Design-Parameters Optimization of a Deep-Groove Ball Bearing for Different Boundary Dimensions, Employing Amended Differential Evolution Algorithm

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Abstract. Rolling-element bearing that is mostly used wherever rotary motion is provided to a shaft in rotating machineries. A deep-groove ball bearing is one type of rolling-element bearing which is used to support radial load, axial load or combination of both. After proper installation and condition, ball bearings usually fail because of fatigue under normal operating conditions. Therefore, the fatigue-life optimization is a prime objective in designing a ball bearing. In the present work, eleven different problems of a deep-groove ball bearing by changing boundary dimensions are optimize to obtain maximum fatigue life. Amended Differential Evolution Algorithm (ADEA), which is modified version of Differential Evolution (DE) algorithm along with constraint handling technique, is applied to these eleven problems and optimum results in the form of optimal design-parameters and fatigue life is reported. The design parameters are bearing pitch diameter, ball diameter, number of balls and curvature coefficient of the outer and inner raceway groove are considered. Further, optimal results are compared with the other researcher's work and standard catalogue for the same problems. Better results for fatigue life are obtained using ADEA.

Keywords: design-parameter optimization, constraint optimization, a ball bearing, Amended Differential Evolution Algorithm (ADEA).

1 Introduction

Rolling element bearings are critical element of any rotating machinery and it is used in variety of applications such as machine tools, electrical equipments, automobile, household appliances, medical equipments and aeroplanes. Rolling element bearings are also called as antifriction bearings. Different types of rolling element bearings are used depending upon the loading condition, i.e., radial load, axial load and combination of both. Radial bearings are used to support the load which is perpendicular to the shaft whereas thrust bearings are used to support the load which acts along axis of the shaft. Among the various types of rolling element bearing, a single-raw, deep-groove ball bearing is most frequently used bearing because it can take radial as well as some thrust

load and withstand a small amount of shaft misalignment or deflection [1]. A single-raw, deep-groove ball bearings rely on balls to support loads between rotating and stationary machine elements. Ball bearings have high load carrying capacity, generates less noise and low resultant temperature because of point contact. Deep-groove ball bearings are available in variety of bore dimensions up to 400 mm [2]. The internal structural-parameters (design parameters) affect the bearing characteristics such as bearing life, lubrication, load distribution, stresses and deflection. Generally, ball bearings are designed to determine the geometric structure and their relative design parameters for smooth operation and bearing life.

Researchers have worked to optimize design parameters of various rolling-element bearing but some important and relevant literatures on optimization of a ball bearing are discussed here. Changsen [3] discussed a design method for rolling element bearing using a gradient based numerical optimization technique; he defined five different non-linear objectives for rolling element bearing design, i.e., maximum frictional moment, maximum wear life, maximum static load rating, maximum spin to roll ratio and maximum fatigue life. Further, he proposed concept of multi-objective optimization of above objectives and also introduced basic concepts and optimization techniques to solve problems. Chakraborty et al. [4] optimized the design parameters of deep-groove ball bearings using Genetic Algorithm (GA), where five design parameters were considered to maximize fatigue life of a bearing. Rao and Tiwari [5] refined the problem proposed by Chakraborty et al. [4] to obtain more realistic optimum design parameters, where five design variables, five various constants of constraints and eight constraints, were used to maximize dynamic load rating of a deep-groove ball bearing using GA.

In the present work, eleven different problems of a deep-groove ball bearing by changing boundary dimensions are considered. These problems are modified by Rao and Tiwari [5] and used to optimize the design parameters using Amended Differential Evolution Algorithm (ADEA) [6]. The optimized results are compared with the results of Genetic Algorithm (GA) [5] and standard catalogue [1]. The rest of the paper is organized as follow: Section 2 describes mathematical model of ball bearing design problem; section 3 presents flow chart of Amended Differential Evolution Algorithm (ADEA); section 4 presents results and discussion and section 5 concludes the work.

2 Mathematical modelling of ball bearing design problem

On the basis of operating requirements, one of the most important objectives is the requirement of the longest bearing life (fatigue life). In normal operating conditions of ball bearings, fatigue failure is the main mode of failure at the surfaces of balls and races. The fatigue life of an individual ball bearing is defined as the number of revolutions, which the bearing runs before the first evidence of fatigue crack in balls or races. The Anti-Friction Bearing Manufacturing Association (AFBMA) standard states that the failure criterion is the first evidence of fatigue. AFBMA sanctioned a term ‘rating life’ of a bearing that is used by most of the bearing manufacturers and it is defined as ‘the number of revolutions or hours at some given constant speed that 90 per cent of

group of a bearings will complete or exceed before the failure criterion develops'. The fatigue life (or rating life) of bearing is based on the dynamic load capacity (dynamic load rating or basic dynamic capacity or specific dynamic capacity [1]). The dynamic load capacity (C) is defined as the dynamic equivalent radial load in deep-groove ball bearings that can be carried for a minimum life of one million revolutions. The fatigue life of bearing can be improved by maximizing dynamic load capacity for a given size of bearing boundary dimensions (i.e., bearing bore, d, and outside diameter, D and bearing width, w). The fatigue life of a bearing (in millions of revolutions) that is subjected to applied dynamic equivalent radial load F is given as:

$$L_{10} = \left(\frac{C}{F}\right)^{a} \text{and } L_{10h} = \frac{10^6}{60n} L_{10} \tag{1}$$

where L_{10} is rated bearing life (in millions of revolutions), C is dynamic load capacity (N), F is applied dynamic equivalent radial load (N) and a is 3 for ball bearings, L_{10h} is rated bearing life (in operating hours) and n is rotational speed in rpm. The bounds of design parameters, objective function and constraints are taken from Rao and Tiwari [5] and briefly discussed below.

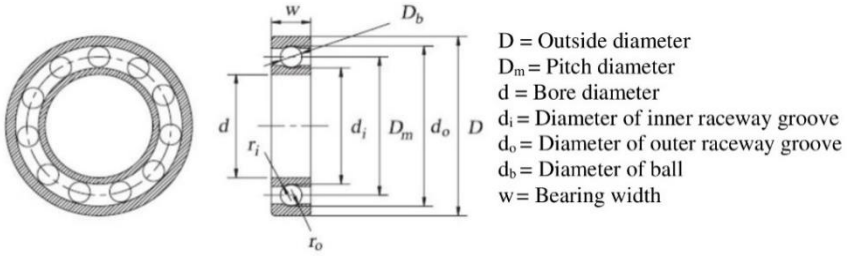

Fig. 1. Internal diagram of ball bearing [4].

2.1 Design parameters

The design parameters are basically internal structural dimensions of a ball bearing as shown in Fig. 1. The five design parameters of ball bearing are: diameter of ball (Db), number of balls (Z), pitch diameter (Dm) and curvature coefficient of the outer raceway groove (fo = ro/Db) and inner raceway groove (fi = ri/Db). ro and ri are the curvature radius of outer and the inner raceway groove, respectively [5]. Table 1 shows the design parameters and their ranges.

Table 1. Design parameters and their ranges [5]

Design parameters	Ranges
Diameter of ball (D_b)	0.15(D-d) to 0.45(D-d)
Pitch diameter (D_m)	0.5(D+d) to 0.6(D+d)
Number of balls (Z)	4 to 50
Curvature coefficient of the inner raceway groove (f_i)	0.515 to 0.52
Curvature coefficient of the outer raceway groove (f_o)	0.515 to 0.53

2.2 Objective function

The objective is to obtain the fatigue life of a ball bearing that is based on the dynamic load capacity (*C*). The dynamic load capacity (*C*) is a function of D_b, D_m, Z, f_i and f_o, and can be expressed as

$$\text{Maximize, } C = \left[f_c\, Z^{2/3}\, D_b^{1.8}\right] \quad \text{when } D_b \le 25.4 \text{ mm,} \qquad (2)$$
$$= 3.647\, f_c\, Z^{2/3} D_b^{1.4} \quad \text{when } D_b > 25.4 \text{ mm.}$$

where f_c for deep-groove ball bearings is given as

$$f_c = 37.91\left[1+\left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i(2f_o-1)}{f_o(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3}\left[\frac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1+\gamma)^{1/3}}\right]\left[\frac{2f_i}{2f_i-1}\right]^{0.41} \qquad (3)$$

where γ is $D_b cos\alpha / D_m$ and α is free contact angle that is zero for deep-groove ball bearings.

2.3 Constraints

Ten constraints that are proposed by Rao and Tiwari [5] are considered in the present work. Out of ten constraints, five constraints are refined by Rao and Tiwari [5] that were proposed by Chakraborty et al [4]. In the present work, constraints for curvature coefficient of inner and outer raceway groove (constraints 9 and 10 of Rao and Tiwari [5]) are in the form of range (bounds) that are already considered as design parameters and given in Table 2. Therefore, constraints for f_i and f_o are not discussed. The remaining eight constraints are discussed below [5].

Constraint 1: For a number of balls

For the convenience of the bearing assembly, the number of balls should satisfy the following equation, $g_1(x)$:

$$g_1(x) = Z \le 1 + \frac{\emptyset_0}{2Sin^{-1}(D_b\,/\,D_m)} \qquad (4)$$

can be written as, $g_1(x) = Z - 1 - \frac{\emptyset_0}{2Sin^{-1}(D_b\,/\,D_m)} \le 0$ (5)

where $\emptyset_0$ is the maximum assembly angle and it is given as

$$\emptyset_0 = 2\pi - 2\,cos^{-1}\left[\frac{U^2+\left(\frac{D}{2}-T-D_b\right)^2-\left(\frac{d}{2}+T\right)^2}{2U\left(\frac{D}{2}-T-D_b\right)}\right], \qquad (6)$$

and *T* and *U* are defined as

$$T = \frac{(D-d-2D_b)}{4} \text{ and } U = \frac{(D-d)}{2} - \frac{3(D-d-2D_b)}{4} \qquad (7)$$

In the constraints 2 to 8, some constraint constants are used and their range (bounds) are shown in Table 2 except constraint 6.

Constraints 2 and 3: For diameter of balls

For the convenience of the bearing assembly, the diameter of balls should be within a following range:

$$g_2(x) = 2D_b \geq K_{D\,min}(D-d), \tag{8}$$

can be written as, $g_2(x) = K_{D\,min}(D-d) - 2D_b \leq 0,$ (9)

$$g_3(x) = 2D_b \leq K_{D\,max}(D-d), \tag{10}$$

can be written as, $g_3(x) = 2D_b - K_{D\,max}(D-d) \leq 0,$ (11)

where $K_{D\,min}$ and $K_{D\,max}$ are the minimum and the maximum values of the ball diameter constants, respectively. The range of $K_{D\,min}$ is 0.4 to 0.5 and range $K_{D\,max}$ is 0.6 to 0.7.

Constraints 4 and 5: For running mobility of a bearing

For the running mobility of bearings, the difference between the pitch diameter and the average diameter in a bearing should satisfy below constraints.

$$g_4(x) = D_m - \frac{D+d}{2} \leq -e(D+d), \tag{12}$$

can be written as, $g_4(x) = D_m - \frac{D+d}{2} + e(D+d) \leq 0, and$ (13)

$$g_5(x) = D_m - \frac{D+d}{2} \geq e(D+d), \tag{14}$$

can be written as, $g_5(x) = e(D+d) - D_m + \frac{D+d}{2} \leq 0$ (15)

where *e* is a constant and its value depends on mobility condition of the balls. The range of *e* is taken here 0.03 to 0.08.

Constraint 6: For relationship between inner and outer ring thickness

In normal operating conditions, the stress on the inner ring is always more than the outer ring of a ball bearing. Therefore, outer ring thickness should be less than or equal to the inner ring thickness.

$$g_6(x) = \frac{D-d_0}{2} \leq \frac{d_i-d}{2}, \tag{16}$$

can be written as, $g_6(x) = \frac{D-d_0}{2} - \frac{d_i-d}{2} \leq 0,$ (17)

where d_i and d_o are the inner and outer raceway diameters at the grooves.

Constraint 7: For thickness of bearing ring at outer raceway bottom

The average diameter of bearing is usually less than the pitch diameter. Therefore, thickness of bearing ring at outer raceway bottom should not be less than εD_b and the following equation, $g_7(x)$ should satisfy:

$$g_7(x) = \varepsilon D_b \leq 0.5(D - D_m - D_b) \tag{18}$$

can be written as, $g_7(x) = \varepsilon D_b - 0.5(D - D_m - D_b) \leq 0$ (19)

where ε is a constant that is obtained using simple strength consideration of outer ring and the range of ε is 0.3 to 0.35.

Constraint 8: For relation between ball bearing width and ball diameter

$$g_8(x) = D_b \leq \beta w \tag{20}$$

can be written as, $g_8(x) = D_b - \beta w \leq 0$ (21)

where β is a constant and the range of β is 0.7 to 0.85. The ranges of various constants used in constraints are given in Table 2.

Table 2. Range of various constants [5]

Constants	Range [Lower Bound (LB), Upper Bound (UB)]
$K_{D\,min}$	0.4 to 0.5 [0.4,0.5]
$K_{D\,max}$	0.6 to 0.7 [0.6,0.7]
e	0.03 to 0.08 [0.03,0.08]
ε	0.3 to 0.35 [0.3,0.35]
β	0.7 to 0.85 [0.7,0.85]

3 Amended Differential Evolution Algorithm (ADEA)

Amended Differential Evolution Algorithm is the modified version of Differential Evolution (DE) algorithm and it uses 'Σ_{const}-constraint handling method' to solve constrained optimization problems [6]. Fig. 2 shows the flow chart of ADEA that is adapted from reference [6].

In the present work, maximization problems of deep-groove ball bearing are converted into minimization problems and all the constraints are converted into ≤ 0 type. The design parameters of deep-groove ball bearing problems are optimized using ADEA. Following parameters are used in ADEA: Population Size (*PS*) = 50, number of generations (*G*) = 500, number of runs = 30, scale factor: F_{min} = 0.5, F_{max} = 0.8, crossover rate: CR_{min} = 0.85, CR_{max} = 0.95. The program is written in MATLAB® for ADEA and results are reported.

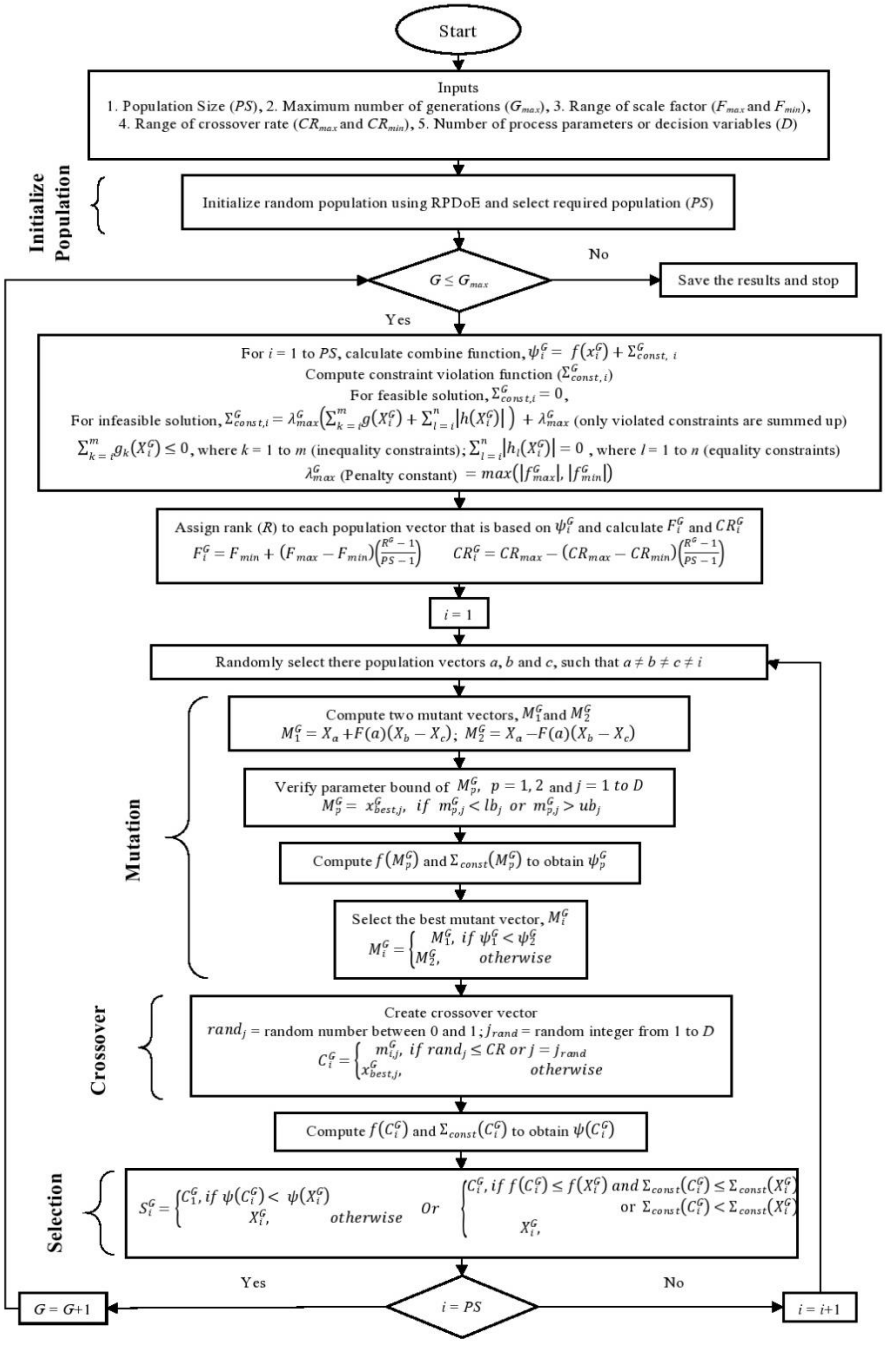

Fig. 2. Flow chart of ADEA (adopted from [6])

4 Results and Discussion

ADEA is successfully applied to optimize design-parameters of single raw, deep-groove ball bearing problems for maximizing dynamic load capacity (C). Results obtained using ADEA are reported in Tables 3-5.

Table 3 shows the optimum design-parameters and value of constants (used in constraints) that are obtained using ADEA. When optimal value of any parameter is exactly either the start value (lower bound) or the end value (upper bound) of range, algorithm requires greater number of generations (iterations) to converge. However, ADEA algorithm converges to lower bound value for two design parameters, namely, f_o and f_i, within 500 generations (Table 3). Further, ADEA attains lower bound value for $\boldsymbol{\varepsilon}$ (constant-constraint). Thus, ADEA is capable to converge the bound value within reasonable generations.

Table 4 presents the values of optimum design parameters, constraints ($g_1(x)$ to $g_8(x)$) and dynamic load capacity (C). The value of design parameters, i.e., D_b and D_m, are reported up to 3 decimal accuracy and accordingly other results are reported. It is clear from Table 4 that $g_6(x)$ and $g_7(x)$ constraints are active constraints (i.e., $g_6(x) = 0$ and $g_7(x) = 0$) for some boundary dimensions. Two constraints, $g_1(x)$ and $g_7(x)$, are difficult constraints for some boundary dimensions because value of constraint is neither zero nor negative value but very nearer to zero value, i.e., 0.0001.

Table 5 shows the comparison of the results with the results of Genetic Algorithm (GA) [5] and standard catalogue [1] for dynamic load capacity (C). The comparison shows that ADEA provides better results for all boundary dimensions. Further, per cent improvement in dynamic load capacity with respect to GA and standard catalogue is presented in Table 5. The maximum and minimum per cent improvement over standard catalogue are 68.50 % and 8.24 %, respectively. Similarly, the maximum and minimum per cent improvement over GA are 11.11 % and 1.35 %, respectively.

Table 3. Results obtained using ADEA in terms of optimized design-parameters and value of constants (used in constraints) for eleven boundary dimensions.

Boundary Dimension (mm)			**Design Parameters**					**Values of constants obtained in constraints**				
D	***d***	***w***	D_b **(mm)**	D_m **(mm)**	***Z***	f_i	f_o	$K_{D\ min}$	$K_{D\ max}$	***e***	ε	β
30	10	9	6.213	20.059	7	0.515	0.515	0.442	0.649	0.072	0.3	0.793
35	15	11	6.25	25	8	0.515	0.515	0.428	0.681	0.055	0.3	0.708
47	20	14	8.438	33.5	8	0.515	0.515	0.435	0.664	0.065	0.3	0.762
62	30	16	10	46	9	0.515	0.515	0.403	0.653	0.069	0.3	0.810
80	40	18	12.5	60	9	0.515	0.515	0.412	0.667	0.048	0.3	0.812
90	50	20	12.5	70	10	0.515	0.515	0.455	0.678	0.08	0.3	0.758
110	60	22	15.625	85	10	0.515	0.515	0.456	0.656	0.044	0.3	0.750
125	70	24	16.732	98.228	11	0.515	0.515	0.420	0.609	0.069	0.3	0.741
140	80	26	18.75	110	11	0.515	0.515	0.402	0.648	0.05	0.3	0.768
160	90	30	21.426	125.719	11	0.515	0.515	0.417	0.674	0.055	0.3	0.787
170	95	32	22.752	133.597	11	0.515	0.515	0.435	0.682	0.051	0.3	0.786

Table 4. Results obtained using ADEA in terms of optimized design-parameters, value of constraints and dynamic load capacity (C) for eleven boundary dimensions.

Boundary Dimension (mm)			**Design Parameters**					**Value of Constraints**								**Dynamic load capacity, C (N)**
D	d	w	D_b (mm)	D_m (mm)	Z	f_i	f_o	$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$	$g_5(x)$	$g_6(x)$	$g_7(x)$	$g_8(x)$	
30	10	9	6.213	20.059	7	0.515	0.515	0*	-3.59	-0.55	-2.94	-2.82	-0.06	0*	-0.92	6032.33
35	15	11	6.25	25	8	0.515	0.515	-0.15	-3.94	-1.12	-2.65	-2.75	0	0	-1.54	7059.09
47	20	14	8.438	33.5	8	0.515	0.515	-0.10	-5.13	-1.05	-4.36	-4.36	0	0*	-2.23	12100.67
62	30	16	10	46	9	0.515	0.515	-0.06	-7.10	-0.90	-6.35	-6.35	0	0	-2.96	18113.18
80	40	18	12.5	60	9	0.515	0.515	-0.37	-8.52	-1.68	-5.76	-5.76	0	0	-2.12	27144.31
90	50	20	12.5	70	10	0.515	0.515	-0.60	-6.80	-2.12	-11.2	-11.2	0	0	-2.66	29174.48
110	60	22	15.625	85	10	0.515	0.515	-0.35	-8.45	-1.55	-7.48	-7.48	0	0	-0.88	43620.92
125	70	24	16.732	98.228	11	0.515	0.515	0*	-10.36	-0.03	-14.18	-12.77	-0.73	0*	-1.05	52452.58
140	80	26	18.75	110	11	0.515	0.515	-0.01	-13.38	-1.38	-11	-11	0	0	-1.22	64386.86
160	90	30	21.426	125.719	11	0.515	0.515	0*	-13.66	-4.33	-14.47	-13.03	-0.72	0*	-2.18	81862.64
170	95	32	22.752	133.597	11	0.515	0.515	0*	-12.88	-5.65	-14.61	-12.42	-1.10	0*	-2.4	91203.51

* indicates constraint value less than or equal to 1×10^{-4} (1E-4) and considered as zero (0).

Table 5. Comparison of results of ADEA with GA and catalogue for dynamic load capacity (C)

Boundary Dimension (mm)			Dynamic load capacity, C (N)			% improvement	
D	d	w	ADEA	GA [5]	Catalogue [1]	$\frac{\text{ADEA} - \text{GA}}{\text{GA}} \times 100$	$\frac{\text{ADEA} - \text{Catalogue}}{\text{Catalogue}} \times 100$
30	10	9	6032.33	5942.36	3580	1.51	68.50
35	15	11	7059.09	6955.35	5870	1.49	20.26
47	20	14	12100.67	10890.9	9430	11.11	28.32
62	30	16	18113.18	16387.4	14900	10.53	21.56
80	40	18	27144.31	26678.4	22500	1.75	20.64
90	50	20	29174.48	28789.3	26900	1.35	8.46
110	60	22	43620.92	42695.3	40300	2.17	8.24
125	70	24	52452.58	51117.4	47600	2.61	10.19
140	80	26	64386.86	59042.9	55600	9.05	15.80
160	90	30	81862.64	75466.8	73900	8.48	10.77
170	95	32	91203.51	89244.7	83700	2.21	8.96

ADEA: results obtained using Amended Differential Evolution Algorithm, GA: Genetic Algorithm and results are taken from reference [5], Catalogue: values of dynamic load capacity for 02-series deep-groove ball bearing are taken from bearing catalogue [1]

5 Conclusions

ADEA is used to optimize the design-parameters of single raw, deep-groove ball bearing for maximizing dynamic load capacity (fatigue life) for eleven boundary dimensions. From the optimization results, following conclusions are drawn:

- Comparison of the results shows that ADEA provides better results than GA and catalogue for all boundary dimensions of deep-groove ball bearing.
- The maximum per cent improvement in dynamic load capacity is 68.50 % over catalogue and 11.11 % over GA.
- ADEA is capable to obtain optimum design-parameters or constants that are extreme value of the range within reasonable generations.
- ADEA can be used to solve other design problems such as angular contact bearing, journal bearing problem, heat exchanger problems and truss problem.

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