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USING TORQUE-ANGLE AND TORQUE-VELOCITY MODELS TO CHARACTERIZE ELBOW MECHANICAL FUNCTION: MODELING AND APPLIED ASPECTS

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ABSTRACT

Characterization of muscle mechanism through the torque-angle and torque-velocity relationships is critical for human movement evaluation and simulation. In-vivo determination of these relationships through dynamometric measurements and modelling is based on physiological and mathematical aspects. However, no investigation regarding the effects of the mathematical model and the physiological parameters underneath these models was found. The purpose of the current study was to compare the capacity of various torque-angle and torque-velocity models to fit experimental dynamometric measurement of the elbow and provide meaningful mechanical and physiological information. Therefore, varying mathematical function and physiological muscle parameters from the literature were tested. While a quadratic torque-angle model seemed to increase predicted to measured elbow torque fitting, a new power-based torque-velocity parametric model gave meaningful physiological values to interpret with similar fitting results to a classical torque-velocity model. This model is of interest to extract modeling and clinical knowledge characterizing the mechanical behavior of such a joint.

1 **Keywords:** maximal joint torque - isokinetic dynamometer – torque-angle-velocity
2 relationship, maximal power velocity, muscle mechanics.

3

4 INTRODUCTION

5 Joint strength models are valuable representations of the torque generation
6 capacities of a human, useful in direct assessment as well as in musculoskeletal modeling
7 and analyses of human body. These models assume that muscles are viscoelastic
8 actuators [1-3], resulting at the joint level in Joint Torque-Angle and Torque-Velocity
9 Relationships (JTAR and JTVR respectively, and their coupling JTAVR). Fitting such models
10 to specific subjects while keeping their physiological meaning relevant remains an issue.
11 Basically, models are fitted to isokinetic measurements of joint torques in different angle
12 and angular velocity conditions [1–3].

13 At sarcomere scale, force-length relationship is asymmetrical piecewise linear due
14 to actin and myosin cross-bridge dynamics [4]. At muscle scale, the inter-fiber variability
15 blurs the transient states [5]. At joint scale, muscle-specific non-linear moment arms [6]
16 bring additional transformation into the torque-angle relationship. These observations
17 make it difficult to choose between various JTAR models, and no consensus exists in the
18 literature: normal curve [7,8], quadratic spline [9,10], cubic spline [11], cosinus wave [12],
19 or sinus exponential wave [13]. In a previous study, differences between those models in
20 fitting experimental isokinetic measures have been observed [14], particularly in the
21 eccentric portion of JTAR, evidencing the interaction between angle and velocity in such
22 models. Meanwhile, JTVR is mostly represented with hyperbolic functions [15,16],
23 although it might not cover all the joint velocity range [17].

24 JTAR and JTVR integrate parameters supposed to be physiologically meaningful.
25 At joint scale, parameters reflect partially the muscle physiology, even if the joint reflects

26 the interaction between multiple muscles. For JTAR, maximal isometric torque F_{max} ,
27 optimal isometric angle α_0 , and maximal range of motion RoM , are recurrent parameters.
28 For JTVR, Yeadon et al. 2006 [18] introduced maximal eccentric torque, maximal
29 concentric velocity and technical parameters. Anderson, et al. 2007 [12] added eccentric
30 to concentric force ratio and velocities at 75% and 50% of maximal isometric force within
31 the physiological range. In this last model, the two concentric parameters are dependent;
32 and the model lacks derivative continuity between concentric and eccentric portions that
33 can lead to unrealistic JTAVR fitting to data, particularly exhibiting continuity jumps. As
34 proposed in [19], maximal range of velocity can be useful for JTVR extrapolation in high
35 velocity regions.

36 Besides models fitting issues, physiological significance is useful for interpretation.
37 For example, maximal strength and muscle compositions are useful in ergonomics
38 [20,21]. In sports, specific velocity at maximal power can be a training focus [22]. In clinics,
39 eccentric to concentric strength ratio is an indicator for pathology [23]. Last,
40 musculoskeletal analysis needs such parameters to calibrate models to subjects [24,25].
41 A direct in-vivo estimation of these parameters remains an issue, since it requires
42 cadaveric, invasive or expensive measurements [24–27]. Joint strength models are
43 therefore useful to get these values indirectly [12,24,28].

44 The purpose of the current study is to investigate the effects of mathematical
45 models and muscle parameters on JTAR and JTVR from modeling and applied points of
46 views. We assumed that:

- 47 • **[H1]** an asymmetrical JTAR can reduce torque prediction errors;

- 48 • **[H2]** a new power-based JTVR can improve the physiological relevance of such
49 models.

50 **METHODS**

51 **Ethics and Participants**

52 Under INRIA national ethics committee agreement (COERLE #2017-002), twenty-two
53 male participants (33±6 years; 1.81±0.07 m; 78±9 kg) gave their informed consent to
54 participate in the study.

55 **Isokinetic measurement**

56 Participants seated upright with the arm alongside on a Con-Trex MJ® isokinetic
57 dynamometer (CMV AG, Dübendorf, Switzerland) according the manual guidelines
58 (fig. 1). The dynamometer axis was aligned with the elbow flexion axis at 90° for maximum
59 precision. Maximal distance between handle and arm brace without hampering elbow
60 flexion was chosen to minimize elbow displacement away from the dynamometer axis.
61 Similarly, tight straps were used to immobilize thorax. Range of motion was adjusted to
62 the subject.

63 Goniometric measurement was used to calibrate angular values. Three passive flexion-
64 extension trials at 60°.s⁻¹, 120°.s⁻¹, or 180°.s⁻¹ were recorded for gravity and passive
65 components compensation.

66 After a ten minutes sub-maximal warm-up, five voluntary flexion or extension hold for
67 five seconds at angles evenly spaced throughout the range of motion were recorded as
68 isometric trials. Then, three repetitions of concentric-passive cycles at 60°.s⁻¹, 120°.s⁻¹, or
69 180°.s⁻¹ or eccentric-passive trials at 60°.s⁻¹, 120°.s⁻¹, or 180°.s⁻¹ in flexion and extension

70 were recorded as isokinetic trials. Within subject ranges of motion, reaching isokinetic
71 state limited the maximal velocity to $180^{\circ} \cdot s^{-1}$. During $180^{\circ} \cdot s^{-1}$ trials, subjects were asked
72 to anticipate their effort at the end of the previous passive cycle to decrease delay and
73 ensure a sufficient maximal contraction time during trials. To decrease fatigue effects,
74 trials order was randomized within subjects, flexion and extension trials were alternated,
75 and a 45 seconds rest was respected between each trial. All measurements were collected
76 by the same experimenter. Angle, angular velocity and torque were recorded at 256 Hz.
77 Only data corresponding to isokinetic states were used for analysis, discarding the first
78 milliseconds of trials (215 ms to 355 ms) during which muscle activation is not maximal
79 [18]. For each condition, the repetition with the largest average torque was selected.

80

81 INSERT FIGURE 1 HERE

82

83 **JTAR models**

84 Five JTAR mostly encountered in the literature were implemented as Normal [3,10],
85 Quadratic [11,12], Cosinus [13], Cubic [14] and Sinus-exponential [15] models.
86 Parameters of all these models are the maximal isometric torque Γ_{max} , the optimal joint
87 angle α_0 and the maximal range of isometric force production RoM . These models are
88 extensively described in the supplementary material and presented in figure 2.

89

90 INSERT FIGURE 2 HERE

91

92 **JTVR models**

93 Two JTVR models were compared. The Anderson-based model is an adapted version of
94 [12]. The power-based model is a new polynomial function of the maximal power velocity
95 and other physiological parameters selected from the literature.

96 The Anderson-based model introduces 3 parameters: $\omega_{\Gamma_{.75}}$ - Velocity at 75% of maximal
97 isometric torque, E - Eccentric to concentric torque index and an additional $\omega_{\Gamma_{.5}}/\omega_{\Gamma_{.75}}$
98 ratio as an optimization constraint rather than an arbitrary value as in the original version.

99 The power-based model depends on the concentric velocity at maximal power, $\omega_{P_{max}}$,
100 because of its unique correlation with muscle composition [29,30]. $\omega_{P_{max}}$ is also used as
101 an inflexion constraint for the concentric part of the JTAR. The model also depends on
102 $\omega_{min}/\omega_{max}$ - Max. eccentric to concentric velocity ratio, $\Gamma_{ECC}/\Gamma_{CON}$ - Max. eccentric to
103 concentric torque ratio and ω_{max} - Maximum concentric velocity at which muscle sustains
104 no more tension [31].

105 Both models are extensively described in the supplementary material and presented in
106 figure 3.

107

108 INSERT FIGURE 3 HERE

109

110 **Fitting models to data**

111 A least-square-curve-fitting method (trust-region algorithm, Matlab® Optimization
112 Toolbox™) minimized the quadratic distance between modelled and measured torques
113 by optimizing models parameters.

114 Isometric, concentric and eccentric parameters were optimized in successive steps as
115 recommended by [12]. The five JTAR models were tested in the isometric step and were
116 combined with both JTVR models in the 2nd and 3rd steps.

117 Since range of motion and acquisition rate were constant for all trial, duration and frame
118 number varied for each velocity. To guarantee equal weight of all velocities on fitting,
119 samples of equal number of frames were selected within the algorithm.

120 **Models comparison**

121 First, models were compared in terms of ability to fit data. Adjusted correlation and linear
122 regression coefficients were compared between all combinations of JTAR and JTVR
123 models. Additionally, a one-way repeated measures Anova was performed to test the
124 effects of JTAR models on isometric torque prediction errors; and a two-way repeated
125 measures Anova was performed to test the effects of JTAR and JTVR models on isokinetic
126 (concentric + eccentric) torque prediction errors. Mauchly normality and sphericity test
127 was performed. Then, the Distribution Cumulative Differences (Matlab[®] Statistics &
128 Machine Learning Toolbox[™]) was performed for the Anovas. Results are presented with
129 significance level set to $p \leq .05$ and significance power F.

130 Second, the optimized parameters of all models were compared and confronted to
131 literature values.

132 **RESULTS**

133 The significant effects of JTAR and JTVR were not different between flexion or extension
134 motions. Results for both motions are presented together in this section.

135 Torque predicted by cosinus, quadratic, and cubic models displayed larger correlation
136 with experimental data than normal and sinus-exponential models (table 1). Highest
137 correlations were obtained for isometric data. Correlation for concentric data was higher
138 with the power-based model than with the Anderson-based model but lower for eccentric
139 data.

140

141 INSERT TABLE 1 HERE

142

143 Sphericity was verified for all data ($p < 0.01$). ANOVA revealed that JTAR had significant
144 effects on isometric prediction errors ($p < 0.01$, $F = 3.86$). Post-hoc tests attributed the
145 smallest errors to the quadratic model ($p < 0.001$, Fig 4a). Error increased significantly of
146 0.19, 0.53, 2.67, and 4.25 N.m between cosinus, cubic, sinus-exponential and normal
147 models respectively.

148 The ANOVA showed an ordinal interaction between these models on average error ($p <$
149 0.01 , $F = 18.36$), plus significant effects of JTAR ($p < 0.01$, $F = 13.15$) and JTVR ($p <$
150 0.05 , $F = 4.66$). Normal and sinus-exponential models still displayed the largest errors -
151 10% larger than other models - in combination with both JTVR models (Fig. 4b). The
152 power-based model displayed larger overall error than Anderson's model only when
153 combined with sinus-exponential and normal torque-angle models.

154

155 INSERT FIGURE 4 HERE

156

157 Average isometric parameters obtained with each model are presented in table 2. In
158 flexion, Γ_{max} varied between 63 N.m and 69 N.m, RoM varied between 155° and 175°, and
159 α_0 varied between 59° and 102°. In extension, Γ_{max} between 60 N.m and 66 N.m, RoM
160 between 164° and 179°, and α_0 between 56° and 99° were obtained in extension.
161 Significant effects of the model were found for Γ_{max} and α_0 only in extension ($p < 0.05$,
162 $F = 2.93$, and $p < 0.01$, $F = 3.90$ respectively). Average Γ_{max} obtained with cubic and
163 sinus-exponential models, and average α_0 obtained with the normal model, differed by
164 more than 10% from values found in the literature.

165

166 INSERT TABLE 2 HERE

167

168 Optimal ω_{max} , ω_{min} , and $\omega_{P_{max}}$ obtained from the new model combined to each isometric
169 model are presented in table 3. No statistical effect of the model was found for these
170 parameters.

171

172 INSERT TABLE 3 HERE

173

174 Optimal $\Gamma_{ECC}/\Gamma_{CON}$ ratios obtained with both JTAR are presented in table 4. The ANOVA
175 showed an effect of the JTAR, but no effect of the JTAR and an interaction between JTAR
176 and JTAR. Differences identified through the post-hoc tests were about 1% of the average
177 ratio values.

178

179 INSERT TABLE 4 HERE

180

181 **Discussion**

182 The purpose of the current study was to investigate the capacity of JTAR and JTVR models
183 to represent the elbow mechanical function from modeling and applied points of views.

184 We first compared their ability to fit the experimental data and then interpreted their
185 physiological parameters in comparison to the literature.

186 Differences in fitting experimental data were observed. Both choices of JTAR and JTVR
187 showed statistical effects and interaction on maximal torque prediction. For JTAR, **[H1]**

188 was not verified, since the asymmetrical models displayed significantly larger errors than
189 two of the symmetrical models (quadratic and cosinus). This may be explained by

190 interactions between maximal forces and moment arms of multiple muscles crossing the
191 elbow [6]. For the elbow joint, the quadratic JTAR appears as the most accurate and

192 adaptable model on a large cohort. Those results might be joint-specific. As reported in
193 our preliminary study [14], average prediction error is increased by the JTVR. Associated

194 with one of the two best JTAR (i.e. quadratic, cosinus, cubic), the power-based and
195 Anderson-based models gave similar prediction levels. Although the new physiological

196 parameters may improve the model meaningfulness, it did not improve its compliance to
197 fit measured data. Especially, the additional constraint added to ensure derivative

198 continuity between concentric and eccentric parts of the model seemed to decrease
199 eccentric fitting efficiency of the model. However, it ensured that the torque envelope

200 generated by the fitting method was continuous, that may not be the case with the

201 Anderson-based one. Globally, correlations between measured and predicted JTAVR
202 were weaker in the current study than in other studies in the literature that focused on
203 lower limbs [14,21]. This result may be due to the larger anatomical variability of the
204 upper limb. Moreover, larger misalignment between elbow and dynamometer axes may
205 arise during motion because of the equipment, since the arm position cannot be as
206 controlled as the thigh on the dynamometer. Quantification and correction of this
207 problem could improve the fitting quality and the subsequent parameter estimation.
208 Specific joint strengths models dedicated to fitting may also have been of interest to be
209 tested here [5,36]

210

211 For applied perspectives, physiological parameters obtained through optimization of the
212 models seem coherent with the literature. For a group of young healthy men, maximal
213 isometric torque Γ_{max} between 63 N.m and 69 N.m in flexion [2,32], and balanced flexion-
214 extension ratios between .95 and .97 are similar to literature [2,33]. Range of motion *RoM*
215 obtained with all models is larger than the anatomical reference [34], probably due to an
216 extrapolation of muscle strength beyond realistic elbow configuration due to bony limits.
217 Average optimal angle, α_0 , obtained for elbow flexion and extension with all models are
218 consistent with the literature [35–37]. For both flexion and extension, α_0 found for normal
219 ($79^\circ, 76^\circ$), cosinus ($77^\circ, 72^\circ$), and quadratic ($77^\circ, 72^\circ$) models was close to classically
220 observed average angles [38,39], while cubic ($59^\circ, 56^\circ$) and sinus-exponential ($112^\circ, 99^\circ$)
221 models values were at boundaries. Only Γ_{max} obtained with cubic and sinus-exponential

222 models, and α_0 obtained with the normal model differed by more than 10% from
223 literature values [32,38].

224 Concerning concentric and eccentric parameters, concentric velocity at maximal power,
225 $\omega_{P_{max}}$, was our focus. Due to its relationship with muscle composition [30], linking
226 mechanical and physiological muscle functions, the implementation of this parameter in
227 the model seemed interesting for applications in sports, rehabilitation or ergonomics [40].
228 Optimized $\omega_{P_{max}}$ values, between $404^\circ \cdot s^{-1}$, and $561^\circ \cdot s^{-1}$, are about two times larger than
229 $\omega_{P_{max}}$ values only based on isokinetic dynamometer for thigh muscles [29]. Previous study
230 showed that measured $\omega_{P_{max}}$ correlated better ($r=.55$) with fiber composition when
231 corrected with a Hill-type model as in our study [41] and could have values between
232 $215^\circ \cdot s^{-1}$ and $539^\circ \cdot s^{-1}$. Although correlations are seen, only 51.8% of the fiber composition
233 variance was explained by $\omega_{P_{max}}$ [42]. For further work, a combination of $\omega_{P_{max}}$
234 measurements with Hill-model correction and electromyography could be investigated
235 [43] and a validation of muscle composition prediction through this technique should be
236 performed.

237 For the other concentric and eccentric parameters, maximal concentric velocity, ω_{max} ,
238 between $1268^\circ \cdot s^{-1}$ and $1531^\circ \cdot s^{-1}$ in flexion and, $1368^\circ \cdot s^{-1}$ and $1667^\circ \cdot s^{-1}$ in extension were
239 found. That lays below values reported for baseball players, middle-aged, and elderly men
240 respectively [43,44] . The normal and sinus-exponential models produced the smallest
241 values. For maximal eccentric velocity, ω_{min} , no conclusive reference data were found. In
242 general, physiological parameters related to velocity were obtained by extrapolation of
243 our model beyond highest velocity measured in this study. Since no direct measurement

244 for much higher velocities was possible with such dynamometers, these values cannot be
245 directly validated.

246 Since the degree of meaningfulness of the power-based JTVR was higher, without
247 decreasing significantly the data fitting, [H2] seemed verified.

248
249 To conclude, five JTAR and two JTVR were compared when fitting experimental
250 dynamometric measurements from modelling and applied perspectives. While a
251 quadratic torque-angle model fitted best the data, a new proposed JTVR increased
252 physiological transparency and clinical relevance without decreasing significantly the data
253 fitting. The study highlights the needs for improvement of dynamometric measurement
254 accuracy for the upper limb and the importance of the meaningfulness of the
255 physiological parameters to be optimized when fitting these models to data.

256

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258

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260 project.

261

262 **CONFLICT OF INTEREST**

263

264 The authors encountered no conflict of interest for the current study.

265

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389**Figure Captions List**

- Fig. 1 Experimental set up. The participant is seated and attached to the ConTrex dynamometer in upright position with the arm along his side. The axis of the dynamometer is aligned with the epicondylitis axis with the elbow flexed at 90°.
- Fig. 2 Normalized torque-angle relationship as defined by the five mathematical models: normal, cosinus, and quadratic models are symmetrical; cubic and sinus-exponential are asymmetrical.
- Fig. 3 Normalized torque-velocity models and constraint parameters: in the Anderson-based model (A), we find one derivative constraint at ω_{max} , two independent constraints at $-\omega_{max}$ and ω_0 , and two dependant constraints at $\omega_{\Gamma_{.5}}$ and $\omega_{\Gamma_{.75}}$; in our power-based model (B), we defined three derivative constraints and three independent constraints at ω_{max} , ω_{max} , and ω_0 , and an additional derivative constraints at $\omega_{P_{max}}$ on the power-velocity relationship.
- Fig. 4 Effects of torque-angle models (A) and interaction between the torque-angle and torque-velocity models (B) on average prediction errors. Colored stars represent individuals (one color = one subject). Black dots, blue dots and red diamonds represent the average of all individuals. Asterisks indicate significant difference between means.

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Table Caption List

- Table 1 Adjusted correlation coefficients between measured and predicted maximal torque for all models in elbow flexion and extension for each type of data: isometric, concentric and eccentric.
- Table 2 Optimal elbow torque-angle parameters obtained with the five isometric models.
- Table 3 Optimal elbow torque-velocity parameters of the new model obtained with the five isometric models.
- Table 4 Optimal elbow eccentric to concentric ratios obtained with Anderson-based model and the new model combined with each of the five isometric models.

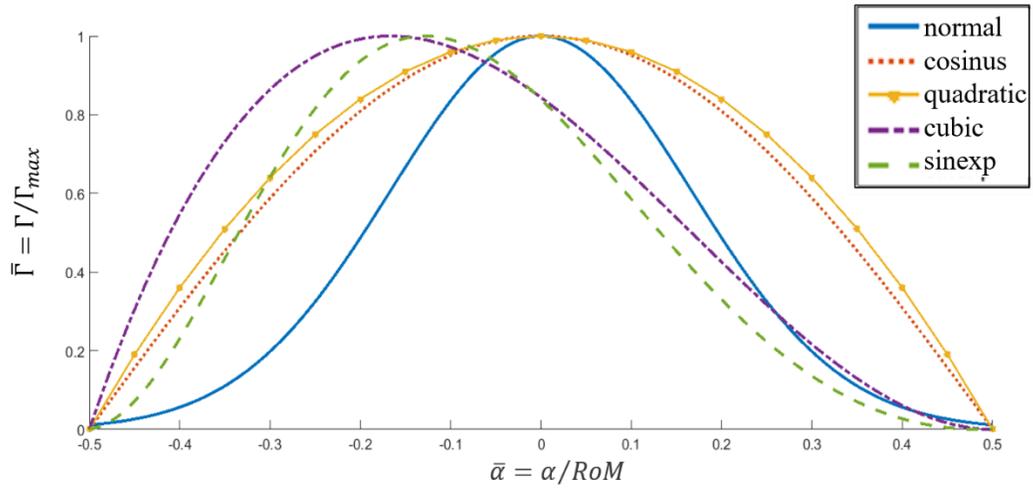
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395 Figure 1– Experimental set up. The participant is seated and attached to the ConTrex
396 dynamometer in upright position with the arm along his side. The axis of the
397 dynamometer is aligned with the epicondylitis axis with the elbow flexed at 90°.



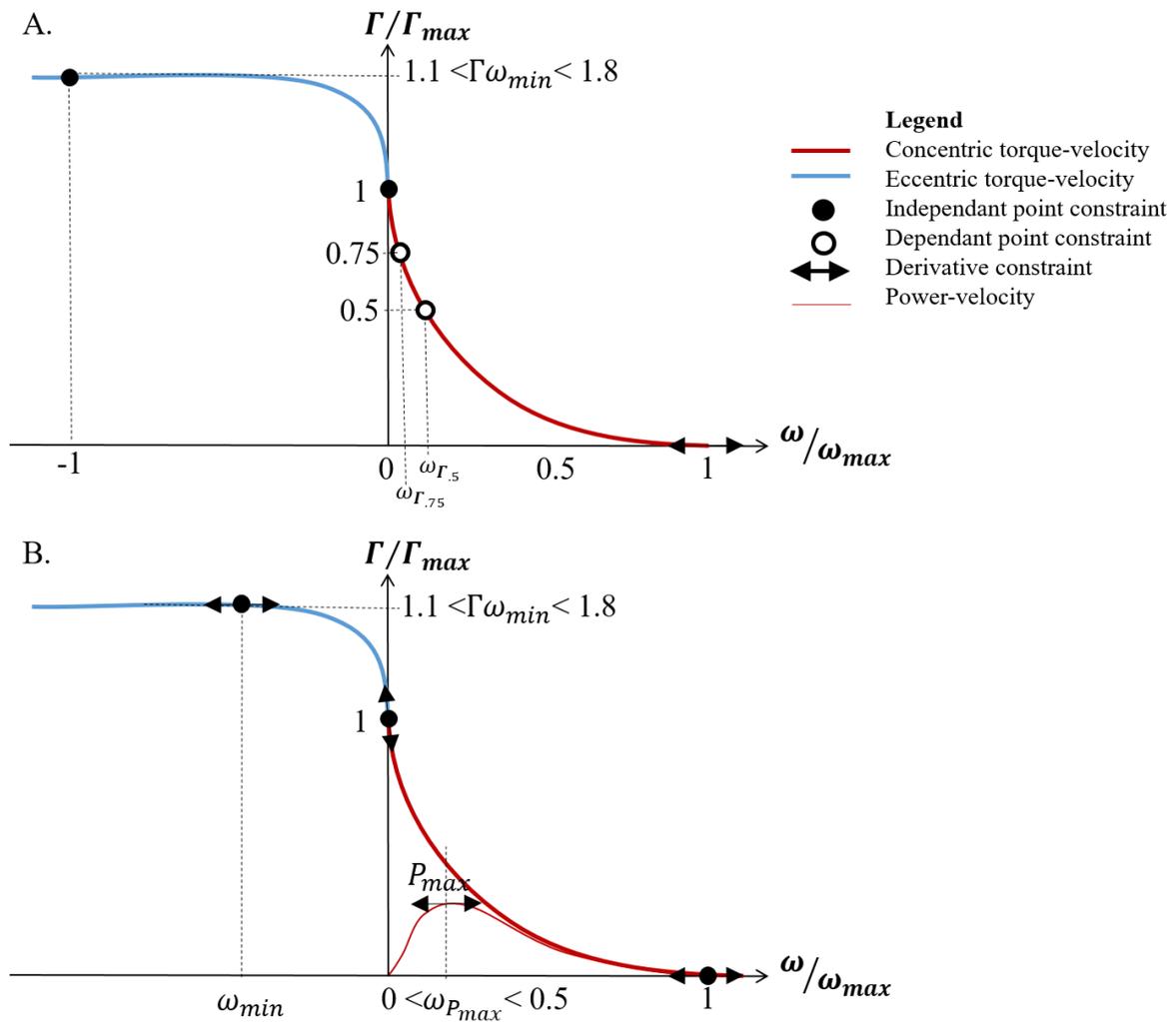
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400 Figure 2 – Normalized torque-angle relationship as defined by the five mathematical
401 models: normal, cosinus, and quadratic models are symmetrical; cubic and sinus-
402 exponential are asymmetrical.



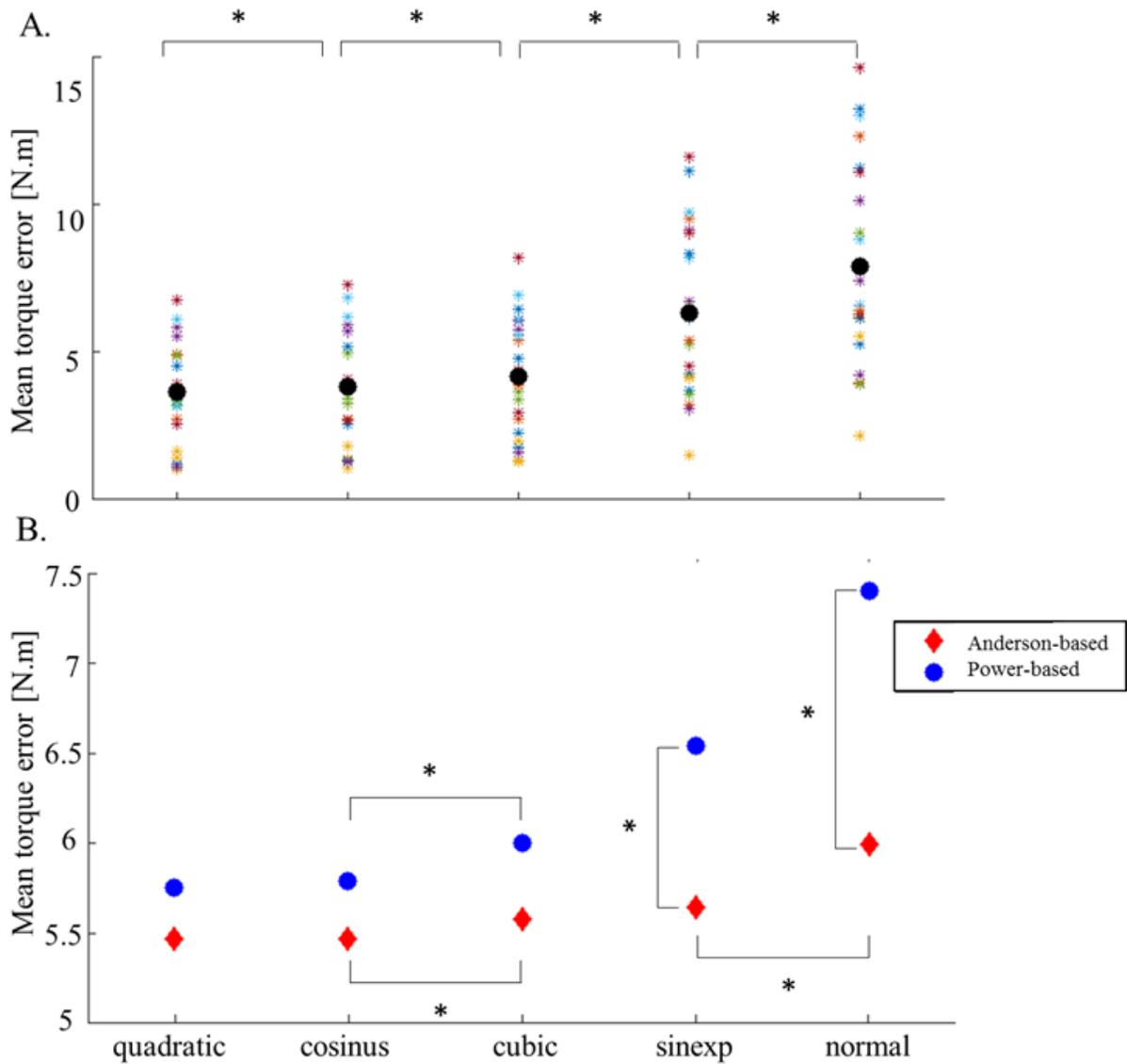
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405 Figure 3 – Normalized torque-velocity models and constraint parameters: in the
 406 Anderson-based model (A), we find one derivative constraint at ω_{max} , two independent
 407 constraints at $-\omega_{max}$ and ω_0 , and two dependant constraints at $\omega_{\Gamma.5}$ and $\omega_{\Gamma.75}$; in our
 408 power-based model (B), we defined three derivative constraints and three independent
 409 constraints at ω_{max} , ω_{max} , and ω_0 , and an additional derivative constraints at $\omega_{P_{max}}$ on
 410 the power-velocity relationship.



411

412 Figure 4 – Effects of torque-angle models (A) and interaction between the torque-angle
 413 and torque-velocity models (B) on mean errors computed as the average difference
 414 between maximal torque measured on the dynamometer and maximal torque predicted
 415 by the models. Colored stars represent individuals (one color = one subject). Black dots,
 416 blue dots and red diamonds represent the average of all individuals. Asterisks indicate
 417 significant difference between means.
 418



419
 420

421 Table 1. Adjusted correlation coefficients between measured and predicted maximal
 422 torque for all models in elbow flexion and extension for each type of data: isometric,
 423 concentric and eccentric.

R ²			Normal	Cosinus	Quadratic	Cubic	Sinus-exponential
Anderson-based model	<i>flexion</i>	Isometric	.62	.86	.87	.84	.74
		Concentric	.58	.62	.62	.61	.61
		Eccentric	.75	.75	.75	.74	.77
	<i>extension</i>	Isometric	.66	.88	.89	.87	.75
		Concentric	.74	.75	.75	.76	.74
		Eccentric	.71	.77	.77	.76	.74
Power-based model	<i>flexion</i>	Isometric	.62	.86	.87	.84	.74
		Concentric	.64	.70	.70	.68	.68
		Eccentric	.60	.64	.64	.62	.64
	<i>extension</i>	Isometric	.66	.88	.89	.87	.75
		Concentric	.81	.85	.85	.85	.82
		Eccentric	.64	.70	.70	.70	.67

424

425 Table 2. Optimal elbow torque-angle parameters obtained with the five isometric models.

426 Values displayed in the table correspond to: means \pm standard deviations.

		Normal	Cosinus	Quadratic	Cubic	Sinus-exp
FLEXION	Γ_{max} [N.m]	69 \pm 13	63 \pm 11	63 \pm 11	64 \pm 11	66 \pm 12
	RoM [°]	175 \pm 17	160 \pm 33	155 \pm 34	167 \pm 28	173 \pm 24
	α_0 [°]	79 \pm 10	77 \pm 13	77 \pm 14	102 \pm 12	59 \pm 10
EXTENSION	Γ_{max} [N.m]	66 \pm 18	60 \pm 16	60 \pm 16	61 \pm 17	64 \pm 18
	RoM [°]	179 \pm 4	167 \pm 27	164 \pm 30	169 \pm 23	173 \pm 20
	α_0 [°]	76 \pm 9	72 \pm 11	72 \pm 11	99 \pm 10	56 \pm 9

427

428

429 Table 3. Optimal elbow torque-velocity parameters of the new model obtained with the
 430 five isometric models. Values displayed in the table correspond to: means \pm standard
 431 deviations.

Torque-angle model		Normal	Cosinus	Quadratic	Cubic	Sinus-exp
FLEXION	ω_{max} [$^{\circ} \cdot s^{-1}$]	1268 \pm 514	1517 \pm 544	1531 \pm 548	1492 \pm 567	1331 \pm 507
	ω_{min} [$^{\circ} \cdot s^{-1}$]	-173 \pm 87	-301 \pm 276	-369 \pm 500	-330 \pm 477	-238 \pm 241
	$\omega_{P_{max}}$ [$^{\circ} \cdot s^{-1}$]	404 \pm 231	495 \pm 242	502 \pm 500	490 \pm 255	426 \pm 225
EXTENSION	ω_{max} [$^{\circ} \cdot s^{-1}$]	1368 \pm 647	1636 \pm 747	1667 \pm 754	1605 \pm 738	1458 \pm 683
	ω_{min} [$^{\circ} \cdot s^{-1}$]	-297 \pm 329	-509 \pm 464	-531 \pm 482	-536 \pm 530	-391 \pm 410
	$\omega_{P_{max}}$ [$^{\circ} \cdot s^{-1}$]	432 \pm 291	551 \pm 343	563 \pm 347	537 \pm 337	473 \pm 305

432

433

434 Table 4. Optimal elbow eccentric to concentric ratios obtained with Anderson-based
 435 model and the new model combined with each of the five isometric models. Values
 436 displayed in the table correspond to: means \pm standard deviations.

			Normal	Cosinus	Quadratic	Cubic	Sinus-exp
FLEXION	$\Gamma_{ECC}/\Gamma_{CON}$	Anderson-based	1.17 \pm 0.07	1.18 \pm 0.08	1.18 \pm 0.08	1.18 \pm 0.08	1.18 \pm 0.08
		ω_{max} -based	1.16 \pm 0.15	1.20 \pm 0.21	1.20 \pm 0.22	1.20 \pm 0.21	1.17 \pm 0.16
EXTENSION	$\Gamma_{ECC}/\Gamma_{CON}$	Anderson-based	1.21 \pm 0.10	1.22 \pm 0.10	1.22 \pm 0.10	1.22 \pm 0.10	1.21 \pm 0.10
		ω_{max} -based	1.17 \pm 0.15	1.34 \pm 0.30	1.32 \pm 0.30	1.31 \pm 0.29	1.21 \pm 0.20

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439 **SUPPLEMENTARY MATERIAL**
 440 **USING TORQUE-ANGLE AND TORQUE-VELOCITY MODELS TO**
 441 **CHARACTERIZE ELBOW MECHANICAL FUNCTION: MODELING**
 442 **AND APPLIED ASPECTS**

443 Diane Haering, Charles Pontonnier, Nicolas Bideau, Guillaume Nicolas, Georges
 444 Dumont

445 submitted to Journal of Biomechanical Engineering.

446
 447 For the purpose of concision, the mathematical models used in the paper are extensively
 448 presented here.

449

450 **1. Joint Torque Angle Relationships (JTAR)**

451

Normal [8] $\Gamma(\bar{\alpha}) = e^{-\frac{1}{2}(6\bar{\alpha})^2}$ (1)

Cosinus [12] $\Gamma(\bar{\alpha}) = \cos(\pi\bar{\alpha})$ (2)

Quadratic [9,15] $\Gamma(\bar{\alpha}) = -4\bar{\alpha}^2 + 1$ (3)

Cubic [11] $\Gamma(\bar{\alpha}) = \frac{27}{4}\bar{\alpha}^3 - \frac{27}{8}\bar{\alpha}^2 - \frac{27}{16}\bar{\alpha} + \frac{27}{32}$ (4)

Sinus-exponential [13] $\Gamma(\bar{\alpha}) = \frac{1}{2}\sin(1.919\pi e^{\bar{\alpha}}) + \frac{1}{2}$ (5)

452 In all of these models, the normalized maximal torque ($\bar{\Gamma} = \frac{\Gamma}{\Gamma_{max}}$) depended on the maximal
 453 isometric torque and on the joint angle to optimal joint angle distance normalized by the maximal
 454 range of isometric force production ($\bar{\alpha} = \frac{\alpha - \alpha_0}{ROM}$) as presented in table 1. Those models are shown
 455 in figure 1. All coefficients used for the normal, quadratic, cubic and sinus-exponential models
 456 were obtained by solving the system of equations expressing the following constraints:

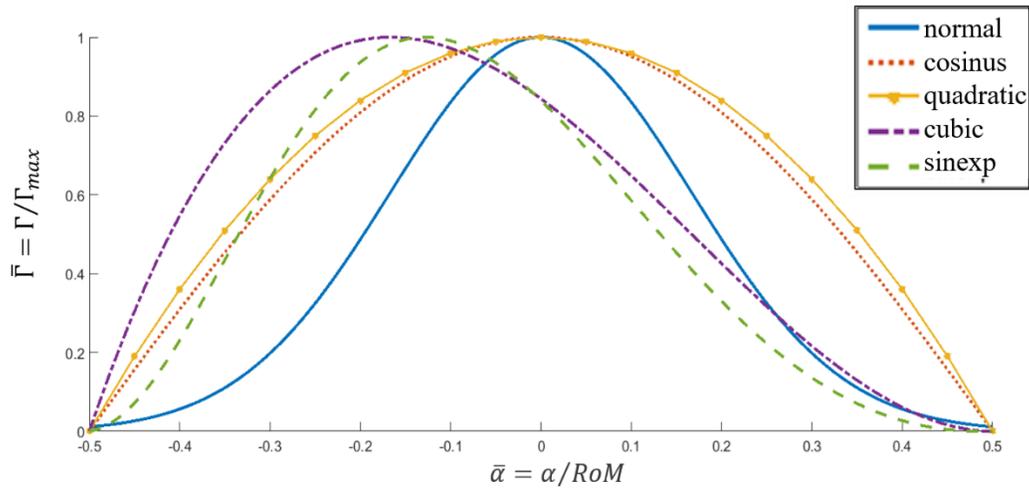
457 $\bar{\Gamma}(\bar{\alpha}_0) = 1,$

458 $\bar{\Gamma}(\bar{\alpha}_{min}) = 0,$

459 $\bar{\Gamma}(\bar{\alpha}_{max}) = 0,$ and

460 $\bar{\alpha}_{max} - \bar{\alpha}_{min} = \mathbf{1};$ $\bar{\alpha}_{max}$ and $\bar{\alpha}_{min}$ being normalized values of maximal and minimal angles.

461



462

463 Figure 1 – Normalized torque-angle relationship as defined by the five mathematical
 464 models: normal, cosinus, and quadratic models are symmetrical; cubic and sinus-
 465 exponential are asymmetrical.

466

467 Table 1: JTAR parameters

Parameters		Limits	
Γ_{max}	Max. isometric torque	$0.75 \Gamma_{meas}$	$1.25 \Gamma_{meas}$
RoM	Max. isometric range of motion	0	π
α_0	Isometric optimal angle	$\pi/6$	$5\pi/6$

480

481

2. Joint Torque Velocity Relationships (JTVR)

2.1. JTVR parameters

This section presents the parameters exploited in the models presented below.

Table 2. JTVR parameters

Model	Parameters			Limits	
Anderson-based torque-velocity model	P_1	$\omega_{\Gamma_{.75}}$	Velocity at 75% of maximal isometric torque	0	π
	P_2	$\omega_{\Gamma_{.5}}/\omega_{\Gamma_{.75}}$	Ratio between velocities at 50% and 75% of maximal isometric torque	1.9	2.1
	P_3	E	Eccentric to concentric torque index	.1	.8
Power-based torque-velocity model	P_1	ω_{max}	Max. concentric velocity	$\pi/3$	5π
	P_2	$\omega_{P_{max}}$	Velocity at maximal power	0.25	0.4
	P_3	$\omega_{min}/\omega_{max}$	Max. eccentric to concentric velocity ratio	-1	-0.1
	P_4	$\Gamma_{ECC}/\Gamma_{CON}$	Max. eccentric to concentric torque ratio	1.1	1.8

2.2. Anderson-based model

$$\begin{cases} \Gamma(\omega) = \frac{2P_1P_2 + \omega(P_2 - 3P_1)}{2P_1P_2 + \omega(2P_2 - 4P_1)}, & \omega \geq 0 \\ \Gamma(\omega) = \left(\frac{2P_1P_2 + \omega(P_2 - 3P_1)}{2P_1P_2 + \omega(2P_2 - 4P_1)} \right) (1 - P_3\omega), & \omega < 0 \end{cases} \quad (1)$$

where P_1 is the velocity at 75% of maximal isometric torque, P_2 is the ratio between velocities at 50% and 75% of maximal isometric torque, and P_3 is a maximal eccentric to maximal concentric torque index.

496 **2.3.Power-based torque-velocity model**

$$497 \quad \begin{cases} \Gamma(\bar{\omega}) = 0, & \mathbf{1} \leq \bar{\omega} \\ \Gamma(\bar{\omega}) = a_1\bar{\omega}^3 + b_1\bar{\omega}^2 + c_1\bar{\omega} + d_1, & \mathbf{0} \leq \bar{\omega} < \mathbf{1} \\ \Gamma(\bar{\omega}) = a_2\bar{\omega}^3 + b_2\bar{\omega}^2 + c_2\bar{\omega} + d_2, & \mathbf{P}_3 \leq \bar{\omega} < \mathbf{0} \\ \Gamma(\bar{\omega}) = P_4, & \bar{\omega} < \mathbf{P}_3 \end{cases} \quad (2)$$

498 with:

499
$$\bar{\omega} = \frac{\omega}{P_1},$$

500
$$a_1 = -\frac{3P_2^2 - 4P_2 + 1}{4P_2^3 - 6P_2^2 + 2P_2},$$

501
$$b_1 = \frac{2P_2^3 - 3P_2 + 1}{2P_2^3 - 3P_2^2 + P_2},$$

502
$$c_1 = \frac{8P_2^3 - 9P_2^2 + 1}{4P_2^3 - 6P_2^2 + 2P_2},$$

503
$$d_1 = 1,$$

504 and

505 a_2

506
$$= -\frac{P_3 - 4P_2 - 9P_2^2P_3 - 12P_2^2P_4 + 8P_2^3P_3 + 8P_2^3P_4 + 12P_2^2 - 8P_2^3 + 4P_2P_4}{2P_3^3(2P_2^3 - 3P_2^2 + P_2)},$$

507
$$b_2 = \frac{2P_2^3 - 3P_2 + 1}{2P_2^3 - 3P_2^2 + P_2},$$

508
$$c_2 = \frac{8P_2^3 - 9P_2^2 + 1}{4P_2^3 - 6P_2^2 + 2P_2},$$

509
$$d_2 = 1,$$

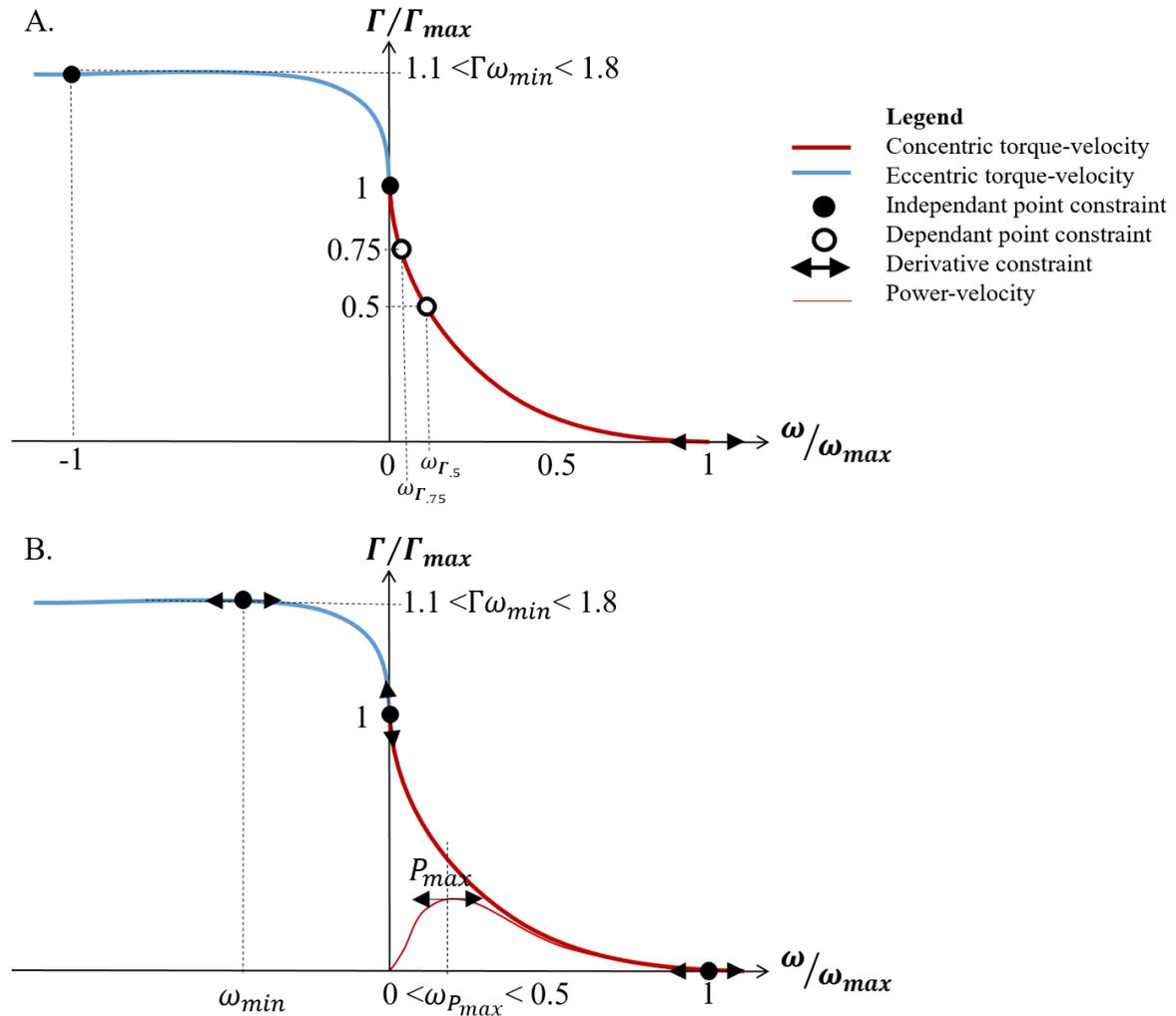
510 where P_1 is the maximal concentric velocity, P_2 is the velocity at maximal power, P_3
 511 is maximal eccentric velocity to maximal concentric velocity ratio, and P_4 is the
 512 maximal eccentric to maximal concentric torque ratio.

513

514

515
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517

2.4. Model representations



518

519 Figure 2 – Normalized torque-velocity models and constraint parameters: in the Anderson-based
520 model (A), we find one derivative constraint at ω_{max} , two independent constraints at $-\omega_{max}$ and
521 ω_0 , and two dependant constraints at $\omega_{\Gamma_{.5}}$ and $\omega_{\Gamma_{.75}}$; in the power-based model (B), we defined
522 three derivative constraints and three independent constraints at ω_{max} , ω_{max} , and ω_0 , and an
523 additional derivative constraints at $\omega_{P_{max}}$ on the power-velocity relationship.

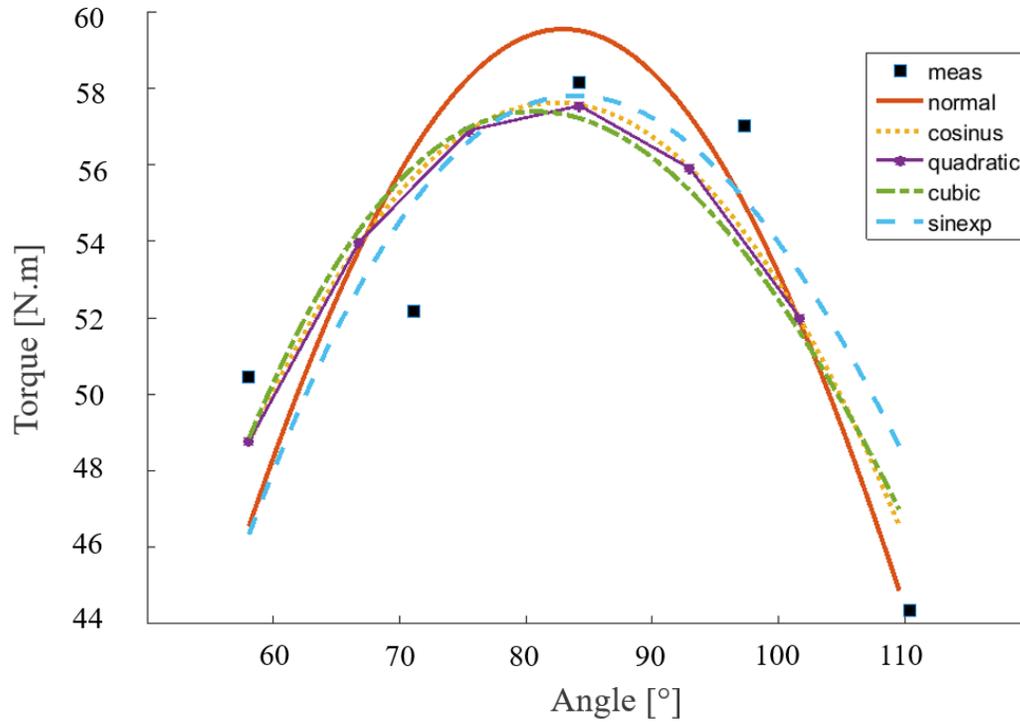
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525 **3. Visualization of model to measurement fitting**

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527

3.1. Joint Torque Angle Relationships



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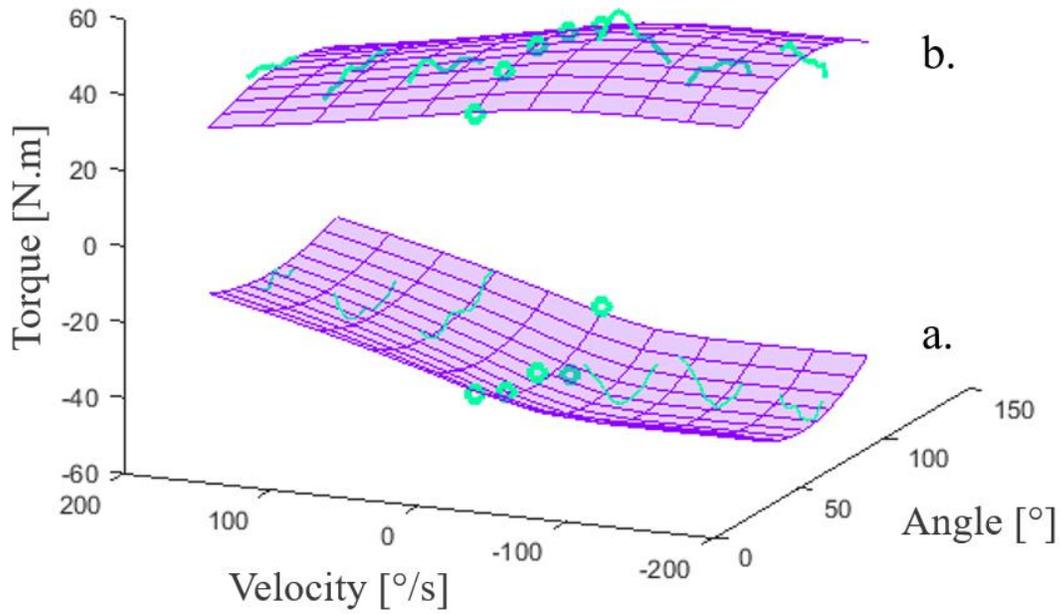
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Figure 3 – Example of comparison between normal, cosines, quadratic, cubic and sinus exponential JTAR models fitting on isometric maximum torque measurements (black squares).

532
533

3.2. Joint Torque Angle & Joint Torque Velocity Relationships



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535
536
537

Figure 4 – Example of a combined JTAR and JTVR model mesh fitting on isometric maximum (dots) and isokinetic (lines) torque measurements for a. elbow flexion, and b. elbow extension.