Sampled-Data Observers: Scarce Arbitrarly Large Sampling Intervals

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Abstract. Continuous-time systems with discrete measurements are considered. It is shown that the technique of [7] can be used to design converging observers in the case where the size of some intervals between 2 measurements is larger than the upper bound ensuring convergence of the observer that is provided in [7]. A scarcety condition on these intervals is exhibited. This result is established through a recent stability analysis technique called trajectory based approach.

Key Words: Observer, continuous-discrete, asynchronous sampling.

I. Introduction

In many engineering applications, the measurements are available at discrete instants only. Moreover, digital implementation sometimes affects continuous-time observers. These facts are obstacles to the design or the use of continuous observers. This motivated many works on the design of observers using discrete outputs, as illustrated for instance by the papers [2], [9] and [14] and the references therein. In the pioneering work [7], an important approach was initiated for the design of observers for continuous-time nonlinear systems with discrete outputs. In this paper, a system of the type:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \mathcal{F}(x(t)) \ , \\ y(t) & = & Cx(t_i) \quad , \quad \forall t \in [t_i, t_{i+1}) \ , \end{array} \right. \tag{1}$$

with $x \in \mathbb{R}^n$, $y \in \mathbb{R}^q$ and where $\mathcal{F}$ is a nonlinear function was considered. A standard assumption on the sequence t_i was imposed: the existence of a constant $\overline{T} > 0$ such that for all $i \in \mathbb{N}$, $0 \leq t_{i+1} - t_i \leq \overline{T}$ was assumed. Then the main result was the design of a new type of observer and the determination of a constant $T_\star > 0$ ensuring that if $\overline{T} \leq T_\star$, then the proposed observer converges exponentially to the solutions of (1). A key aspect of this result is that the size of all the sampling intervals must be smaller than a constant which depends on $\mathcal{F}$ and C, otherwise the convergence of the observer is not guaranteed. But in practice, due for instance to a temporary loss of measurements, the outputs are sometimes not available during long time intervals. Then the condition $\overline{T} \leq T_\star$ mentioned above may be violated. But this fact does not imply that the solutions of the observer of [7] do not converge to the solutions of the system (2): the intuition suggests that if, for many integers i, $t_{i+1} - t_i$ is sufficiently small and $t_{i+1} - t_i$ is large for a sufficiently small number of integers i, then the solutions of the observer can still converge to the solutions of the studied system.

The present work is build upon this intuitive idea. For systems with asynchronous sampling belonging to the subfamily of the one studied in [7], we show that the assumption $\overline{T} \leq T_\star$ can be relaxed and that an arbitrarily large value for $\overline{T}$ is allowed, provided that $t_{i+1} - t_i$ is sufficiently small for a sufficiently large number of integers i.

We prove the main result of our work via the trajectory based approach, which is a stability analysis for nonlinear time-varying systems introduced in [10], developed in [13] and applied to the design of observers for continuous-time switched systems in [1]. This approach makes it possible to establish the main result under assumptions which can be checked in practice.

The notation will be simplified whenever no confusion can arise form the context. The Euclidean norm is denoted by $|\cdot|$. We denote by I the identity matrix of any dimension.

The paper is organized as follows. The main result is stated and proved in Section II. An illustrative example is given in Section III. Concluding remarks are drawn in Section IV.

II. Main result

A. Studied system

We consider the system

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Hx(t) + \varphi(Cx(t)) \ , \\ y(t) & = & Cx(t_i) \quad , \quad \forall t \in [t_i, t_{i+1}) \ , \end{array} \right. \tag{2}$$

with $i \in \mathbb{N}$, $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^q$ is the output, $C \in \mathbb{R}^{q \times n}$, $C \neq 0$ is a constant matrix, φ is a nonlinear function, $H \in \mathbb{R}^{n \times n}$ is a constant Hurwitz matrix and t_i is an increasing sequence with $t_0 = 0$ and $\lim_{i \to +\infty} t_i = +\infty$.

Let $\sigma : [0, +\infty) \to [0, +\infty)$ be the function defined by $\sigma(t) = t_i$ when $t \in [t_i, t_{i+1})$.

Remark. The assumption that H in (2) is Hurwitz is not restrictive. Indeed, for any system $\dot{x}(t) = Ax(t) + \phi(Cx(t))$ such that (A, C) is detectable, there is a matrix L so that the matrix $A + LC$ is Hurwitz. Then the system $\dot{x}(t) = Ax(t) + \phi(Cx(t))$ can be rewritten as $\dot{x}(t) = Hx(t) + \varphi(Cx(t))$ with $H = A + LC$ and $\varphi(\mathfrak{y}) = \phi(\mathfrak{y}) - L\mathfrak{y}$ and this system is of the type (2).

Remark. Since the matrix H is Hurwitz, there are constants $c_1 > 0$, $p_1 > 0$ and $p_2 > 0$ and a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ such that the LMIs:

$$PH + H^\top P \leq -2c_1 P \tag{3}$$

and

$$p_1 I \leq P \leq p_2 I \tag{4}$$

are satisfied.

We introduce three assumptions.

Assumption A1. *There is a constant $k_\varphi > 0$ such that for all $y_1 \in \mathbb{R}^q$ and $y_2 \in \mathbb{R}^q$, the inequality*

$$|\varphi(y_1) - \varphi(y_2)| \leq k_\varphi |y_1 - y_2| \tag{5}$$

holds.

Assumption A2. *There are 2 constants $\overline{T} > 0$ and $\underline{T} > 0$ such that for all $i \in \mathbb{N}$, the inequality*

$$\underline{T} \leq t_{i+1} - t_i \leq \overline{T} \tag{6}$$

is satisfied.

Let us introduce the constant

$$b_1 = \frac{k_\varphi^2 |P||CH|^2}{c_1 p_1} \tag{7}$$

and the function

$$b_2(t) = \frac{b_1}{2|C|k_\varphi}(t - \sigma(t)) \left(e^{2|C|k_\varphi (t-\sigma(t))} - 1 \right) . \tag{8}$$

Assumption A3. *There are constants $g > 0$, $\delta \in (0,1)$ and $t_\sharp \geq g$ such that*

$$e^{-gc_1} + \int_{t-g}^{t} e^{c_1(r-t)} b_2(r) dr \leq \delta \tag{9}$$

for all $t \geq t_\sharp$.

Remark. The condition (9) is not a restriction on the constant $\overline{T}$ in (6): it can be satisfied for an arbitrarily large constant $\overline{T}$, as illustrated in Section III.

B. Observer

Let us introduce the dynamic extension:

$$\left\{ \begin{array}{rcl} \dot{\omega}(t) & = & CHz(t) + C\varphi(\omega(t)) \quad , \quad t \in [t_i, t_{i+1}) \\ \omega(t_i) & = & Cx(t_i) \quad , \quad i \in \mathbb{N} \quad , \\ \dot{z}(t) & = & Hz(t) + \varphi(\omega(t)) \quad , \quad t \geq 0 \end{array} \right. \tag{10}$$

with $i \in \mathbb{N}$, $\omega \in \mathbb{R}^q$. This dynamic extension was proposed in [7].

We are ready to state and prove the main result:

Theorem 1. *Assume that the system (2) satisfies Assumptions A1 to A3. Then the system (10) is an asymptotic observer for the system (2).*

Proof. Since the system (2) satisfies Assumptions A1 to A3, the solutions of the system (2)-(10) are well-defined and the system (2)-(10) is forward complete.

Now, let us introduce the variables:

$$e_\omega = \omega - Cx \quad , \quad e_x = z - x \, . \tag{11}$$

Elementary claculations give:

$$\left\{ \begin{array}{rcl} \dot{e}_\omega(t) & = & CHz(t) + C\varphi(\omega(t)) \\ & & -CHx(t) - C\varphi(Cx(t)) \quad , \quad \forall t \in [t_i, t_{i+1}) \, , \\ e_\omega(t_i) & = & 0 \, , \\ \dot{e}_x(t) & = & Hz(t) + \varphi(\omega(t)) - Hx(t) \\ & & -\varphi(Cx(t)) \quad , \quad \forall t \in [t_i, t_{i+1}) \end{array} \right. \tag{12}$$

for $i \in \mathbb{N}$. Thus, we have

$$\left\{ \begin{array}{rcl} \dot{e}_\omega(t) & = & CHe_x(t) + C\varphi(\omega(t)) \\ & & -C\varphi(Cx(t)) \quad , \quad \forall t \in [t_i, t_{i+1}) \, , \\ e_\omega(t_i) & = & 0 \, , \\ \dot{e}_x(t) & = & He_x(t) + \varphi(\omega(t)) \\ & & -\varphi(Cx(t)) \quad , \quad \forall t \in [t_i, t_{i+1}) \, . \end{array} \right. \tag{13}$$

Assumption A1 ensures that there is a function θ, bounded in norm by k_φ, such that

$$\left\{ \begin{array}{rcl} \dot{e}_x(t) & = & He_x(t) \\ & & +\theta(x(t), \omega(t)) e_\omega(t) \quad , \quad \forall t \in [t_i, t_{i+1}) \, , \\ \dot{e}_\omega(t) & = & C\theta(x(t), \omega(t)) e_\omega(t) \\ & & +CHe_x(t) \quad , \quad \forall t \in [t_i, t_{i+1}) \, , \\ e_\omega(t_i) & = & 0 \, . \end{array} \right. \tag{14}$$

By integrating the e_ω-subsystem of (14) and using the bound of θ, we deduce that there is a function $\zeta_1(t,s)$, bounded by $\mu(t,s) = |CH| e^{|C|k_\varphi (t-s)}$, such that

$$e_\omega(t) = \int_{t_i}^{t} \zeta_1(t,s) e_x(s) ds \quad , \quad \forall t \in [t_i, t_{i+1}) \, . \tag{15}$$

This equality and the e_x-subsystem in (14) give

$$\dot{e}_x(t) \quad = \quad He_x(t) + \int_{t_i}^{t} \zeta_2(t,s) e_x(s) ds \quad , \quad t \in [t_i, t_{i+1}) \, , \tag{16}$$

with $\zeta_2(t,s) = \theta(x(t), \omega(t)) \zeta_1(t,s)$.

Now, we analyze the stability properties of (16) by using the positive definite quadratic function:

$$V(e_x) = e_x^\top P e_x \, . \tag{17}$$

The LMI (3) ensures that the derivative of this function along the trajectories of (16) satisfies

$$\dot{V}(t) \leq -2c_1 V(e_x(t)) + \int_{t_i}^{t} 2e_x(t)^\top P \zeta_2(t,s) e_x(s) ds \, . \tag{18}$$

Since the matrix P is symmetric and positive definite, for any $a > 0$ the inequality

$$\begin{array}{l} \displaystyle\int_{t_i}^{t} 2e_x(t)^\top P \zeta_2(t,s) e_x(s) ds \leq \\ \displaystyle a \int_{t_i}^{t} e_x(t)^\top P e_x(t) ds \\ \displaystyle + \frac{1}{a} \int_{t_i}^{t} e_x(s)^\top \zeta_2(t,s)^\top P \zeta_2(t,s) e_x(s) ds \end{array} \tag{19}$$

holds. We deduce that the inequality

$$\begin{array}{rcl}\dot{V}(t) & \leq & [-2c_1 + a(t-t_i)]V(e_x(t)) \\ & & +\frac{1}{a}\displaystyle\int_{t_i}^{t} e_x(s)^\top \zeta_2(t,s)^\top P \zeta_2(t,s) e_x(s) ds\end{array} \quad (20)$$

is satisfied. Since $|\zeta_2(t,s)| \leq k_\varphi \mu(t,s)$ for all $(t,s) \in \mathbb{R}^2$, we obtain

$$\begin{array}{rcl}\dot{V}(t) & \leq & [-2c_1 + a(t-t_i)]V(e_x(t)) \\ & & +\frac{1}{a}\displaystyle\int_{t_i}^{t} k_\varphi^2 \mu(t,s)^2 |P||e_x(s)|^2 ds \\ & \leq & [-2c_1 + a(t-t_i)]V(e_x(t)) \\ & & +\frac{k_\varphi^2|P|}{ap_1}\displaystyle\int_{t_i}^{t} \mu(t,s)^2 V(e_x(s)) ds \\ & = & [-2c_1 + a(t-t_i)]V(e_x(t)) \\ & & +\frac{k_\varphi^2|P||CH|^2}{ap_1}\displaystyle\int_{t_i}^{t} e^{2|C|k_\varphi(t-s)} V(e_x(s)) ds ,\end{array} \quad (21)$$

where the second inequality above is a consequence of (4). Let $\epsilon > 0$ be any constant and let

$$a = \frac{c_1}{t - t_i + \epsilon} .$$

Then

$$\begin{array}{rcl}\dot{V}(t) & \leq & \left[-2c_1 + \frac{c_1}{t-t_i+\epsilon}(t-t_i)\right] V(e_x(t)) \\ & + & \frac{t-t_i+\epsilon}{c_1}\frac{k_\varphi^2|P||CH|^2}{p_1}\displaystyle\int_{t_i}^{t} e^{2|C|k_\varphi(t-s)} V(e_x(s)) ds \\ & \leq & -c_1 V(e_x(t)) \\ & + & \frac{t-t_i+\epsilon}{c_1}\frac{k_\varphi^2|P||CH|^2}{p_1}\displaystyle\int_{t_i}^{t} e^{2|C|k_\varphi(t-s)} V(e_x(s)) ds .\end{array} \quad (22)$$

Since ϵ is arbitrarily small, it follows that

$$\begin{array}{rcl}\dot{V}(t) & \leq & -c_1 V(e_x(t)) \\ & & +b_1(t-t_i)\displaystyle\int_{t_i}^{t} e^{2|C|k_\varphi(t-s)} V(e_x(s)) ds ,\end{array} \quad (23)$$

where b_1 is the constant defined in (7). Since $t_{i+1} - t_i \leq \overline{T}$ for all $i \in \mathbb{N}$, we have

$$\begin{array}{c}\dot{V}(t) \leq -c_1 V(e_x(t)) \\ +b_1(t-\sigma(t))\displaystyle\int_{\sigma(t)}^{t} e^{2|C|k_\varphi(t-s)} ds \sup_{s\in[t-\overline{T},t]} V(e_x(s)) .\end{array} \quad (24)$$

Thus

$$\dot{V}(t) \leq -c_1 V(e_x(t)) + b_2(t) \sup_{s\in[t-\overline{T},t]} V(e_x(s)) , \quad (25)$$

where b_2 is the function defined in (8). Now, one cannot apply Halanay's inequality [4] or Razumikhin's theorem [3, Chapt. 1] to (25) because for some instants $t \geq 0$, $b_2(t)$ may be larger than c_1. But we can apply the trajectory based approach, initiated in [10].

By integrating (25) over $[t-g, t]$, where g is the constant in Assumption A3, we obtain:

$$\begin{array}{c}V(e_x(t)) \leq e^{-gc_1} V(e_x(t-g)) \\ +\displaystyle\int_{t-g}^{t} e^{c_1(r-t)} b_2(r) dr \sup_{s\in[t-\overline{T}-g,t]} V(e_x(s))\end{array} \quad (26)$$

for all $t \geq \overline{T} + g$. It follows that

$$\begin{array}{rcl}V(e_x(t)) & \leq & \left[e^{-gc_1} + \displaystyle\int_{t-g}^{t} e^{c_1(r-t)} b_2(r) dr\right] \\ & & \times \sup_{s\in[t-\overline{T}-g,t]} V(e_x(s)) \\ & \leq & \delta \sup_{s\in[t-\overline{T}-g,t]} V(e_x(s))\end{array} \quad (27)$$

for all $t \geq \overline{T} + g$, where the last inequality is a consequence of Assumption A3. This inequality and [10, Lemma 1] allow us to conclude that $e_x(t)$ converges exponentially to the origin. Now, from (15) and (6), we deduce that $e_\omega(t)$ converges exponentially to the origin. This concludes the proof.

Remark. One can apply the main result of [11] to establish a result similar to the one of Theorem 1. But we have chosen to conclude via the trajectory based approach because it leads to Assumption A3 which is simpler than the assumption that we could derived from [11].

III. ILLUSTRATION

We illustrate Theorem 1 by applying it to the pendulum model, studied for instance in [9].

Let $\underline{T} > 0$ and $\overline{T} > \underline{T}$ be two positive numbers. Let t_i be the sequence periodic of $k > 1$, i.e. for all i, $t_i = t_{i+k}$, such that: $t_1 - t_0 = \overline{T}$ and for all $i \in \{1, ..., k-1\}$, $t_{i+1} - t_i = \underline{T}$.

Now, consider the two dimensional system:

$$\left\{\begin{array}{rcl}\dot{x}_1(t) & = & x_2(t) \\ \dot{x}_2(t) & = & \sin(x_1(t)) ,\end{array}\right. \quad (28)$$

with the output

$$y(t_i) = x_1(t_i) . \quad (29)$$

Let us check that Assumptions A1 and A2 are satisfied.

- The system (28) can be rewritten as

$$\dot{x}(t) = Hx(t) + \varphi(x_1(t)) \quad (30)$$

with

$$H = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} \quad (31)$$

and

$$\varphi(x_1) = \begin{bmatrix} 2x_1 \\ x_1 + \sin(x_1) \end{bmatrix} . \quad (32)$$

Then Assumption A1 is satisfied with $k_\varphi = 2\sqrt{2}$.

- The definition of the sequence t_i implies that Assumption A2 is satisfied.

Now, with the notation of the previous section, we can take:

$$P = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} , \quad (33)$$

and $c_1 = \frac{1}{4}$, $p_1 = \frac{9}{20}$, $p_2 = 1$. We have

$$b_2(t) = \frac{200}{3}\sqrt{2}(t-\sigma(t))\left(e^{4\sqrt{2}(t-\sigma(t))} - 1\right) . \quad (34)$$

Then for any constant $g > 0$, the function

$$\gamma(g,t) = e^{-gc_1} + \int_{t-g}^{t} e^{c_1(r-t)} b_2(r) dr \tag{35}$$

satisfies

$$\begin{aligned} \gamma(g,t) = e^{-\frac{g}{4}} \\ + \tfrac{200}{3}\sqrt{2} \int_{t-g}^{t} e^{\frac{1}{4}(r-t)} (r-\sigma(r)) \left(e^{4\sqrt{2}(r-\sigma(r))} - 1\right) dr \ . \end{aligned} \tag{36}$$

Since the sequence t_i is periodic of period k, the function $t - \sigma(t)$ is periodic of period

$$\varpi = (k-1)\underline{T} + \overline{T} \ . \tag{37}$$

Now, we choose $g = \varpi$. The fact that $t - \sigma(t)$ is periodic of period ϖ implies that the function $\gamma(\varpi, t)$ is identiquality equal to the constant

$$\begin{aligned} \Lambda = e^{-\frac{\varpi}{4}} \\ + \tfrac{200\sqrt{2}}{3} \int_{0}^{\varpi} e^{\frac{1}{4}(r-\varpi)} (r-\sigma(r)) \left(e^{4\sqrt{2}(r-\sigma(r))} - 1\right) dr \ . \end{aligned} \tag{38}$$

Let us write Λ as the sum of three terms:

$$\Lambda = e^{-\frac{\varpi}{4}} + \tfrac{200\sqrt{2}}{3} e^{-\frac{1}{4}\varpi} \zeta_a + \tfrac{200\sqrt{2}}{3} e^{-\frac{1}{4}\varpi} \zeta_b \tag{39}$$

with

$$\zeta_a = \int_0^{\overline{T}} e^{\frac{1}{4}r} (r-\sigma(r)) \left(e^{4\sqrt{2}(r-\sigma(r))} - 1\right) dr \tag{40}$$

and

$$\zeta_b = \int_{\overline{T}}^{\varpi} e^{\frac{1}{4}r} (r-\sigma(r)) \left(e^{4\sqrt{2}(r-\sigma(r))} - 1\right) dr \ . \tag{41}$$

From the definition of σ, we deduce

$$\begin{aligned} \zeta_a &= \textstyle\int_0^{\overline{T}} e^{\frac{1}{4}r} r \left(e^{4\sqrt{2}r} - 1\right) dr \\ &= \left(\tfrac{4\overline{T}}{16\sqrt{2}+1} - \tfrac{16}{277+32\sqrt{2}}\right) e^{(4\sqrt{2}+\frac{1}{4})\overline{T}} \\ &\quad + (16 - 4\overline{T}) e^{\frac{1}{4}\overline{T}} - \tfrac{4416+512\sqrt{2}}{277+32\sqrt{2}} \end{aligned} \tag{42}$$

and

$$\begin{aligned} \zeta_b &= \sum_{l=1}^{k-1} \int_{t_l}^{t_{l+1}} e^{\frac{1}{4}r} (r-\sigma(r)) \left(e^{4\sqrt{2}(r-\sigma(r))} - 1\right) dr \\ &= \sum_{l=1}^{k-1} e^{\frac{1}{4}t_l} \int_{t_l}^{t_{l+1}} e^{\frac{1}{4}(r-t_l)} (r-t_l) \left(e^{4\sqrt{2}(r-t_l)} - 1\right) dr \ . \end{aligned} \tag{43}$$

Using $t_1 = \overline{T}$ and $t_{l+1} = t_l + \underline{T}$ for all $l \in \{1, ..., k-1\}$ and the inequality $e^a - 1 \leq a e^a$ for all $a \geq 0$, we deduce that

$$\begin{aligned} \zeta_b &= e^{\frac{\overline{T}}{4}} \frac{e^{\frac{(k-1)\underline{T}}{4}} - 1}{e^{\frac{\underline{T}}{4}} - 1} \textstyle\int_0^{\underline{T}} e^{\frac{1}{4}r} r \left(e^{4\sqrt{2}r} - 1\right) dr \\ &\leq e^{\frac{\overline{T}}{4}} \frac{e^{\frac{(k-1)\underline{T}}{4}} - 1}{e^{\frac{\underline{T}}{4}} - 1} 4\sqrt{2} \textstyle\int_0^{\underline{T}} r^2 e^{\frac{1}{4}r + 4\sqrt{2}r} dr \\ &\leq \frac{4\sqrt{2} e^{\frac{k\overline{T}}{4}}}{e^{\frac{\underline{T}}{4}} - 1} \textstyle\int_0^{\underline{T}} r^2 e^{\left(\frac{1}{4}+4\sqrt{2}\right)r} dr \ . \end{aligned} \tag{44}$$

Since for all $r \in [0, \underline{T}]$, $e^{\left(\frac{1}{4}+4\sqrt{2}\right)r} \leq e^{\left(\frac{1}{4}+4\sqrt{2}\right)\underline{T}}$, the inequality

$$\zeta_b \leq \frac{4\sqrt{2} e^{\frac{k\overline{T}}{4}}}{3\left(e^{\frac{\underline{T}}{4}} - 1\right)} e^{\left(\frac{1}{4}+4\sqrt{2}\right)\underline{T}} \underline{T}^3 \tag{45}$$

is satisfied. Since $\varpi = (k-1)\underline{T} + \overline{T}$, we deduce that

$$\begin{aligned} \Lambda &\leq e^{-\frac{(k-1)\underline{T}+\overline{T}}{4}} \\ &\quad \times \Big[1 + \tfrac{200\sqrt{2}}{3} \Big(\left(\tfrac{4\overline{T}}{16\sqrt{2}+1} - \tfrac{16}{277+32\sqrt{2}}\right) e^{(4\sqrt{2}+\frac{1}{4})\overline{T}} \\ &\quad + (16 - 4\overline{T}) e^{\frac{1}{4}\overline{T}} - \tfrac{4416+512\sqrt{2}}{277+32\sqrt{2}}\Big)\Big] \\ &\quad + \tfrac{200\sqrt{2}}{3} e^{-\frac{(k-1)\underline{T}+\overline{T}}{4}} \frac{4\sqrt{2} e^{\frac{k\overline{T}}{4}}}{3\left(e^{\frac{\underline{T}}{4}} - 1\right)} e^{\left(\frac{1}{4}+4\sqrt{2}\right)\underline{T}} \underline{T}^3 \\ &\leq e^{-\frac{(k-1)\underline{T}+\overline{T}}{4}} q_a(\overline{T}) + q_b(\underline{T}) \end{aligned} \tag{46}$$

with

$$q_a(s) = 1 + \tfrac{200\sqrt{2}}{3} \xi(s) \tag{47}$$

with

$$\begin{aligned} \xi(s) &= \left(\tfrac{4s}{16\sqrt{2}+1} - \tfrac{16}{277+32\sqrt{2}}\right) e^{(4\sqrt{2}+\frac{1}{4})s} \\ &\quad + (16 - 4s) e^{\frac{1}{4}s} - \tfrac{4416+512\sqrt{2}}{277+32\sqrt{2}} \end{aligned} \tag{48}$$

and

$$q_b(s) = \frac{1600 e^{\frac{1+16\sqrt{2}}{4}s}}{9\left(e^{\frac{s}{4}} - 1\right)} s^3 \ . \tag{49}$$

Now, let $\overline{T}$ be any positive real number. Let us choose $\underline{T} > 0$ such that

$$q_b(\underline{T}) \leq \frac{1}{4} \ . \tag{50}$$

Next, one can choose an integer k such that

$$e^{-\frac{(k-1)\underline{T}+\overline{T}}{4}} q_a(\overline{T}) \leq \frac{1}{4} \ . \tag{51}$$

These inqualities and (46) give:

$$\Lambda \leq \frac{1}{2} \ , \tag{52}$$

for all $t \geq 0$. Thus assumption A3 is satisfied. We conclude that Theorem 1 applies. We conclude that, with the constants we have selected, the solutions of

$$\left\{ \begin{array}{rcll} \dot{\omega}(t) &=& -2z_1(t) + z_2(t) + 2\omega(t) \quad , & t \in [t_i, t_{i+1}) \\ \omega(t_i) &=& x_1(t_i) & \\ \dot{z}_1(t) &=& -2z_1(t) + z_2(t) + 2\omega(t) & \\ \dot{z}_2(t) &=& -z_1(t) + \omega(t) + \sin(\omega(t)) & \end{array} \right. \tag{53}$$

asymptotically converge to the solution of the system (28).

Remark. In this example, $\overline{T}$ can take arbitrarily large values. This is not allowed by the stability conditions given in [9] or [7].

IV. Conclusion

In this paper, for a family of nonlinear systems, we have relaxed the assumption on the largest allowable sampling interval that is imposed in [7] by establishing the convergence of the proposed observer to the studied system via the technique called trajectory based approach [10].

Our result can be extended to many other cases, which include time-varying systems with delay in the spirit of what is done in [8] and [5], interconnected sub-observers proposed in [12] and in [6] and ordinary differential equations in interconnection with Partial Differential Equations.

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