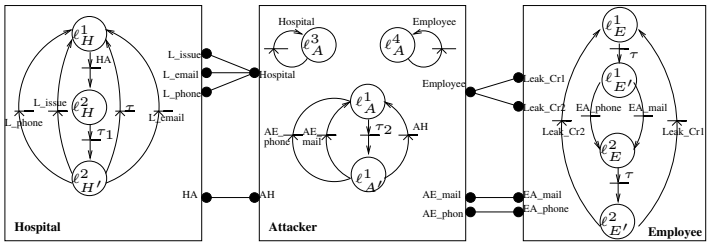


Fig. 6: Transformation of the Smart-Hospital example

As an example, we show in Figure 6 the transformation of the IoT model of the Smart Hospital in an $\mathcal{S}$BIP component.

6 Evaluating the Probability of an Attack

In this section we use executions of an IoT system to evaluate the probability of an attack. Thanks to Theorem 1, instead of reasoning on an IoT system, we can use the corresponding $\mathcal{S}$BIP system.

A successful attack can be a *rare* event, meaning that it occurs with probability 10^{-3} or smaller. We employ two SMC techniques. The first is Monte Carlo which consists of sampling executions and then estimating the probability of an attack, based on the number of executions for which the attack was successful. The method requires a large number of simulations for a correct estimate of a rare events which is not always feasible.

Secondly, we use an SMC technique tailor-made for rare events, called *importance splitting* [12]. This technique requires the decomposition of an execution leading to an attack into a sequence of elements, called levels, denoted l_i, for $i \leq m$ and for a decomposition in m levels. The first level is reached by all executions, while the last level is reached only if the attack succeeds. The levels are ordered $l_0 < \cdots < l_m$ meaning that level l_i is reached only if the previous levels $l_{j<i}$ have been reached before. We write $\mathbb{P}(\sigma > l_i)$ for the probability that l_i was reached during an execution σ. Then $\mathbb{P}(\sigma > l_i) = \mathbb{P}(\sigma > l_i \mid \sigma > l_{i-1})\mathbb{P}(\sigma > l_{i-1})$. Therefore we can compute the probability of the attack as follows: $\prod_{i=1}^{n} \mathbb{P}(\sigma > l_i \mid \sigma > l_{i-1})$. To infer the levels, importance splitting uses a *score* function defined on executions. Intuitively the closer we get to successful attack, the higher the score.

Attack trees provide an initial decomposition of the attack, on which the score function is defined. The attack tree is transformed into a $\mathcal{S}$BIP component, called a *monitor*. The leaves of the tree are some of the interactions between the Attacker and the other components in the model. The branches of the tree are internal transitions to the monitor component. In a monitor obtained from an attack tree t we associate a Boolean variable, denoted v_{n}, for every node (or leaf) n of t. The variable associated to a leaf are set to *true* when the associated event occurred in the monitored execution. The variables of each other node are updated according to their corresponding Boolean formula. Our approach is similar to [9], where monitors are Boolean formula that can be evaluated using executions of BIP systems.

We write $h(\mathsf{t})$ for the height of a tree t and $d(\mathsf{n}, \mathsf{t})$ for the depth of node n in t. The score of an execution is computed as $\mathsf{score} = h(\mathsf{t}) - d(\mathsf{n}, \mathsf{t})$, where n is the highest node for which v_{n} is *true*.

Definition 9 (Monitor). *The monitor $M_{\mathsf{t}} = (P, V, N)$ of an attack tree t is defined as follows:*

- *$P = \{p_{SR}, p_{LC}, p_{score}\}$ consists of two ports used for observing the SR and LC interactions and of a third port used for an internal transition that updates the score;*
- *$V = V^d \cup V^p$ where $V^d = \{\mathsf{score}\} \cup \{v_{\mathsf{n}} \mid \mathsf{n}$ is a node of $\mathsf{t}\}$ and $V^p = \emptyset$;*
- *$N = (\{l_0\}, \{l_0\}, T)$ is a Petri-Net with only one place l_0 and with $T = \{(l_0, \langle p, true, f\rangle, l_0) \mid p \in P\}$ where f updates the variables v_{n} and score.*

For each interaction in an $\mathcal{S}$BIP system, we add either the port p_{SR} or p_{LC} to the interaction. In this manner, the monitor can observe the system and update its Boolean variables accordingly.

In [17] the authors apply rare event techniques to *fault-trees*, a variant of attack trees. Importance sampling consists of simulating a system using an *importance sampling distribution* which makes the rare event more frequent. The results are adjusted w.r.t. the difference between the normal distribution and the importance sampling one. While this method is suited for attack trees, it is less convenient as it requires an additional step: the search for an importance sampling measure. We argue therefore that importance splitting is best suited for attack trees as it does not require the additional steps prior to the simulations.

7 Implementation and Experiments

In this section we describe the tool chain we implemented and some experiments based on the Smart Hospital example. In the diagram of Figure 7, the user provides an IoT system and an attack tree in the form of a *json* file. We implemented two parsers, "IoT-to-BIP" and "json-to-BIP", that transform the IoT model and the attack tree into two $\mathcal{S}$BIP files. The $\mathcal{S}$BIP files are compiled into a BIP executable. The BIP simulation engine runs the executable and interacts with Plasma [7], the statistical model checker we used. Plasma provides both Monte Carlo and importance splitting as SMC techniques.

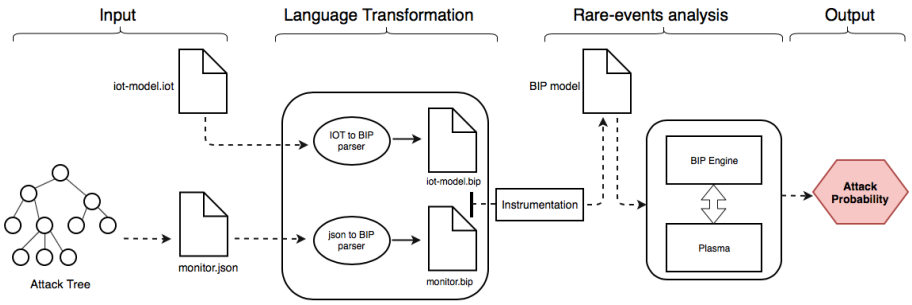


Fig. 7: Tools

The results of our experiments are shown in Figure 8. The model we used is based on the Smart Hospital, but is more complex in order to make successful attacks less probable. We provide the complete example and all resources necessary for replicating the experiments at `http://iot-modeling.gforge.inria.fr`.

	Model A				Model B			
	Monte Carlo		Importance Splitting		Monte Carlo		Importance Splitting	
Nb of Simulations	Result	Time (s)	Result	Time (s)	Result	Time (s)	Result	Time (s)
1000	0	6,29	6,738E-05	8,82	0	6,22	6,403E-06	8,76
10000	8,0E-05	15,00	5,240E-05	30,22	0	15,74	1,190E-05	30,46
100000	6,7E-05	255,60	7,033E-05	1349,196	0	251,61	5,824E-06	1088,802
1000000	6,4E-05	10410,11	4,033E-10	26330,33	7,5E-05	10423,48	4,348E-12	27013,48

Fig. 8: Experiments: In model B the *leaks* are twice less probable than in model A.

In Figure 8 we used two variants of our IoT model to calculate the probabilities of the success of an attack using the Monte Carlo and the importance splitting methods. The results and times presented in Figure 8 are obtained by averaging outcomes of 10 iterations. We observe that importance splitting gives a correct estimate from 1000 simulations, whereas the Monte Carlo method needs 100 times (1000 times for Model B) more simulations. However the importance splitting methods does not behave well when running on a large number of simulations. Therefore we argue that both methods are useful and complement each other in our analysis: Monte Carlo for estimating a probability n when we can produce around $10/n$ simulations, and importance splitting for experiments with fewer simulations.

8 Conclusion

In this paper, we proposed a sound probabilistic framework for modeling IoT systems and verifying its security using attack trees. The approach consists on transforming a high level IoT model and its attack tree into a $\mathcal{S}$BIP model. We validated our approach by showing on a complex example how to estimate the probability of success of an attack, using Monte Carlo and rare events techniques.

References

[1] *Anatomy of an Attack, MEDJACK (Medical Device Attack)*. Tech. rep. May 2015.

[2] Manos Antonakakis et al. "Understanding the mirai botnet". In: *USENIX Security Symposium*. 2017.

[3] Ananda Basu, Marius Bozga, and Joseph Sifakis. "Modeling Heterogeneous Real-time Components in BIP". In: *Fourth IEEE International Conference on Software Engineering and Formal Methods (SEFM 2006), 11-15 September 2006, Pune, India*. 2006. DOI: 10.1109/SEFM.2006.27.

[4] Delphine Beaulaton et al. "A Language for Analyzing Security of IOT Systems". submitted.

[5] Delphine Beaulaton et al. "A Modeling Language for Security Threats of IoT Systems". In: *Formal Methods for Industrial Critical Systems*. Ed. by Falk Howar and Jiří Barnat. Cham: Springer International Publishing, 2018, pp. 258–268. ISBN: 978-3-030-00244-2.

[6] Saddek Bensalem et al. "Statistical Model Checking Qos Properties of Systems with SBIP". In: *Proceedings of the 5th International Conference on Leveraging Applications of Formal Methods, Verification and Validation: Technologies for Mastering Change - Volume Part I*. 2012. DOI: 10.1007/978-3-642-34026-0_25.

[7] Benoît Boyer et al. "PLASMA-lab: A Flexible, Distributable Statistical Model Checking Library". In: *Quantitative Evaluation of Systems*. Ed. by Kaustubh Joshi et al. 2013. ISBN: 978-3-642-40196-1.

[8] ENISA. *Smart Hospitals, Security and Resilience for Smart Health Service and Infrastructures*. Tech. rep. 2016.

[9] Yliès Falcone et al. "Runtime Verification of Component-Based Systems". In: *Software Engineering and Formal Methods*. Ed. by Gilles Barthe, Alberto Pardo, and Gerardo Schneider. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011, pp. 204–220.

[10] Imene Ben Hafaiedh, Susanne Graf, and Sophie Quinton. "Building Distributed Controllers for Systems with Priorities". In: *J. Log. Algebr. Program.* 80.3 (2011), pp. 194–218.

[11] H. Holm et al. "P^2 CySeMoL: Predictive, Probabilistic Cyber Security Modeling Language". In: *IEEE Transactions on Dependable and Secure Computing* 12.6 (Nov. 2015), pp. 626–639. ISSN: 1545-5971. DOI: 10.1109/TDSC.2014.2382574.

[12] Cyrille Jegourel, Axel Legay, and Sean Sedwards. "Importance Splitting for Statistical Model Checking Rare Properties". In: *Computer Aided Verification*. Ed. by Natasha Sharygina and Helmut Veith. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013, pp. 576–591. ISBN: 978-3-642-39799-8.

[13] Bartek Klin and Vladimiro Sassone. "Structural Operational Semantics for Stochastic and Weighted Transition Systems". In: *Inf. Comput.* 227 (June 2013), pp. 58–83. ISSN: 0890-5401. DOI: 10.1016/j.ic.2013.04.001. URL: http://dx.doi.org/10.1016/j.ic.2013.04.001.

[14] Barbara Kordy, Marc Pouly, and Patrick Schweitzer. "Computational Aspects of Attack–Defense Trees". In: *Security and Intelligent Information Systems*. Ed. by Pascal Bouvry et al. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012, pp. 103–116.

[15] Sjouke Mauw and Martijn Oostdijk. "Foundations of Attack Trees". In: *Proceedings of the 8th International Conference on Information Security and Cryptology*. ICISC'05. Seoul, Korea: Springer-Verlag, 2006, pp. 186–198. ISBN: 3-540-33354-1, 978-3-540-33354-8. DOI: 10.1007/11734727_17. URL: http://dx.doi.org/10.1007/11734727_17.

[16] R. Milner. *A Calculus of Communicating Systems*. Springer-Verlag New York, Inc., 1982.

[17] Enno Ruijters et al. "Rare Event Simulation for Dynamic Fault Trees". In: *Computer Safety, Reliability, and Security*. Ed. by Stefano Tonetta, Erwin Schoitsch, and Friedemann Bitsch. Cham: Springer International Publishing, 2017, pp. 20–35. ISBN: 978-3-319-66266-4.

[18] T. Sommestad, M. Ekstedt, and H. Holm. "The Cyber Security Modeling Language: A Tool for Assessing the Vulnerability of Enterprise System Architectures". In: *IEEE Systems Journal* 7.3 (Sept. 2013), pp. 363–373. ISSN: 1932-8184. DOI: 10.1109/JSYST.2012.2221853.

[19] R.J. Vanglabbeek, S.A. Smolka, and B. Steffen. "Reactive, Generative, and Stratified Models of Probabilistic Processes". In: *Information and Computation* 121 (1995). DOI: https://doi.org/10.1006/inco.1995.1123.

Appendix

In this section, we present additional examples and definitions. We start with two definitions, for the counting functions used in Figure 1.

Definition 10 (Counting τ transitions from a state). *The functions* $\mathsf{count}_\tau : \mathit{State} \to \mathbb{N}$ *and* $\mathsf{count\_proc}_\tau : \mathit{Proc} \to \mathbb{N}$ *are defined as follows:*

$$\begin{aligned}
\mathsf{count}_\tau(s|t) &= \mathsf{count}_\tau(s) + \mathsf{count}_\tau(t)\\
\mathsf{count}_\tau(\langle P, k\rangle) &= \mathsf{count_proc}_\tau(P)\\
\mathsf{count_proc}_\tau(0) &= 0\\
\mathsf{count_proc}_\tau(\alpha.P) &= 1 \text{ if } \alpha \neq \tau\\
&\quad\ 0 \text{ if } \alpha = \tau\\
\mathsf{count_proc}_\tau(\textstyle\sum \alpha_i.P_i) &= 1\\
\mathsf{count_proc}_\tau(P \mid Q) &= \mathsf{count_proc}_\tau(P) + \mathsf{count_proc}_\tau(Q).
\end{aligned}$$

For counting the number of interactions, we have first to rewrite a state into a *canonical* form:

$$s \equiv s_S \mid s_R \mid s_L \mid s_C \qquad \text{where} \qquad \begin{aligned} s_S &= \langle P_1^{\mathrm{S}}, k_1^{\mathrm{S}} \rangle \mid \cdots \langle P_{nS}^{\mathrm{S}}, k_{nS}^{\mathrm{S}} \rangle \\ s_R &= \langle P_1^{\mathrm{R}}, k_1^{\mathrm{R}} \rangle \mid \cdots \langle P_{nR}^{\mathrm{R}}, k_{nR}^{\mathrm{R}} \rangle \\ s_L &= \langle P_1^{\mathrm{L}}, k_1^{\mathrm{L}} \rangle \mid \cdots \langle P_{nL}^{\mathrm{L}}, k_{nL}^{\mathrm{L}} \rangle \\ s_C &= \langle P_1^{\mathrm{C}}, k_1^{\mathrm{C}} \rangle \mid \cdots \langle P_{nC}^{\mathrm{C}}, k_{nC}^{\mathrm{R}} \rangle \end{aligned}$$

and where $P_i^{\mathrm{S}} \equiv a.P$ and the action a is a send; nS is the number of processes of the form above in s. Similarly we define the rest of the processes. Note that if we cannot rewrite a state in this form then the rule ParState_Interaction cannot be applied (any internal or sum transitions have priority over the interactions). Moreover entities can only communicate with other entities, that is interactions are not defined internally to an entity. We therefore only need to count interactions between entities.

The function $\mathsf{count}_{\mathrm{SR,LC}}$ uses an auxiliary function $\bar{\cdot} : \text{action} \to \text{action}$ which defines an action $\overline{a}$ which can synchronise with a using the rules SendReceive or LeakCollect.

Definition 11. *Let $s \equiv s_S \mid s_R \mid s_L \mid s_C$ be a state in a canonical form. The function $\mathsf{count}_{SR,LC} : State \to \mathbb{N}$ is defined on s as follows:*

$$\begin{aligned} \mathsf{count}_{SR,LC}(s_S \mid s_R \mid s_L \mid s_C) &= \mathsf{count}_{SR}(s_S, s_R) + \mathsf{count}_{LC}(s_L \mid s_C) \\ \mathsf{count}_{SR}(\langle a.P, k \rangle \mid s, t) &= \mathsf{count}(a, t) + \mathsf{count}_{SR}(s, t) \\ \mathsf{count}_{LC}(\langle a.P, k \rangle \mid s, t) &= \mathsf{count}(a, t) + \mathsf{count}_{LC}(s, t) \\ \mathsf{count}(a, \langle b.P, k \rangle \mid t) &= 1 + \mathsf{count}(a, t) \ \textit{if } a = \overline{b} \\ &= \mathsf{count}(a, t) \ \textit{otherwise} \end{aligned}$$

The following example is a IoT trace for which we compute its probability, as described at the end of Section 2.

Example 2. Let us compute the probability of the following trace:

$$\begin{aligned} &\langle [n_1]e_1 \xrightarrow[\mathrm{v'}]{\mathrm{c}} e_2 + [n_1']a_1, k_1 \rangle | \langle [n_2]e_2 \xleftarrow{\mathrm{c}} e_1 + [n_2']a_2, k_2 \rangle \ \xrightarrow[\tau]{[n_1]} \\ &\langle e_1 \xrightarrow[\mathrm{v'}]{\mathrm{c}} e_2, k_1 \rangle | \langle [n_2]e_2 \xleftarrow{\mathrm{c}} e_1 + [n_2']a_2, k_2 \rangle \ \xrightarrow[\tau]{[n_2]} \\ &\langle e_1 \xrightarrow[\mathrm{v'}]{\mathrm{c}} e_2, k_1 \rangle | \langle e_2 \xleftarrow{\mathrm{c}} e_1, k_2 \rangle \ \xrightarrow[\mathtt{LC}:v]{[1]} \ \langle 0, k_1 \rangle | \langle 0, k_2^{c'} \uplus \{v\} \rangle \end{aligned}$$

where we omit the 0 process and write a_1, a_2 for some actions. In the trace above, first a choice is made between the actions $e_1 \xrightarrow[\mathrm{v'}]{\mathrm{c}} e_2$ and a_1, then, in a second transition, between the actions $e_2 \xleftarrow{\mathrm{c}} e_1$ and a_2. Only after the two probabilistic choices are resolved that the interaction can proceed. The probability of the trace is then $n_1 \cdot n_2 \cdot 1$.

The next example shows how the random variables engender a probabilistic behavior over transitions of a $\mathcal{S}$BIP component.

Example 3. The random variables engender a probabilistic behavior over transitions of $\mathcal{M}$. Let us consider the atomic component $\mathcal{B}$ in Figure 9a. Component $\mathcal{B}$ has a unique transition going from place l_1 to place l_2 using port p, with a guard that is trivially true, and which updates the random variable v according to a distribution μ. Assuming the initial value of v is x_0, when executing $\mathcal{B}$, there will be several possible transitions, from state $(\{l_1\}, x_0)$ to states $(\{l_2\}, x_i)$ for all $x_i \in D$, where D is the valuation domain of v, as illustrated in Figure 9b.

(a) A Stochastic Atomic Component $\mathcal{B}$ (b) Behavior of $\mathcal{B}$

Fig. 9: Example of a stochastic atomic component $\mathcal{B}$ and its behavior.

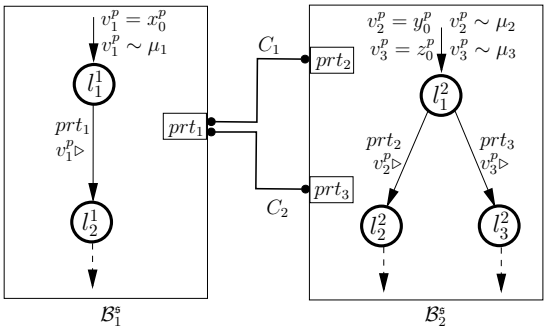
The definition of priorities, informally described after Definition 2 is as follows:

Definition 12 (Priorities [10]). *Let $<$ be a priority order on the ports P of a component $\mathcal{B} = (P, V, N)$, with $N = (L, L_0, T)$. We define $\langle < \rangle(\mathcal{B})$ as the component (P, V, N') where $N' = (L, L_0, T')$ with*

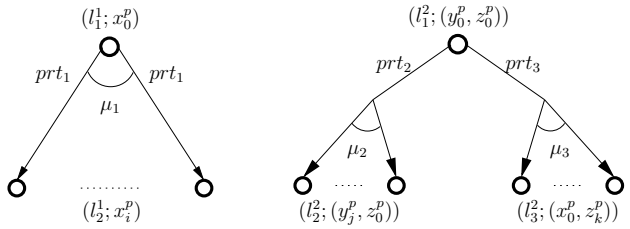
$$T' = \{t \mid t \in T \text{ and } \nexists t' \in T, \text{ s.t. } p_t < p_{t'} \text{ and } {}^\bullet t =^\bullet (t')\}.$$

Example 4. The two $\mathcal{S}$BIP components compose into a $\mathcal{S}$BIP component as shown in Figure 10.

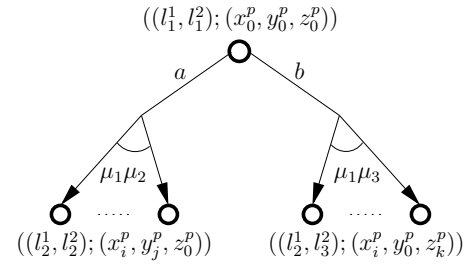
Remark 1 (BIP transitions are deterministic once the port is chosen). In $\mathcal{S}$BIP only one transition $q \xrightarrow{p} q'$ is allowed at anytime, which corresponds to the semantic constraint of the IoT language (Remark 1.) In order to ensure that any IoT system can be translated into an $\mathcal{S}$BIP system, as in Section 5, we have to impose a similar semantic constraint on the IoT language. We impose that whenever $\sum_{i \in I} a_i.T_i$ appears in a process, if $a_i = a_j$ then $T_i \not\equiv_P T_j$, for any $i \neq j$, $i, j \in I$. Informally the constraint says in an IoT transition system there cannot be two transitions with the same label and leading to the same state.



(a) Composition of two $\mathcal{S}$BIP components



(b) Underlying Behavior of $\mathcal{B}_1$ (c) Underlying Behavior of $\mathcal{B}_2$



(d) Underlying Behavior of $\mathcal{B} = \gamma(\mathcal{B}_1, \mathcal{B}_2)$, with $\gamma = \{a = \{prt_1, prt_2\}, b = \{prt_1, prt_3\}\}$

Fig. 10: Illustration of the stochastic semantics of composition in $\mathcal{S}$BIP.

The following two definitions are a formal treatment of notions related to attack tress and used in Definition 9.

Definition 13 (Height and depth in an attack tree for). *The height of* t*, denoted* $h(\mathsf{t})$*, is defined recursively as follows:*

$$h(\mathsf{t}) = \begin{cases} 1 & \textit{if } \mathsf{t} \in \Delta \\ 1 + max(h(\mathsf{t}_1), h(\mathsf{t}_2)) & \textit{if } \mathsf{t} = \mathsf{t}_1 \wedge \mathsf{t}_2 \textit{ or } \mathsf{t} = \mathsf{t}_1 \vee \mathsf{t}_2 \end{cases}$$

Each node n *in a tree* t *has a* depth*, defined as follows:*

$$d(\mathsf{n},\mathsf{t}) = \begin{cases} 0 & \text{if } \mathsf{n} = \mathsf{t} \\ 1 + d(\mathsf{n},\mathsf{t}_1) & \text{if } \mathsf{t} = \mathsf{t}_1 \wedge \mathsf{t}_2 \text{ or } \mathsf{t} = \mathsf{t}_1 \vee \mathsf{t}_2 \text{ and } \mathsf{n} \in \mathsf{t}_1 \\ 1 + d(\mathsf{n},\mathsf{t}_2) & \text{if } \mathsf{n} = \mathsf{n}_1 \wedge \mathsf{n}_2 \text{ or } \mathsf{n} = \mathsf{n}_1 \vee \mathsf{n}_2 \text{ and } \mathsf{n} \in \mathsf{t}_1 \end{cases}$$

Definition 14 (Attack Tree Semantics). *Let* $t \in \mathcal{T}$ *an attack tree. The semantics of* t*, denoted* $[\![t]\!]$*, consists of a propositional formula defined by recursion on* t *as follows:*

- *if* $t \in \Delta$ *then* $[\![t]\!] = X_t$*;*
- *if* $t = t_1 \wedge t_2$ *then* $[\![t]\!] = [\![t_1]\!] \wedge [\![t_2]\!]$*;*
- *if* $t = t_1 \vee t_2$ *then* $[\![t]\!] = [\![t_1]\!] \vee [\![t_2]\!]$*.*

Remark 2 (Semantics of an attack tree). An attack tree can also be seen as a Boolean expression, by associating to every event in $e \in \Delta$ a Boolean variable v_e such that $e \neq e' \iff v_e \neq v_{e'}$. The semantics of an attack tree is given with respect to a valuation of the Boolean variables: Let $\mathbf{X} : \Delta \to \{true, false\}$ be a valuation for Δ, then the semantics of t w.r.t. $\mathbf{X}$, denoted $[\![\mathsf{t}]\!](\mathbf{X}) \in \{true, false\}$, consists in evaluating the associated Boolean formula [14].

In order to assess whether an attack was successful, we monitor the executions of a system. Given an execution trace σ, its corresponding valuation $\mathbf{X}(\sigma)$ sets v_e to *true* if the event e occurred in σ. If $[\![t]\!](\mathbf{X}(\sigma))$ is true then the execution σ is a successful attack of t.

Definition 15 (Interaction between a monitor and a $\mathcal{S}$BIP system). *Let* Γ *be a set of interactions of a* $\mathcal{S}BIP$ *system. The set of interactions* Γ' *between the* $\mathcal{S}BIP$ *system and a monitor* $M_{\mathsf{t}} = (P, V, N)$ *is defined as follows: for all* $\gamma{=}(\{a, a'\}, G_\gamma, F_\gamma) \in \Gamma$*, let* $(\{a, a', p\}, G_\gamma, F_\gamma \sqcup f) \in \Gamma'$ *where* p *and* f *are defined as follows:*

- *if* $a = e_1 \xrightarrow[v]{c} e_2$ *then* $p = p_{SR}$ *and* $f = \{v_{\mathsf{n}} := true, \text{ if } v_{\mathsf{n}} \in V\}$*, for* $\mathsf{n} = (SR, v)$*;*
- *if* $a = e_1 \xrightarrow[v]{\twoheadrightarrow} e_2$ *then* $p = p_{SR}$ *and* $f = \{v_{\mathsf{n}} := true, \text{ if } v_{\mathsf{n}} \in V\}$ *for* $\mathsf{n} = (LC, v)$*;*

Remark 3 (Comparison of the IoT language with the Probabilistic CCS). In Figure 11 we show an alternative LTS for an IoT system inspired by the probabilistic CCS as defined in [19]. In the alternative LTS we redefine the rules for SendReceive, LeakCollect and Internal, we remove the Choice rule, and keep the rest as in Figure 2.

The LTS of a process is translated into a *probabilistic transition system* by normalizing the probabilities of transitions, denoted as follows:

$$\rho(P \xrightarrow[l]{[n]} P') = P \xrightarrow[l]{[p \cdot r]} P'$$

where r is a normalizing factor, called *deadlock eliminating*, computed as follows:

$$r = \{p_i \mid \sum_i P \xrightarrow[l_i]{[p_i]} P_i', l_i = \tau\}.$$

Informally, probabilities are normalized w.r.t. all possible (non deadlocking) transitions from the same process P. For such a semantics we lose the correspondence with $\mathcal{S}$BIP. To see this consider the process $P = [n_a]a.P + [n_b]b.Q \mid a_1.P_1 \mid a_2.P_2$, where a can synchronize with either a_1 or a_2, but b cannot synchronize with neither of the two. In $\mathcal{S}$BIP we can reach deadlock with probability $n_b \cdot 0.5 + n_b \cdot 0.5$ whereas the *deadlock eliminating* normalization removes deadlocks from the behaviour of P. Indeed $r = 2 \cdot n_b$ and therefore $P \xrightarrow[\tau]{[0.5]} P \mid P_1$ or $P \xrightarrow[\tau]{[0.5]} P \mid P_2$.

Another possibility is to take into account deadlocks when normalizing. However in order to do this, we need to change the semantics, by expliciting the deadlock transitions in the LTS. We use the CHOICE rule instead, which is equivalent, and has the additional benefit of being closer to the semantics of $\mathcal{S}$BIP.

Lastly note that other approaches to probabilistic CCS consists of giving weights or rates [13] to transitions could also have been used in our language.

SENDRECEIVE

$$\frac{\exists v \in k_1^c \text{ s.t. } v \in k_2^c \qquad c' = \mathsf{protocol}(v')}{\langle [p_1]e_1 \xrightarrow[\mathrm{v'}]{\mathrm{c}} e_2.P_1, k_1\rangle | \langle [p_2]e_2 \xleftarrow{\mathrm{c}} e_1.P_2, k_2\rangle \xrightarrow[\mathrm{SR}:v]{p_1 \cdot p_2} \langle P_1, k_1\rangle | \langle P_2, k_2^{c'} \uplus \{v'\}\rangle}$$

LEAKCOLLECT

$$\frac{c' = \mathsf{protocol}(v')}{\langle [p_1]e_1 \xrightarrow[\mathrm{v'}]{} e_2.P_1, k_1\rangle | \langle [p_2]e_2 \nleftarrow e_1.P_2, k_2\rangle \xrightarrow[\mathrm{LK}:v]{p_1 \cdot p_2} \langle P_1, k_1\rangle | \langle P_2, k_2^{c'} \uplus \{v'\}\rangle}$$

INTERNAL $\quad \langle [p]\tau.P, k\rangle \xrightarrow[\tau]{p} \langle P, k\rangle$

Fig. 11: The operational semantics of an IoT system *a la PCCS*

Lemma 1. *Any two congruent IoT states have the same transformation in $\mathcal{S}$BIP systems.*

Proof. We proceed by cases on the congruence relation. First consider the congruence relation on states: For the monoid laws on $|$, note that the transformation results in a set of atomic components and therefore the order of states in the parallel composition does not matter. In the case where processes are congruent, we distinguish two subcases: (i) Threads in a parallel composition translate into tuples of states in the transformation of a process (Definition 7) where the order of the states does not matter; (ii) For the rest we use the fact that inside an atomic component the states that have congruent labels are identified.

Lemma 2. *Let $s = \langle P_1, k_1\rangle \mid \cdots \mid \langle P_n, k_n\rangle$ be an IoT state and let $(P, Q, \pi, q_0 = (m, \mathbf{X}))$ be the semantics of $\langle\ll\rangle\Gamma(\mathcal{B}_{e_1}, \cdots \mathcal{B}_{e_n})$, where $\mathcal{B}_{e_i}$ is the transformation of each state $\langle P_1, k_1\rangle$, for $i \leq n$. Then*

1. $\mathsf{count}_\tau(s) = |\mathsf{Enabled}(m; \mathbf{X})|$;
2. *if* $\mathsf{count}_\tau(s) = 0$ *then* $\mathsf{count}_{SR,LC}(s) = |\mathsf{Enabled}(m; \mathbf{X})|$.

Proof. The function $\mathsf{count}_\tau(s)$ counts the transitions labeled τ that can be triggered from state s (Definition 10). On the contrary, the function $\mathsf{count}_{\mathtt{SR},\mathtt{LC}}(s)$ counts the transitions that are not labeled τ and that can be triggered from s.

We have that $\mathsf{Enabled}(m; \mathbf{X}) = \{t \in T \mid {}^\bullet t \leq m \text{ and } \mathbf{X}(g_t) \text{ is true } \}$. From the definition of priorities (Definition 12) we distinguish two cases: either there are no τ transitions enabled or there is at least one τ transition. In the latter case, by structural induction on s and using Definition 6, we can show that $\mathsf{count}_\tau(s) = |\mathsf{Enabled}(m; \mathbf{X})|$.

If there are no τ transitions enabled, then $\mathsf{count}_\tau(s) = 0$. Again, by an induction on s and using the transformation in Definition 6, we can show that $\mathsf{count}_{\mathtt{SR},\mathtt{LC}}(s) = |\mathsf{Enabled}(m; \mathbf{X})|$.

Proof (Theorem 1). Let $e_1, \cdots, e_n$ be n entities of an IoT system (S, L, T, s_0) with the initial states $\langle P_1, k_1\rangle, \cdots \langle P_n, k_n\rangle$. $\mathcal{B}_{e_i} = (P_i, V_i, N_i)$ with $N_i = (L_i, L_{i,0}, T_i, F_i)$, is the transformation of the current state of the entity e_i, for $i \leq n$. Also let $V_i = V_i^p \cup V_i^d$. We write (P, Q, π, q_0) for the semantics of $\langle\ll\rangle\Gamma(\mathcal{B}_{e_1}, \cdots \mathcal{B}_{e_n}) = (\Gamma, \mathbf{V}, \mathbf{N})$ with $\mathbf{N} = (\mathbf{L}, \mathbf{L_0}, \mathbf{T}, \mathbf{F})$. Lastly $\mathbf{X_{init}}$ is the initial valuation.

To construct the relation $\mathcal{R} \subseteq S \times Q$ required by the theorem, we first set some notations and constraints below. Informally, these constraints establish the relation between the processes and knowledge functions in states of S and the markings and the valuations, respectively, in states of Q.

1. **Correspondence between Processes and Markings.** For a thread T let us write m_T for the marking associated with T and defined as follows:

$$m_T(l) = \begin{array}{l} 1 \text{ if } \ell(l) = T \text{ or } \ell(l) = U^\star, U = [n]T + T', \quad \text{for some threads } T', U \\ 0 \text{ otherwise} \end{array}$$

   where (P, V, N) and $N = (\mathcal{L}, \mathcal{L}_0, \mathcal{T}, \mathcal{F})$ is obtained as in Definition 6 and where $l \in \mathcal{L}$. For a process $P = T_1 \mid \cdots \mid T_m$, let us write m_P for the marking associated with P and defined as $m_{T_1} + \cdots + m_{T_m}$.
2. **Correspondence between Knowledge and the Deterministic Variables.** From Definition 6 it follows that for each thread T_j in a process P_i we define the set $V_i^d = \{v_c \mid c \text{ is a protocol used in } T_j\}$. From Definition 7 then the set of variables of $P_i = T_1 \mid \cdots \mid T_m$ is $\cup_{j \leq m} V_j = \{v_c \mid c \text{ is a protocol used in } P_i\}$.
   Then, if $\mathbf{X_i}$ the current valuation of entity e_i, we require that $\mathbf{X_i}(v_c) = k_i(c)$, for $i \leq n$, $c \in C$ and $v_c \in V_i^d$. Recall that we write C for the set of protocols used in the IoT system and k_i for the knowledge function of an entity e_i.
3. **Correspondence between Probabilistic Choices in Processes and the Random Variables.** For every summation thread U in a process P_i, we have that there exists a random variable $v_U \in V_i^p$, by Definition 6. Moreover, if T a thread of P_i, belongs to a summation, i.e. $U = [n]T + T'$, for some threads T', U, then for the current valuation $\mathbf{X_i}$ we have that $\mathbf{X_i}(v_U) = T$.

For a process $P = T_1 \mid \cdots \mid T_m$ we use Definition 7 and have that V^p is the disjoint union of all V_j^p, where V_j^p is the set of random variables for T_j, $j \leq m$.

We define the following relation between the states of S and the states of Q:

$$\mathcal{R} = \big\{(\langle P_1, k_1\rangle \mid \cdots \mid \langle P_n, k_n\rangle, (m = m_{P_1} + \cdots m_{P_n}, \mathbf{X} = \mathbf{X_1} \sqcup \cdots \sqcup \mathbf{X_n})) \mid \text{the conditions 1-3 above hold}\big\}.$$

We show that $\mathcal{R}$ is the relation required in Theorem 1. First we have to show that $(s_0, q_0) \in \mathcal{R}$.

We use Definition 6 from which we have that $\mathcal{L}_0 = \{l_T\}$ is the initial place in the transformation of a thread T. Then, by Definition 7, $L_0 = \uplus_{j \leq m} \mathcal{L}_0^j = \uplus_{j \leq m} \{l_{T_j}\}$ is the initial set of places in the transformation of a process $P = T_1 \mid \cdots \mid T_m$. From Definition 5 it follows that $\mathbf{L}_0 = \uplus_{i \leq n} L_{0,i}$. By Definition 3 the initial marking in $q_0 = (m_0, \mathbf{X_{init}})$ is defined as $m_0(l) = 1 \iff l \in \mathbf{L}_0$ and 0 otherwise. Hence we can write $m_0 = m_{P_1} + \cdots + m_{P_n}$. This shows condition 1 of $\mathcal{R}$.

From Definition 7 we have that for each entity e_i, $\mathbf{X_{init}}(v_c) = k_i(c)$, for all protocols c used by e_i. From Definition 5 the set of variables of the composed $\mathcal{B}_{e_i}$ components is the disjoint union V_i, i.e. $\mathbf{V} = \uplus_{i \leq n} V_i$, in particular $\mathbf{V}^d = \uplus_{i \leq n} V_i^d$. Then a valuation for $\mathbf{V}$ is the disjoint composition of the individual valuations for V_i, from which it follows the required decomposition of $\mathbf{X_{init}}$ in $q_0 = (m_0, \mathbf{X_{init}})$. Therefore condition 2 of $\mathcal{R}$ holds. For condition 3 to hold suffices to note that there is no probabilistic choice made yet in any process and therefore there is no correspondence to show. We can take any initial valuation we want for the random variables.

Let us now suppose that $(s, q) \in \mathcal{R}$ and that $s \xrightarrow[l]{[n]} s'$, for some label $l \in L$, some probability n and state $q' \in Q$. We have to show that there exists $q' \in Q$ and $q \xrightarrow{p} q' \in \pi$ with $\mathbb{P}(q \xrightarrow{p} q') = n$ such that $(s', q') \in \mathcal{R}$. We reason by cases on the label l of the transition $s \xrightarrow[l]{[n]} s'$.

– Let $l = SR : v$ or $l = LC : v$; then let e_1 and e_2 be the two communicating entities. Using Lemma 1 we can rewrite the transition as follows:

$$s = \langle P_1, k_1\rangle \mid \langle P_2, k_2\rangle \mid \langle P_3, k_3\rangle \mid \ \ldots \ \mid \langle P_n, k_n\rangle \xrightarrow[l]{[1/m]}$$
$$s' = \langle Q_1, k_1'\rangle \mid \langle Q_2, k_2'\rangle \mid \langle P_3, k_3\rangle \mid \ \ldots \ \mid \langle P_n, k_n\rangle$$

where we can decompose $P_1 \equiv_P a_1.T_1 \mid P_1'$ and $P_2 \equiv_P a_2.T_2 \mid P_2'$, $Q_1 \equiv_P T_1 \mid P_1'$ and $Q_2 \equiv_P T_2 \mid P_2'$, again by Lemma 1 and from the rules of Figure 2. Here we suppose w.l.o.g. that a_1 and a_2 are the two synchronizing actions in P_1 and P_2, respectively. Also suppose w.l.o.g. that a_1 is a send (or a leak) and that a_2 is a receive (or a collect). Let c be the protocol used for the communication in case $l = SR : v$.

From $(s,q) \in \mathcal{R}$ we have that $q = (m_{P_1} + \cdots m_{P_n}, \mathbf{X_1} \sqcup \cdots \sqcup \mathbf{X_n})$ and that $m_{P_i} = m_{a_i.T_i} + m_{P_i'}$, for $i \leq 2$. Also from condition 1 of $\mathcal{R}$, $m_{a_i.T_i} = \{l_i\}$ with either $\ell(l_i) = a_i.T_i$, or $\ell(l_i) = U_i^\star$, for some summation threads U_1, U_2.

- If $\ell(l_1) = a_1.T_1$ then we use the transformation of Definition 6 to show that there exists the place $l_1' \in L_1$, with $\ell(l_1') = T_1$ and the transition $t_1 = (\{l_{a_1.T_1}\}, \langle a_1, g_1 = \mathit{true}, f_1\rangle, \{l_{T_1}\})$ in $\mathcal{B}_1$.
  - If $\ell(l_2) = a_2.T_2$ then as above, there exists $l_2' \in L_2$, with $\ell(l_2') = T_2$ and the transition $t_2 = (\{l_{a_2.T_2}\}, \langle a_2, g_2 = \mathit{true}, f_2\rangle, \{l_{T_1}\})$ in $\mathcal{B}_2$.
  - $\ell(l_2) = U_2^\star$, with $U_2 = [n_2]a_2.T_2 + U_2'$, for some threads U_2, U_2'. As in the case above, from Definition 6 we have that there exists the places $l_2' \in L_2$ with $\ell(l_2') = T_2$. We also have, from condition 3 of $\mathcal{R}$ that there exists a random variable $v_{U_2} \in V_2^p$ with $\mathbf{X}(v_{U_2}) = a_2.T_2$. Moreover we have the transition $t_2 = (\{l_{U_2^\star}\}, \langle a_2, g_2 = (v_{U_2} == a_2.T_2), f_2\rangle, \{l_{T_2}\})$ in $\mathcal{B}_2$.
- the other case is similar.

Note that in all cases above, $f_i = \{v := v \mid v \in V^d\}$ with $R_i^p = \emptyset$, $i \leq n$.
Using Definition 8 we have that there exists an interaction $\gamma = (\{a_1, a_2\}, G, F)$ such that

- If $l = SR : v$ then $G = (\exists x \in v_c^1$ such that $x \in v_c^2)$ for $v_c^1 \in V_1$ and $v_c^2 \in V_2$.
- If $l = LC : v$ then $G = \mathit{true}$.

Also, $F = \{v_{c'}^2 := v_{c'}^2 \cup \{v'\} \mid \mathsf{protocol}(v') = c', v_{c'}^2 \in V_2\}$ for both $l = SR : v$ and $l = LC : v$.
We now use Definition 5 and have that there exists the transition

$$\underline{T} = (\{l_1, l_2\}, \langle \gamma, g_1 \wedge g_2 \wedge G, (f_1 \sqcup f_2) \circ F\rangle, \{l_1', l_2'\}) \in \mathbf{T}.$$

We have to show that the guard $g = g_1 \wedge g_2 \wedge G$ holds for the current valuation $\mathbf{X}$:

- If $g_1 = (v_{U_1} == a_1.T_1)$ then $\mathbf{X}(g_1)$ holds from condition 3 of $\mathcal{R}$; otherwise $g_1 = \mathit{true}$. We proceed similarly for g_2.
- If $l = SR : v$ then $G = (\exists x \in v_c^1$ such that $x \in v_c^2)$ for $v_c^1 \in V_1$ and $v_c^2 \in V_2$. From condition 2 of $\mathcal{R}$ we have that $\mathbf{X}(v_c^i) = k_i(c)$, $i \leq n$. Then the guard holds as it is the condition of rule SendReceive in Figure 2. If $l = LC : v$ then $G = \mathit{true}$.

The transitions above are allowed to proceed by the priority order $\ll$ (see Definition 12) only if there is no internal transition available. This is the case as ensured by the rule ParState_Interaction in Figure 2.
Therefore, by Definition 3, there exists the transition

$$q = (m_{P_1} + m_{P_2} + \cdots m_{P_n}, \mathbf{X_1} \sqcup \cdots \sqcup \mathbf{X_n}) \xrightarrow{\gamma} q' = (m', \mathbf{X}')$$

where we have to show that conditions 1-3 of $\mathcal{R}$ hold. For condition 1 we have to show that $m' = m_{Q_1} + m_{Q_2} + \cdots m_{P_n}$. Using Definition 3 it follows that

$$m' = m -^\bullet \underline{T} + \underline{T}^\bullet = m - \{l_1, l_2\} + \{l_1', l_2'\}.$$

As $\mathbf{L}_0 = \uplus_{i \leq n} L_{0,i}$, from Definition 5, it follows that

$$m' = (m_{P_1} - \{l_1\} + \{l_1'\}) + (m_{P_2} - \{l_2\} + \{l_2'\}) + \cdots + m_{P_n}.$$

Using condition 1 of $\mathcal{R}$ on m_{P_1} and m_{P_2} we have that $m_{P_1} - \{l_1\} + \{l_1'\} = m_{Q_1}$ and similarly for m_{Q_2}.
Let us now show condition 2, i.e. $\mathbf{X}' = \mathbf{X_1'} \sqcup \mathbf{X_2'} \sqcup \cdots \sqcup \mathbf{X_n}$ and $\mathbf{X_i'}(v_{c'}) = k_i'(c')$, $i \leq 2$. Using the function F above we have that $\mathbf{X_i'}(v_{c'}) = \mathbf{X_i}(v_{c'}) \cup \{v\}$. From rules SendReceive and LeakCollect we also get that $k_i'(c') = k_i(c) \cup \{v\}$, $i \leq 2$.
As $R_1^p = R_2^p = \emptyset$ condition 3 is trivial.
Lastly, the two transitions have the same probability: $|\mathsf{Enabled}(m; \mathbf{X})| = m$ by Lemma 2, and therefore $\mathbb{P}\big(q \xrightarrow{p} q'\big) = 1/m$.

– Let $l = \tau$; let e_1 be the entity that triggers the internal transition. Using Lemma 1 we can rewrite the states in the transition as follows:

$$s = \langle P_1, k_1 \rangle \mid \langle P_2, k_2 \rangle \mid \langle P_3, k_3 \rangle \mid \ \ldots \ \mid \langle P_n, k_n \rangle \xrightarrow[l]{[n]}$$
$$s' = \langle Q_1, k_1' \rangle \mid \langle P_2, k_2 \rangle \mid \langle P_3, k_3 \rangle \mid \ \ldots \ \mid \langle P_n, k_n \rangle.$$

There are two possibilities: either $P_1 \equiv_P \sum_{i \in I_1} a_i.T_i \mid P_1'$ where $Q_1 = a_1.T_1$ w.l.o.g. or $P_1 \equiv_P \tau.T_1 \mid P_1'$ with $Q_1 = T_1$. We write $U = \sum_{i \in I_1} a_i.T_i$ or $U = \tau.T_1$ depending on which of the two cases we are.
From $(s, q) \in \mathcal{R}$ we have that $q = (m_{P_1} + \cdots m_{P_n}, \mathbf{X_1} \sqcup \cdots \sqcup \mathbf{X_n})$ and that $m_{P_1} = \{l\} + m_{P_1'}$, $\ell(l) = U$. We use the transformation of Definition 6 to show that there exists the place $l' \in L_1$ and the transition $t = (\{l\}, \langle \tau, g = \mathit{true}, f \rangle, \{l'\})$ in $\mathcal{B}_1$.

- If $U = \sum_{i \in I_1} a_i.T_i$ then $\ell(l') = U^\star$, $f = \{v := v \mid v \in V_1^d\}$ and $R^p = \{v_U\}$.
- If $U = \tau.T_1$ then $\ell(l') = T_1$, $f = \{v := v \mid v \in V_1^d\}$ and $R^p = \emptyset$.

Using Definition 8 we have that there exists an interaction $\gamma = (\{\tau\}, G = \mathit{true}, F)$ with $F = \{v := v \mid v \in V_1^d\}$.
From Definition 5 there exists the transition

$$\underline{T} = (\{l\}, \langle \gamma, g_1 \wedge G = \mathit{true}, f \circ F \rangle, \{l'\}) \in \mathbf{T}.$$

The guard trivially holds and we obtain the transition

$$q = (m_{P_1} + + \cdots m_{P_n}, \mathbf{X_1} \sqcup \cdots \sqcup \mathbf{X_n}) \xrightarrow{\gamma} q' = (m', \mathbf{X}')$$

where we have to show that conditions 1-3 of $\mathcal{R}$ hold. As in the first case, condition 1 follows from $m' = m - \{l\} + \{l'\} = m_{Q_1} + \cdots m_{P_n}$. Condition 2 trivially hold as the update functions f and F are the identity and therefore $\mathbf{X_1}' = \mathbf{X_1}$. Indeed the knowledge function of k_1 is not modified by the rules Choice or Internal.
To show condition 3 we use Definition 6 from which we have that there exists $v_U \in V_1^p$, $v_U \sim \mu$, where $\mu(a_1.T_1) = n_1$. Then we can take $\mathbf{X}'(v_U) = a_1.T_1$. We also this argument to show that the two transitions have the same probabilities: by Lemma 2, $|\mathsf{Enabled}(m; \mathbf{X})| = m$ and therefore $\mathbb{P}\big(q \xrightarrow{p} q'\big) = 1/m \times n_1$.

Hereafter we prove the similarity of the IoT system to its corresponding $\mathcal{S}$BIP model. Let us suppose that $(q, s) \in \mathcal{R}$ and that $q \xrightarrow{\gamma} q'$, for a transition labelled with γ, where $q, q' \in Q$. We have to show that there is a state $s' \in S$ with $s \xrightarrow[l]{[n]} s'$, for some label $l \in L$, such that $(s', q') \in \mathcal{R}$. We define $s = \langle P_1, k_1\rangle \mid \langle P_2, k_2\rangle \mid \cdots \mid \langle P_n, k_n\rangle$. Here we also reason by cases: whether the transition is an interaction between two components $\mathcal{B}_{e_1}$ and $\mathcal{B}_{e_2}$ or an internal transition.

– We consider the communication is an interaction $\gamma = (\{a_1, a_2\}, G, F)$ between $\mathcal{B}_{e_1}$ and $\mathcal{B}_{e_2}$:

$$q = (m_{P_1} + m_{P_2} + \cdots m_{P_n}, \mathbf{X_1} \sqcup \mathbf{X_2} \sqcup \cdots \sqcup \mathbf{X_n}) \xrightarrow{\gamma} q' = (m', \mathbf{X}')$$

As it is an interaction between two entities, from Definition 5 we have that there exists the transitions $t_i = (m_i, \langle p_i, g_i, f_i\rangle, m_i') \in T_i$, for $i \in \{1, 2\}$. From the Definition 8, $m_i = m_{P_i}$, $p_i = a_i$, $g_i = true$ and f_i are the constant update functions. From $(q, s) \in \mathcal{R}$ we have that $m_{P_1} = m_{a_1.T_1} + m_{P_1'}$, $m_{P_2} = m_{a_2.T_2} + m_{P_2'}$ with $P_1 = a_1.T_1 \mid P_1'$ and $P_2 = a_2.T_2 \mid P_2'$. Moreover, from the Definition 3 there exists the transition

$$\underline{T} = (m_{P_1} + m_{P_2}, \langle\{a_1, a_2\}, g_1 \wedge g_2 \wedge G, (f_1 \sqcup f_2) \circ F\rangle, m_{Q_1} + m_{Q_2}) \in \mathbf{T}$$

with $m_{Q_1} = m_{T_1} + m_{P_1'}$, $m_{Q_2} = m_{T_2} + m_{P_2'}$.
We distinguish between the two types of interactions:

- $a_1 = e_1 \xrightarrow[v']{c} e_2$ and there exists $a_2 \in \mathsf{Actions}$ such that $a_2 = e_2 \xleftarrow{c} e_1$,
- or $a_1 = e_1 \xrightarrow[v']{}\!\!\!\!\rightarrow e_2$ and there exists $a_2 \in \mathsf{Actions}$ such that $a_2 = e_2 \twoheadleftarrow e_1$

Following the Definition 8 we have the following guards:

- if $G = (\exists x \in v_c^1$ such that $x \in v_c^2)$ for $v_c^1 \in V_1$ and $v_c^2 \in V_2$ then $l = SR$
- if $G = true$ then $l = LC$

We can then apply the rules SendReceive or LeakCollect from Figure 2. Hence we derive an interaction between e_1 and e_2 exists for which we have to show that conditions 1-3 of $\mathcal{R}$ hols.

$$s = \langle P_1, k_1\rangle \mid \langle P_2, k_2\rangle \mid \ldots \mid \langle P_n, k_n\rangle \xrightarrow[l]{[n]}$$
$$s' = \langle Q_1, k_1'\rangle \mid \langle Q_2, k_2'\rangle \mid \ldots \mid \langle P_n, k_n\rangle.$$

From above, it follows that $m' = m_{Q_1} + m_{Q_2} + \cdots m_{P_n}$, which is the first condition of $\mathcal{R}$.
In the interaction γ, we apply the update function $F = \{v_{c'}^2 := v_{c'}^2 \cup \{v'\} \mid \mathsf{protocol}(v') = c', v_{c'}^2 \in V_2\}$ for both $l = SR : v$ and $l = LC : v$, then $\mathbf{X_i'}(v_{c'}) = \mathbf{X_i}(v_c) \cup \{v\}$. Therefore we can write $\mathbf{X}' = \mathbf{X_1'} \sqcup \mathbf{X_2'} \cdots \mathbf{X_n}$.
With the interaction $s \xrightarrow[l]{[n]} s'$, we apply rules SendReceive or LeakCollect from Figure 2 where $k_i'(c') = k_i(c) \cup \{v\}$. Hence the condition 2 hols, i.e. $\mathbf{X_i'}(v_{c'}) = k_i'(c')$. With the execution of the γ interaction, the probabilistic

distribution $R_1^p = R_2^p = \emptyset$, and from the SendReceive or LeakCollect from Figure 2 is the same, then the condition 3 trivially hols. The two transitions have the same probability: $\mathbb{P}(q \xrightarrow{p} q') = 1/m$ by Lemma 2, and therefore $|\mathsf{Enabled}(m; \mathbf{X})| = m$.

– We consider the transition to be an internal transition τ in component $\mathcal{B}_{e_1}$. From lemma 1 we can write the transition:

$$q = (m_{P_1} + m_{P_2} + \cdots m_{P_n}, \mathbf{X_1} \sqcup \mathbf{X_2} \sqcup \cdots \sqcup \mathbf{X_n}) \xrightarrow{\gamma} q' = (m', \mathbf{X}')$$

where $s = \langle P_1, k_1\rangle \mid \langle P_2, k_2\rangle \mid \cdots \mid \langle P_n, k_n\rangle$, from $(q, s) \in \mathcal{R}$, we distinguish two cases of transition execution:

- A probabilistic choice: $m_{P_1} = \{l\} + m_{P_1'}$ where $\ell(l) = \sum_{i\in I}[n_i]a_i.T_i$ and $P_1 = \sum_{i\in I} a_i.T_i|P_1'$. From the transformation of Definition 6, the transition

$$t = (\{l_T\}, \langle \tau, \text{true}, f^\star\rangle, \{l_{T^\star}\}) \in T_1$$

can be executed where $f^\star = (\{v := v \mid v \in V^d\}$ and $R^p = \{v_T\})$. From relations of Figure 2, there exists a Choice transition in IoT system such that

$$s = \langle P_1, k_1\rangle \mid \langle P_2, k_2\rangle \mid \ldots \mid \langle P_n, k_n\rangle \xrightarrow[l]{[n_1]}$$
$$s' = \langle Q_1, k_1'\rangle \mid \langle P_2, k_2\rangle \mid \ldots \mid \langle P_n, k_n\rangle$$

where $Q_1 = a_1.T_1$. Now we can verify if the conditions 1-3 of R holds. We have that $m_{Q_1} = \{\ell\}^\star$ and $m' = m_{Q_1} + m_{P_2} + \cdots m_{P_n}$. As the update function f is the identity function the condition 2 trivially hold and the knowledge $k_1' = k_1$. To show condition 3 , we note that there exists $v_{T_1} \in V_1^p$, $v_{T_1} \sim \mu$ such that $\mathbf{X}'(v_{T_1}) = a_1.T_1$. We use Definition 6 from which we have that where $\mu(a_1.T_1) = n_1$.

- An internal transition: $m_{P_1} = m_{\tau.T_1} + m_{P_1'}$ and $P_1 = \tau.T_1|P_1'$. From the transformation of definition 6, the transition $\underline{T} = (\{l_{a.T}\}, \langle a, \text{true}, f\rangle, \{l_T\})$ can be executed where $f = (\{v := v \mid v \in V^d\}$ and $R^p = \emptyset)$. From relations of Figure 2, there exists an Internal transition in IoT system such that

$$s = \langle P_1, k_1\rangle \mid \langle P_2, k_2\rangle \mid \ldots \mid \langle P_n, k_n\rangle \xrightarrow[l]{[n]}$$
$$s' = \langle Q_1, k_1'\rangle \mid \langle P_2, k_2\rangle \mid \ldots \mid \langle P_n, k_n\rangle$$

where $Q_1 = T_1|P_1'$. Now we can verify if the conditions 1-3 of R holds. We have that $m_{Q_1} = m_{T_1} + m_{P_1'}$ and $m' = m_{Q_1} + m_{P_2} + \cdots m_{P_n}$.
As the update function f is the identity function the condition 2 trivially hold and the knowledge $k_1' = k_1$. Then $\mathbf{X}' = \mathbf{X_1'} \sqcup \mathbf{X_2} \sqcup \cdots \sqcup \mathbf{X_n}$. Likewise, since $R^p = \emptyset$ the condition 3 trivially holds.