Load vector based damage localization with rejection of the temperature effect

Md Delwar Hossain Bhuyan, Nicolas Le Touz, Guillaume Gautier, Michael Dohler, Falk Hille, Jean Dumoulin, Laurent Mevel

IOMAC'19

8th International Operational Modal Analysis Conference

2019 May12-14 Copenhagen

LOAD VECTOR BASED DAMAGE LOCALIZATION WITH REJECTION OF THE TEMPERATURE EFFECT

Md Delwar Hossain Bhuyan[1], *Nicolas Le Touz*[2,3], *Guillaume Gautier*[3,2], *Michael Döhler*[3,2], *Falk Hille*[1], *Jean Dumoulin*[2,3], *Laurent Mevel*[3,2]

[1] BAM Federal Institute of Materials Research and Testing, Berlin, Germany, {md-delwar-hossain.bhuyan, falk.hille}@bam.de

[2] Ifsttar, COSYS/SII, Bouguenais, France, {nicolas.le-touz, jean.dumoulin}@ifsttar.fr

[3] Univ. Rennes, Inria, Rennes, France, {guillaume.gautier, michael.doehler, laurent.mevel}@inria.fr

ABSTRACT

The Stochastic Dynamic Damage Locating Vector (SDDLV) approach is a vibration-based damage localization method based on both a finite element model of a structure and modal parameters estimated from output-only measurements in the damage and reference states. A statistical version of the approach takes into account the inherent uncertainty due to noisy measurement data. In this paper, the effect of temperature fluctuations on the performance of the method is analyzed in a model-based approach using a finite element model with temperature dependent parameters. Robust damage localization is carried out by rejecting the temperature influence on the identified modal parameters in the damaged state. The algorithm is illustrated on a simulated structure.

Keywords: damage localization, SDDLV, load vector, temperature rejection, statistical evaluation

1. INTRODUCTION

Vibration-based structural health monitoring (SHM) techniques have been actively developed in the last decades [1–3], for example for the monitoring of bridges, buildings or offshore structures. Monitoring-based structural damage assessment is usually divided into subtasks of increasing difficulty [4]: damage detection (level 1) has been of great effort in the last decade, whereas damage localization (level 2) is more a focus of current research [5–7].

Physical changes in the structure due to damage induce changes in the dynamic properties of the structure, which can be monitored through output-only vibration measurements. Nonetheless, the dynamics of structure can also be affected by some environmental variability e.g. changes in temperature, loads, boundary conditions, winds etc. [8–10].

The Stochastic Dynamic Damage Locating Vector (SDDLV) approach [11] is a vibration-based damage localization method based on both a finite element model of a structure and modal parameters estimated from output-only measurements in the damage and reference states. A vector is obtained in the null space of the changes in the transfer matrix between both states and then applied as a load vector to the model. The damage localization is related to the resulting stress where it is close to zero. In previous work [7, 12, 13], the statistical SDDLV approach has been proposed, taking into account the uncertainties in the measurement data. So far, environmental variability has not been considered, in particular the effect of temperature changes. Since the damage localization approach operates based on the modal parameter estimates, it is proposed to reject the temperature variations by correcting these estimates in a preprocessing step. The objective of this paper is to show that the robustness of the damage localization method is improved significantly with the proposed temperature rejection strategy.

This paper is presented as follows. In Section 2, the SDDLV method and the rejection of the temperature effect are presented, and the modeling issues are derived in Section 3. The proposed approach is applied on a numerical simulation in Section 4.

2. DAMAGE LOCALIZATION WITH S-SDDLV ROBUST TO TEMPERATURE CHANGES

In this section, the dynamic modeling of a structure, modal parameters estimates and the temperature rejection approach on the modal parameters estimates are presented in Sections 2.1., 2.2. and 2.3., respectively. Finally, the deterministic computation of the damage indicator and its statistical evaluation is shown in Section 2.4..

2.1. Dynamic modeling of structure

The behavior of a mechanical structure can be described by a linear time-invariant (LTI) dynamic system

$$M\ddot{\mathcal{X}}(t) + C\dot{\mathcal{X}}(t) + K\mathcal{X}(t) = f(t) \tag{1}$$

where M, C, $K \in \mathbb{R}^{d\times d}$ are the mass, damping and stiffness matrices, respectively, t indicates continuous time and $\mathcal{X} \in \mathbb{R}^d$ denotes the displacements at the d degrees of freedom (DOF) of the structure. The external force $f(t)$ is not measurable and modeled as white noise. The external force $f(t)$ is not measurable. Observing system (1) at r sensor coordinates, it can be transformed to the corresponding continuous-time state-space model

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c e(t) \\ y(t) = C_c x(t) + D_c e(t) \end{cases} \tag{2}$$

with state vector $x \in \mathbb{R}^n$, output vector $y \in \mathbb{R}^r$, the state transition matrix $A_c \in \mathbb{R}^{n\times n}$ and output matrix $C_c \in \mathbb{R}^{r\times n}$, where $n = 2d$ is the system order and r is the number of outputs [11]. The input influence and direct transmission matrices are of size $B_c \in \mathbb{R}^{n\times r}$ and $D_c \in \mathbb{R}^{r\times r}$ respectively. Matrices A_c and C_c contain information on the modal parameters of the structure and can be identified from measurements.

2.2. Modal parameters extraction

From Stochastic Subspace Identification (SSI) [14], modal parameter estimates and subsequently, the estimates of system matrices $\widehat{A}_c$ and $\widehat{C}_c$ can be obtained from output only measurements.

The environmental variability, e.g. due to temperature changes may affect the structural properties or boundary conditions and thus the modal parameter estimates [8]. These effects should be taken into account in the presented models. In the following, the rejection of temperature effect is derived from the modal parameter estimates in the damaged state.

2.3. Temperature rejection

The analysis of temperature variation on the modal parameters of the structure is a subject of increasing interest in the last couple of years [8–10]. Since the considered SDDLV approach operates on the modal parameter estimates, the temperature effect can be rejected from these estimates by a sensitivity analysis.

Assume that $T \in \mathbb{R}^p$ be a temperature parameter vector that describes the monitored system in the current state at p different positions of the structure, and let $T_0 \in \mathbb{R}^p$ be its value in the reference system. To obtain the required sensitivity matrix, the derivative of the modal parameters $(\lambda_{(c,l)}, \varphi_l)$ with respect to the temperature change is computed for each mode l for the eigenvalues $\lambda_{c,l}$ and mode shapes φ_l, $l = 1, \ldots, m$. Denote $\lambda_c = [\lambda_{c,1} ... \lambda_{c,m}]^T$ and $\varphi = [\varphi_1 ... \varphi_m]$ the collection of eigenvalues and mode shapes, respectively, and let

$$\theta = \begin{bmatrix} \lambda_c \\ \mathrm{vec}(\varphi) \end{bmatrix}$$

be the full modal parameter vector of the system. These parameters are temperature dependent, $\theta = \theta(T)$. The Taylor expansion of the parameter vector writes as

$$\theta(T) \approx \theta(T_0) + \mathcal{J}_{\theta,T}(T - T_0), \tag{3}$$

where $\mathcal{J}_{\theta,T} = \frac{\partial \mathrm{vec}(\theta)}{\partial \mathrm{vec}(T)}$ is the sensitivity matrix. Now, the rejection of temperature variation from the considered parameter vector θ in the damaged state can be obtained from (3)

$$\theta(T_0) \approx \theta(T) - \mathcal{J}_{\theta,T}(T - T_0). \tag{4}$$

The derivative $\mathcal{J}_{\theta,T}$ of the modal parameters with respect to the temperature vector is obtained by first deriving the modal parameters with respect to the stiffness parameters of the system as described in detail in [15] at the reference temperature. Then, the relation between the temperature vector and the material properties is used to obtain the derivative of the stiffness parameters with respect to the temperature vector in the second step, e.g. using a finite difference approach.

2.4. Computation of the damage indicator

The SDDLV is an output-only damage localization method based on interrogating changes $\delta G(s) = \tilde{G}(s) - G(s)$ in the transfer matrix $G(s) \in \mathbb{C}^{r \times r}$ of a system at the sensor coordinates in both damaged (with tilde) and reference states [11], where

$$G(s) = R(s)D_c \quad \text{where} \quad R(s) = C_c(sI - A_c)^{-1} \begin{bmatrix} C_c A_c \\ C_c \end{bmatrix}^{\dagger} \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{5}$$

and s is a Laplace variable in the complex plane. Note that $G(s)$ in the reference state is obtained at the reference temperature T_0, and $\tilde{G}(s)$ in the damaged state is obtained with the temperature rejection approach where the temperature effect is removed from the estimates of modal parameters (4) and subsequently the system matrices.

The changes $\delta G(s)$ are linked to physical properties of the structure. The structural damage is indicated by losses of stiffness that are confined to some part or a region of the structure, affecting the flexibility of the system, which is linked to $\delta G(s)$. A load vector $v(s) \in \mathbb{C}^r$ in the null space of $\delta G(s)$ is obtained from the null space of $\delta R(s)^T$ using a singular value decomposition (SVD). Then this load vector is applied to the Finite Element (FE) model of the healthy structure at the reference temperature for the computation of a stress field over the structure. This stress is stacked in vector $S(s)$, which yields a linear relationship to the load, and which can be expressed by a matrix $L_{model}(s) \in \mathbb{C}^{l \times r}$ based on the FE model of the structure satisfying

$$S(s) = L_{model}(s)v(s). \tag{6}$$

The stress vector $S(s)$ indicates potential damage for elements with corresponding entries in $S(s)$ that are close to zero. When estimated, these stresses are not exactly zero but small in practice because of modal truncation, model errors and variance errors from measurements.

To decide if the estimated stress components are zero, the covariance of the stress vector Σ_S has been estimated in [7, 12, 13], since its identification is subject to uncertainty because of unknown excitation, measurement noise and limited data length. The uncertainties in the estimates are penalizing the quality and precision of the damage localization results. For making decisions about damaged elements of the structure, these uncertainties need to be taken into account to decide whether stress of an element is significantly close to zero or not. Then, an appropriate hypothesis test is performed on each structural element t by selecting the respective stress components S_t in $S(s)$ as well as the respective covariance submatrix Σ_t of Σ_S, and computing the test statistic for each structural element t

$$\chi_t^2 = S_t^T \Sigma_t^{-1} S_t \tag{7}$$

tested for damage. Since stress over damaged elements is zero in theory, potential damage is located in some of the elements t corresponding to the lowest value of χ_t^2 among all elements.

Notice that the correction of the modal parameters with respect to the effect of temperature variation is a deterministic step. In that sense, the variances of the modal parameters after correction should not be affected and corrected. Still, the variances of the transfer functions, the load vector and then the stress are affected by the temperature rejection strategy by virtue of the perturbation approach that is used here to derive those covariances and then they should be adjusted accordingly.

2.5. Summary of the algorithm

The proposed algorithm of the damage localization approach runs as follows with the proposed temperature rejection strategy.

1. The FE model of the structure is obtained at reference temperature vector T_0, and matrix $L_{model}(s)$ is extracted from it for the stress computation (Equation (6)).

2. Vibration measurements are recorded in the undamaged state at reference temperature T_0. The modal parameters are identified from these measurements and used for estimation of the system matrices $\widehat{A}_c$ and $\widehat{C}_c$ in the reference state, as well as their covariance is obtained (see [13]).

3. Based on the FE model and a relation between the temperature vector T and the material properties of the structure, the sensitivity $\mathcal{J}_{\theta,T}$ of the modal parameters with respect to temperature changes is obtained (see Equation (3)).

4. Vibration measurements are recorded in the damaged state at an arbitrary temperature vector T. The estimates of the modal parameters are obtained and corrected using the temperature rejection strategy (4) and subsequently, the $\widehat{A}_c$ and $\widehat{C}_c$ are computed in the damaged state rejecting the temperature variation by the sensitivity analysis.

5. In order to compute the stress field $S(s)$ (6) for damage localization, the load vector $v(s)$ in the null space of the transfer matrix difference $\delta G(s)$ between the healthy and damaged systems is applied to the FE model of the structure in the reference state at the reference temperature.

6. Finally, based on this temperature rejection strategy, the robust damage localization framework can be finalized by the computation of stress $S(s)$ (6) and the statistical evaluation by χ_t^2-test (7).

3. NUMERICAL APPLICATION

The damage localization method has been applied on a 3D rectangular beam model in order to investigate the temperature effect on the dynamic properties of the structure in the computation of stress for damage localization.

In this application, the outcome of the localization results with temperature rejection is compared to not using any temperature rejection. To get these results from simulated datasets, the modes of the system are estimated using a stabilization diagram procedure with SSI [14] in both reference and damaged states. Since the modal parameters of the structure are varying due to the temperature variations, these modal parameters are corrected in the damaged state by sensitivity analysis (4). Then, the system matrices are assembled from the modes [13]. Finally, the estimated stress is computed by considering temperature rejection and its statistical χ_t^2 -test is evaluated for each structural element for damage localization.

3.1. Beam model

In this study, a 3D Beam model, matching a designed and realized mock-up by I4S Team [16], has been considered for damage localization under temperature effect. The structure is modeled with 4 main beam elements of length 1 m. The elements are modeled as rectangular hollow structural sections (HSS) with internal 0.142 m $\times$ 0.042 m and external 0.15 m $\times$ 0.05 m sections, as depicted in Figure 1. The mass density, Young modulus (E) and Poisson ratio are 7800 kg.m^{-3}, 70 GPa (at 20°C) and 0.33, respectively. Each section is enclosed by two plates of length 0.1 m with external 0.18 m $\times$ 0.08 m and internal 0.142 m $\times$ 0.042 m sections, respectively. Finally, at the beam ends, two plates of length 0.2m and section 0.3 m $\times$ 0.08 m are mounted. All beam sections have the same physical properties as the main beam elements.

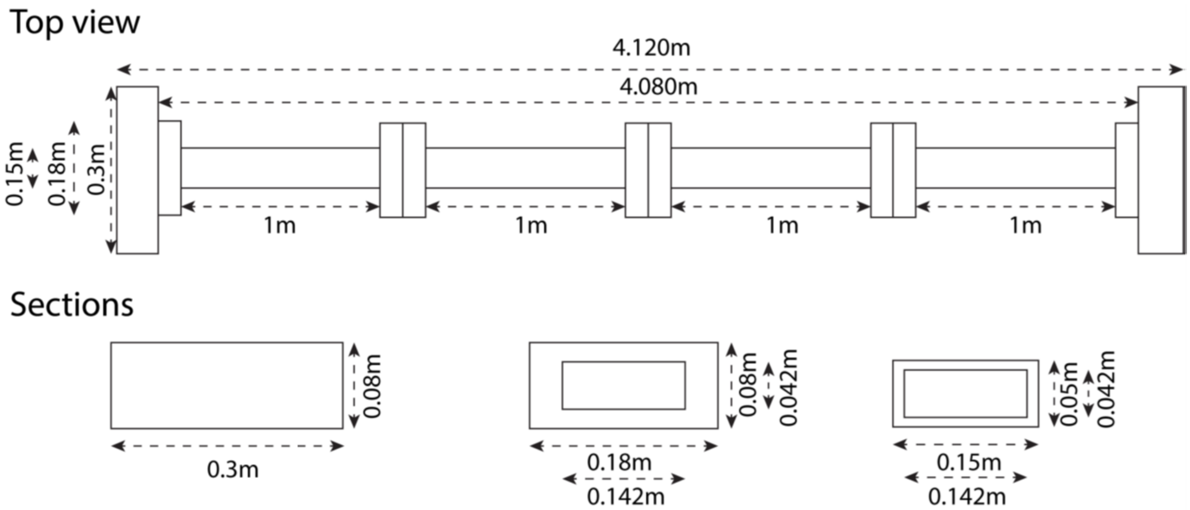


Figure 1: 3D Model with 30 beam elements (180 DOFs)

Each main beam element is discretized into 5 small elements and hence, the total number of elements is 30, and the number of degrees of freedom of the structure is 180. Damping is defined such that the damping ratio of all modes is 0.2%. Note that the damaged position is modeled at elements 12-14 (1.45 m from left end) by 30% stiffness loss. For the damaged and undamaged states, the acceleration data length for each simulated set is N = 100,000, generated from collocated white noise excitation using four sensors in the Y-direction at 0.43 m, 1.49 m, 2.49 m, and 3.47 m from the left support end with a sampling frequency of 2000 Hz, and 5% white noise was added to the output data.

3.2. Temperature model and measurements

A numerical temperature model is used for the computation of a temperature field in the structure from the knowledge of the environmental conditions. It is supposed that the structure is submitted to natural convective exchange with the air, radiative exchange with the environment and outside energy contributions such as solar radiation. Applying a heat source at element 17, the temperature field is computed at the elements of the finite element model. Their stiffness is then computed from the obtained temperatures and used for the data simulation for different magnitudes of the heat flux.

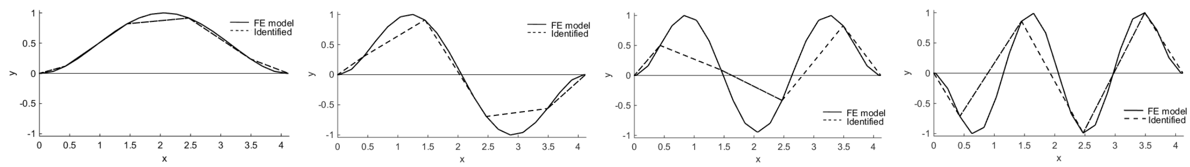
For damage localization, the temperatures at all elements are supposed to be measured, e.g. by an infrared camera in practice, but here computed from the model. In the damaged state, these temperatures are taken into account for the temperature rejection.

3.3. Modal analysis

In the SDDLV approach, the number of considered modes cannot be higher than the number of sensors. The first four modes of the structure are identified (see Table I) from the simulated measurements using stochastic subspace identification (SSI). The identified mode shapes of the first four modes are illustrated in Figure 2 for the respective healthy state together with the mode shapes obtained from the FE model in the healthy state. Moreover, Figure 3(a)-(b) shows that how the first and second modal frequencies are varying in terms of increasing heat flux for both healthy and damaged conditions.

Table I: Frequency (f [Hz]) from healthy state (ref. temperature 293 K) and damaged states (heat flux 40 W.m^{-2}).

Mode	Healthy state	Damaged state
1	22.14	21.03
2	61.10	59.00
3	118.6	113.3
4	204.5	194.2



(a) First mode **(b)** Second mode **(c)** Third mode **(d)** Fourth mode

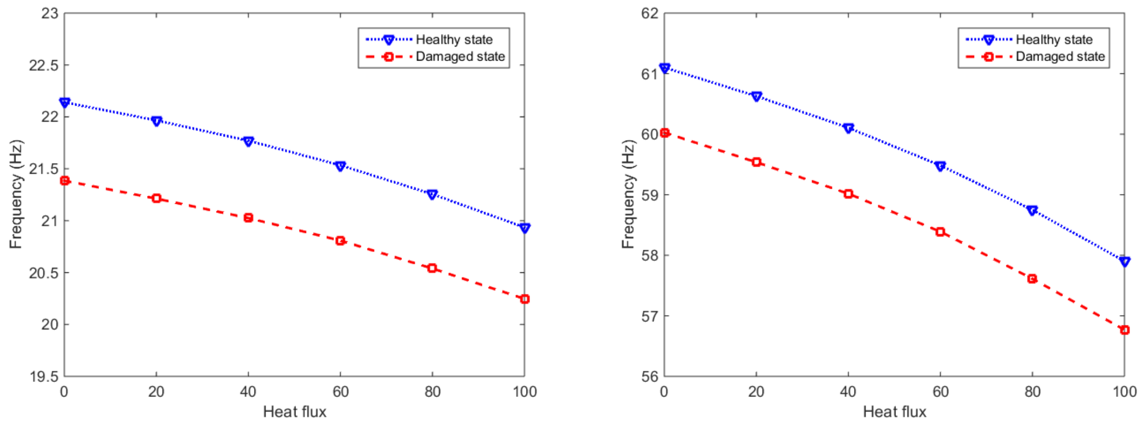
Figure 2: Mode shapes of the first four vertical modes in the healthy state from data and FE model.

In the following, localization results at all structural elements are presented under temperature influence using one dataset and one Laplace variable s.

3.4. Localization results in all elements

To analyze the impact of temperature variation on both the stress computation and its statistical evaluation for damage localization, results for each element are illustrated in Figures 4 and 5 taking into account the temperature rejection approach or not (with/without). The value for the Laplace variable s is chosen as $s_1 = -1 + 700\mathrm{i}$ in the vicinity of the identified poles. To compare the ratios between test results for the healthy and damaged elements, the computed test values are normalized in the figures such that the smallest of the 30 values – indicating damage – is 1.

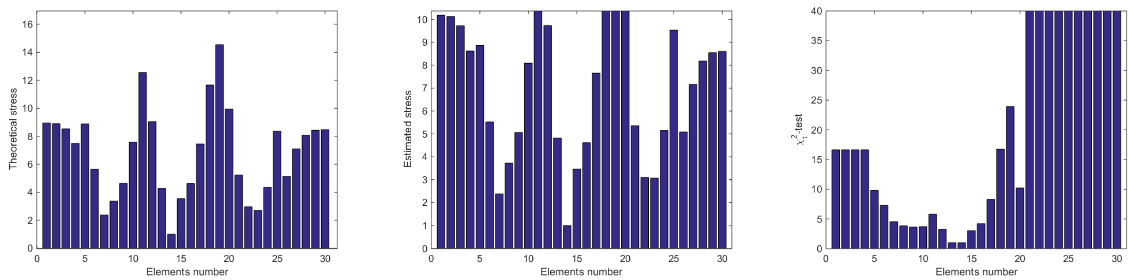
In Figures 4(a)-4(c), the values of the theoretical, estimated stress and its statistical evaluation are displayed when temperature is simulated as varying between healthy and damage states, and similarly, the test values (theoretical, estimated stress and its statistical evaluation) are illustrated in Figures 5(a)-5(c) when temperature rejection is applied. It can be seen that it is difficult to localize the damaged elements



(a) First frequency **(b)** Second frequency

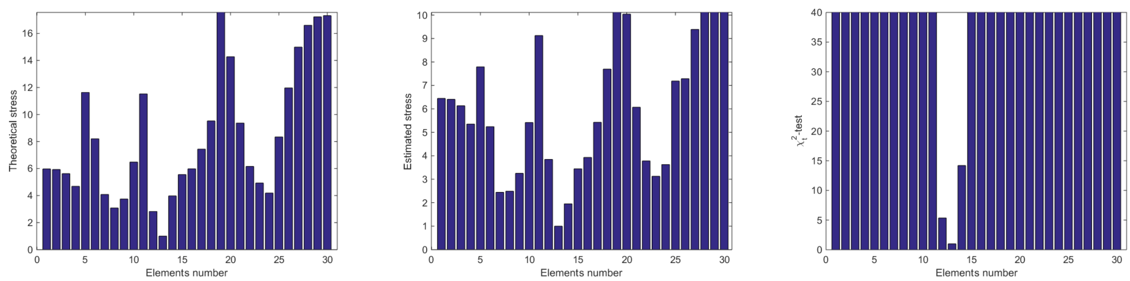
Figure 3: Temperature effect on the first (a) and second (b) frequencies in the healthy and damaged states.

correctly at positions 12, 13 and 14 in Figure 4, while it is easier to discriminate between the damaged and undamaged elements in Figure 5. In addition, it can also be noticed that the contrast ratio between the test values for the damaged and undamaged elements leads to a higher value for the ratio of the statistical evaluation in Figure 5 than in Figure 4, while rejecting the temperature variations.



(a) Theoretical stress **(b)** Estimated stress **(c)** Statistical test

Figure 4: Without temperature rejection – Localization results at all elements for stress computation and its statistical evaluation. Damaged elements are 12, 13 and 14.



(a) Theoretical stress **(b)** Estimated stress **(c)** Statistical test

Figure 5: With temperature rejection – Localization results at all elements for stress computation and its statistical evaluation. Damaged elements are 12, 13 and 14.

3.5. Performance evaluation of damage localization with and without temperature rejection

To investigate the performance of the damage localization method under temperature variation, the success rate (or probability) of damage localization is evaluated for 100 sets of simulated measurement data in a Monte Carlo experiment. In order to indicate if an element is potentially damaged or not, the χ_t^2 value is computed for each structural element. For each Monte Carlo realization, damage localization is seen as successful when the lowest χ_t^2 value among all elements is indeed at the damaged element, and the success rate is the percentage of realizations for which the χ_t^2 value at the damaged element is the smallest χ_t^2 value. The success rate depends on the chosen value for Laplace variable s and serves as the performance indicator for the method. Notice that the generation of several datasets allows the evaluation of the success rate, while in reality usually only one dataset is available.

In order to evaluate the influence of the Laplace variable s on the success rate of the damage localization procedure, each dataset in the Monte-Carlo simulations is evaluated for a set of values of s with different real and imaginary parts in order to obtain the success rate in dependence of s. The range of for the Laplace variable s has been chosen in the vicinity of the identified poles to reduce the effects of modal truncation in the transfer matrix estimates [11].

Considering using temperature rejection or not, the success rates of damage localization are shown in Figure 6 in dependence of the imaginary part of parameter s for a fixed real part $\mathrm{Re}(s) = -1$ (since the choice of the real part of the Laplace variable has little impact on the results), at a heat flux of 80 Wb/m^2. It can be clearly seen that the temperature rejection approach significantly improves the damage localization performance for nearly all values of s in comparison to not using it.

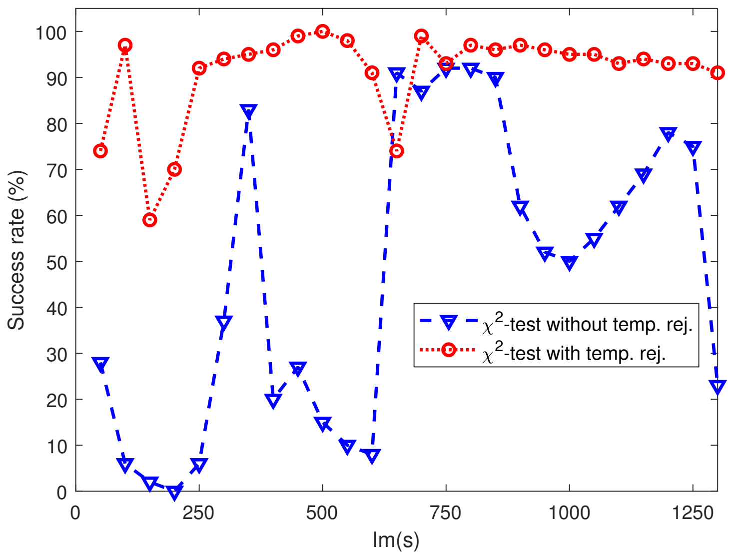


Figure 6: With/without temperature rejection: Success rates of damage localization for different values of s at a heat flux of 80 Wb/m^2. Damaged elements are 12, 13 and 14.

In Figure 7, the mean of the success rates of damage localization for values of s with $\mathrm{Im}(s)$ =100, 150, ..., 1300 are shown in dependence of the heat flux in the damaged state with and without the temperature rejection approach. Without using the rejection, the success rate decreases when the heat flux increases, since no heat flux is present in the healthy state. With the rejection, the localization success rate is constantly high and nearly independent of the heat flux.

4. CONCLUSIONS

In this paper, a temperature rejection strategy has been presented, correcting the modal parameter estimates through a sensitivity analysis before applying a subsequent damage localization approach previously developed. The result shows that temperature rejection on the modal parameters is a useful preprocessing step to improve the performance of the damage localization results. It can be concluded that the risk of poor performance of the damage localization method due to the effect of temperature variation can be reduced using this temperature rejection strategy.

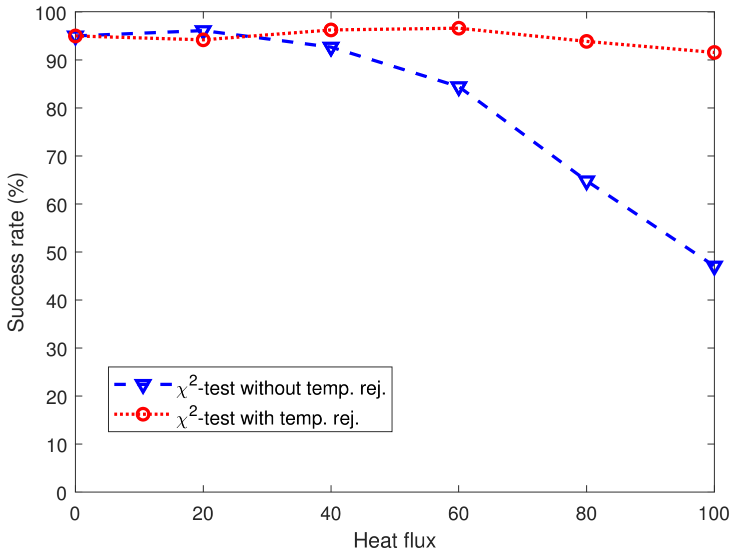


Figure 7: With/without temperature rejection: Success rates of damage localization for different heat flux. Damaged elements are 12, 13 and 14.

ACKNOWLEDGEMENTS

The support from DAAD/German Federal Ministry of Education and Research and the French Ministries of Foreign Affairs (MAEDI) and Education (MENESR) in the PROCOPE program is gratefully acknowledged.

REFERENCES

[1] C.R. Farrar, S.W. Doebling, and D.A. Nix. Vibration-based structural damage identification. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Science*, 359(1778):131–149, 2001.

[2] E.P. Carden and P. Fanning. Vibration based condition monitoring: a review. *Structural Health Monitoring*, 3(4):355–377, 2004.

[3] W. Fan and P. Qiao. Vibration-based damage identification methods: a review and comparative study. *Structural Health Monitoring*, 10(1):83–111, 2011.

[4] C.R. Farrar and K. Worden. An introduction to structural health monitoring. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 365(1851):303–315, 2007.

[5] S. Allahdadian, M. Döhler, C. Ventura, and L. Mevel. Damage localization of a real structure using the statistical subspace damage localization method. In *Proc. 11th International Workshop on Structural Health Monitoring (IWSHM)*, Stanford, CA, USA, 2017.

[6] M.D.H. Bhuyan, G. Gautier, M. Döhler, L. Mevel, Y. Lecieux, and F. Schoefs. Statistical damage localization in mechanical systems based on load vectors. In *International Federation of Automatic Control (IFAC), World Congress*, 9-14 July, 2017, Toulouse, France.

[7] M.D.H. Bhuyan, M. Döhler, Y. Lecieux, L. Mevel, and F. Schoefs. Statistical damage localization with stochastic load vectors using multiple mode sets. *Structural Health Monitoring*, 16(5):518–535, 2017.

[8] H. Sohn. Effects of environmental and operational variability on structural health monitoring. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 365(1851):539–560, 2007.

[9] M. Basseville, F. Bourquin, L. Mevel, H. Nasser, and F. Treyssède. Handling the temperature effect in vibration monitoring: two subspace-based analytical approaches. *Journal of Engineering Mechanics*, 136(3):367–378, 2010.

[10] K. V. Buren, J. Reilly, K. Neal, H. Edwards, and F. Hemez. Guaranteeing robustness of structural condition monitoring to environmental variability. *Journal of Sound and Vibration*, 386:134–148, 2017.

[11] D. Bernal. Load vectors for damage location in systems identified from operational loads. *Journal of Engineering Mechanics*, 136(1):31–39, 2010.

[12] M. Döhler, L. Marin, D. Bernal, and L. Mevel. Statistical decision making for damage localization with stochastic load vectors. *Mechanical Systems and Signal Processing*, 39(1-2):426–440, 2013.

[13] L. Marin, M. Döhler, D. Bernal, and L. Mevel. Robust statistical damage localization with stochastic load vectors. *Structural Control and Health Monitoring*, 22:557–573, 2015.

[14] M. Döhler and L. Mevel. Fast multi-order computation of system matrices in subspace-based system identification. *Control Engineering Practice*, 20(9):882–894, 2012.

[15] W. Heylen, S. Lammens, and P. Sas. *Modal Analysis Theory and Testing*. Katholieke Universiteit Leuven, Belgium, 1998.

[16] G. Gautier, L. Mevel, J. Dumoulin, M. Döhler, M. Le Pen, D. Siegert, and E. Merliot. An experimental setup for subspace based damage identification methods. In *European Geosciences Union, General Assembly*, 09-13 April, 2018, Vienna, Austria.