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Inventory model with the consideration of pricing, product substitution and value deterioration

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Abstract. Nowadays, due to the radical development for the e-commerce means, it becomes a trend to buy the products, especially the electronic products, directly from the manufacturers instead of retailers. It is commonly known that each entity in the market has the autonomy to deciding both the production/inventory plan and the pricing plan. Nevertheless, the existing research focuses either on the production planning or pricing. Using such approach can generate optimal solution for production and pricing, but the global optimal solution for integrated production planning and pricing is not guaranteed. Investigating the literature reveals that there are two literature review studies indicate that it is of great important to study the integrated model in production and inventory control problem with perishable and substitutable product. Therefore, this study aims to discuss the aforementioned problem by solving an inventory model with the consideration of pricing, product substitution and value deterioration. For this purpose, we propose a Mixed Integer Linear Programming model to represent our inventory model and develop a method to find out the best pricing strategy and its corresponding production plan. To demonstrate the validity of the proposed model, we present an example, the results reveal that our model can efficiently handle the proposed problem.

Keywords: Production Planning, Inventory Control, Product Substitution, Product Pricing.

1 Introduction

The process of launching new electronic models to the market happens frequently, owing to the advance technology development. Indeed, this process is a strategy for companies in order to keep up with the customers' changing needs. By doing so, both the new and old models of the product are available in the market. This results in a value deterioration for old models, and hence leads to price reduction of old models to attract customers who were not willing to buy this product with its original price. This situation is quite complex for companies, as it is difficult to set optimal price for both models while considering the customer preference. This difficulty stems from the fact that large price difference might lead in demand drop for new model, whereas small price difference might cause in a slow inventory consumption for old model. This problem has received much attention from researchers. For example, Zhou et al. (2015) [1] discussed how to find out the optimal pricing for fashion product by considering the inventory cost. The challenge faced by companies is not only the price but also the demand of the product, which is affected by the customer preference. For example, with respect to the price sensitive customers, large price difference motivates them to buy the old models, whereas for quality sensitive customers, they tend to buy the new models.

Although the production plan and its corresponding cost account for a large portion of the total company cost, they are seldom considered. Indeed, reasonable production plan or strategy can significantly reduce the inventory cost and production cost as well. Investigating the literature reveal that scholars focused on either the inventory cost with pricing strategy, or production strategy alone. Towards the goal of approaching the reality, it is of great importance to investigate the joint optimization for the production and pricing strategies and see how they are affected by the customer preference. In addition, this study considers one-way substitution, as it is commonly used in practice. Note that substitution reflects the action of using one model in order to substitute another one in order to satisfy the demand.

The above-mentioned problem is handled as follows. First, we model this problem as a multi period dynamic lot size problem, while considering the one-way substitution and value deterioration. Second, we propose a MILP model for this problem and develop a response surface method to derive the optimal production quantity for each period and its corresponding prices. The main objective is to maximize the overall profit, by subtracting the production, inventory and conversion costs from the revenue sales.

The rest of this paper is proceeds as follows. In Section 2, relevant literature review is discussed. Section 3 presents the used notations and assumptions. Then, the mathematical model is presented in Section 4. Numerical experiments are included in Section 5. Finally, the conclusions are given in Section 6.

2 Literature review

This section mainly discusses the research work pertaining to production and inventory control: substitution, deterioration, pricing and customer preference modeling. Indeed, production and inventory control have been extensively discussed in the literature, resulting in models such as Economic Production Quantity (EPQ) and Economical Order Quantity and Dynamic Lot Sizing (DLS). Friedman and Hoch (1978) [2] was the first author to introduce DLS while considering the deterioration rate of perishable products. Later, Hsu et al. (2005) [3] integrated the DLS and products substitution. On the focus of models that consider deteriorating product, Bakker et al.

(2012) [4] provided a review paper, which indicated that modeling the substitution and deterioration together is significant and could contribute to the existing research. This point is also confirmed by another review paper by Shin et al. (2015) [5]. On the other hand, the integration of pricing decision with inventory problems has received much attention in the literature. For example, Dong et al. (2008) [6] studied the dynamic pricing on substitutable products by adoption of stochastic dynamic programming. However, few studies reported the customer preference. Chen et al. (2015) [7] studied the inventory problem assuming a simple form for the customer preference, by proposing linear and exponential price-demand function.

3 Mathematical model

In this section, we present the mathematical model for a multi-period dynamic lot sizing model. Before presenting the model, we first define the notation as follows:

Parameters:

i: Index for production period, $i \in \{1,2,\ldots,n\}$

j,k: Index for production status, $j,k \in \{1,2,\ldots,m\}$

SP_i: Setup cost for production period i

SC_{ijk}: Setup cost for conversion from status j to status k in period i

h_{ij}: Holding cost for product with status j in period i

p: Production cost (\$/unit)

c_{jk}: Conversion cost from status j to status k (\$/unit)

d_{ij}: Demand for product in status j in period i

I_{ij}: Inventory carried over after period i for product with status j

PN_{i1}: Production node in period i

CN_{ij}: Conversion node in period i with product status j

D_i: The total demand of all product statuses in period i

Decision variables:

pr_j: Selling price of product with status j

x_i: Production quantity in period i

q_{ijk}: Conversion quantity from status j to status k in period i

ζ_i: $= 1$ if produces in period i; $= 0$ otherwise

σ_{ijk}: $= 1$ if converts products from status j to status k in period i; $= 0$ otherwise

Based on the predefined notation, the MILP of the proposed problem is formulated as follows:

Max

$$\sum_{i=1}^{n}\sum_{j=1}^{i} pr_j \times d_{ij} - \sum_{i=1}^{n}\sum_{j=1}^{i} h_{ij} \times I_{ij} - \sum_{i=1}^{n}(p \times x_i + SP_i \times \zeta_i) - \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}\left(d_{ij} \times q_{ijk} + SC_{ijk} \times \sigma_{ijk}\right) \tag{1}$$

s.t.:

$$x_i = d_{i1} + I_{i1} - \sum_{j=1}^{i} q_{ij1} \quad \text{where } i \in \{1, \dots, n\} \tag{2}$$

$$I_{i-1,j-1} = d_{ij} + I_{ij} + \sum_{k=1}^{j-1} q_{ijk} \text{ where } i = j,\ i \in \{2, \dots, n\} \tag{3}$$

$$I_{i-1,j-1} = d_{ij} + I_{ij} + \sum_{k=1}^{j-1} q_{ijk} - \sum_{k=j+1}^{i} q_{ijk} \quad \text{where } j \le i, i \in \{2, \dots, n\} \tag{4}$$

$$x_i \le M \times \zeta_i \text{ where } i \in \{1, \dots, n\} \tag{5}$$

$$q_{ijk} \le M \times \sigma_{ijk} \text{ where } k \le j, j \le i, i \in \{2, \dots, n\} \tag{6}$$

$$I_{nj} = 0 \tag{7}$$

The objective function is presented in Eq. (1). First term calculates the sum of revenue made from selling all the products. Second, third and last terms represent the total holding cost, production cost, setup cost and conversion cost, respectively. Eq. (2) defines the flow balance for production node PN_i. Eqs. (3) and (4) require that at the conversion node CN_{ij}, the number of inventory carried from period $i-1$ must equals to the sum of outgoing flow including demand d_{ij}, inventory carry-over in period i and outgoing conversion quantity q_{ijk} minus the sum of incoming conversion quantity q_{ikj}, if any. Eqs. (5) and (6) limit the value of production quantity x_i and conversion quantity q_{ijk} . M is taken as a value large enough to satisfy the production speed. Eq. (7) states the inventory carry-over in the last period n should be zero.

The demand of each model in each period depends on the price of all models, which are given below:

$$D_i = \pi_i \left(1 + \frac{pr_1 - \overline{pr}}{pr_1}\right), \text{where } \overline{pr} = \frac{\sum_{j=1}^{m} pr_j}{m}.$$

Here π_i is a coefficient for demand which can be determined by past sales history. The demand of product model with status j in each period i can be further represented as:

$$d_{ij} = \frac{Q_j - \alpha_j \times pr_j - \sum_{k=1}^{m} \beta_{jk}(pr_j - pr_k)}{\sum_{j=1}^{m}\left[Q_j - \alpha_j \times pr_j - \sum_{k=1}^{m} \beta_{jk}(pr_j - pr_k)\right]} \times D_i \text{ for } k \ne j,$$

where α_j, β_j, Q_j are the coefficients for this demand price function and they are determined by consumer behavior.

4 Solution methodology

Due to the complexity of the problem, instead of using the traditional analytical method, we propose an integrated three-stage approach in this study. Details of each stage are presented below:

- Stage 1: Design of experiments

Central Composite Design is utilized to generate the combinations of decision variables (ΔP_1, ΔP_2, ..., ΔP_{n-1}). Note that ΔP is the price difference between each model, i.e.

$\Delta P_1 = pr_2 - pr_1, \cdots, \Delta P_{n-1} = pr_n - pr_{n-1}$.

Each control variable is divided into a number of levels and the value of α is selected according to the design.

- Stage 2: MILP optimization

In this study, CPLEX is employed to solve the problem. The MILP for the network flow problem is input and coded in CPLEX. In each run, the data of price and demand are changed according to the design of experiments. Obtained optimal solutions and data are recorded for further analysis in Stage 3.

- Stage 3: Response surface method

We utilize full quadratic regression function is selected to find the relationship between control variables and response.

5 Numerical experiments

In this section, we conduct one numerical example to evaluate the performance of the proposed method. This example investigates a model with 4 periods, each decision variables are divided into five levels, which are set at (-1.682, -1, 0, 1, 1.682). In total, we conduct 15 runs of experiments. For the basic demand in each period, it is given as $\pi_i = 500{,}800{,}1300{,}700$. The coefficients for the demand price function are $Q_j = 240{,}240{,}20{,}10$;$\alpha_j = 0.3{,}0.6{,}0.05{,}0.025$; $\beta_{j,k} = (0.1{,}0.6{,}0.05{,}0.025)$, $(0.02{,}0.01{,}0.2{,}0.02)$, $(0.01{,}0.01{,}0.01{,}0.05)$. The price of new product for all the period is set at $400. The range of each price difference ΔP_1, ΔP_2, ΔP_3 is assumed to be from 1 to 100, which means discount rate within range from 0% to 25%. After conducting the design of experiments, and solving the MILP using CPLEX, the results data for profit are analyzed using Minitab by adoption of full quadratic regression analysis. For illustration simplicity, pr_1, pr_2, and pr_3 have been replaced by A, B, and C, respectively.

The result of Analysis of Variance shows that R-sq, R-sq(adj) and R-sq(pred) equals to 99.77%, 99.56% and 98.22% respectively, which means the regression function can well describe the behavior of the model proposed. Fig. 1 illustrates the response surface of the regression model. As can be observed from the surface plot, the function shows concavity in all the three sub-figures. Through the regression model shown in (14), the optimal combination of (ΔP_1, ΔP_2, ΔP_3) is determined as (14, 34, 1). Fig. 2 shows the optimal solution for pricing strategy. The optimal prices for all the four models are ($400, $386, $352, $351). To be noticed, the optimal value of C, which represents ΔP_3, lies beyond the minimum range designed. It means that the optimal price difference between third and fourth product status should be smaller than $1. Under this circumstance, the results suggest that the price of fourth model should be kept as the same as the third model which leads to zero demand for the fourth model. It can also be interpreted that the fourth model should be removed from the product line.

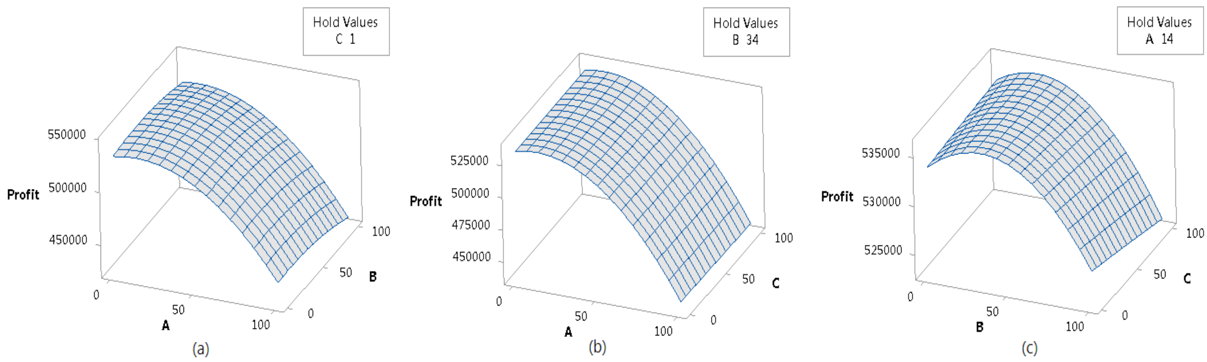

Fig. 1. Response surface of the regression model in Example 1 (a) *A* vs *B* (b) *A* vs *C* (c) *B* vs *C*.

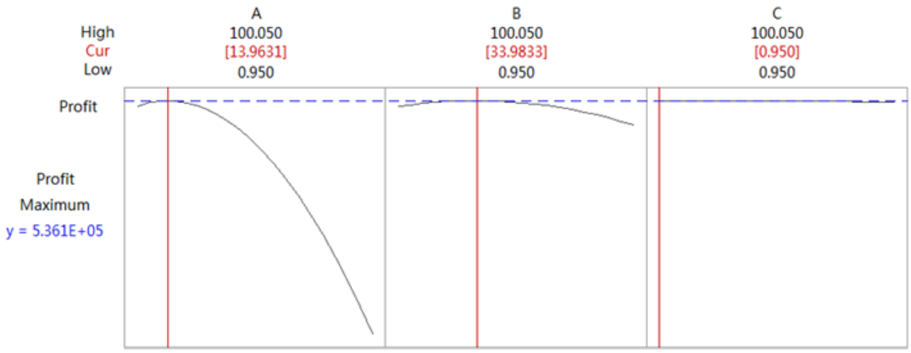

Fig. 2. Predicted optimal pricing strategy.

6 Conclusions

This research specifically looked at a problem inspired by a practical example of electronic product, a multi-period, multi-item dynamic lot sizing model is used to formulate the problem. An integrated three-stage method is utilized to solve the problem proposed which consists of Design of Experiment, MILP and Response Surface method. One example also shows the impact of changing cost structure on the joint strategy. The results show that the proposed method can provide satisfactory performance to this type of problem. Both the production and pricing strategies are optimized. This study can provide managerial implication and practical guidance to decision makers on the selection of optimal pricing and production strategies.

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