A Net Neutrality Perspective for Content Down-Streaming over the Internet

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Chapter 15

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15.1 Introduction

The *Network Neutrality* issue has been at the center of debate worldwide lately. Some countries have established laws so that principles of network neutrality are respected. Among the questions that have been discussed in these debates there is whether to allow agreements between service and content providers, i.e. to allow some preferential treatment by an operator to traffic from some providers (identity-based discrimination).

Network neutrality is an approach of providing network access without unfair discrimination between applications, nor between content, nor between the specific source of the traffic. Hahn and Wallsten [127] wrote that net neutrality "usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users." there are two applications or services or providers that require the same network resources, and one is offered a better quality of service (delays, speed, etc.) or is cheaper to access, then there is discrimination. Our study in this paper is related to the latter point but taking into account also the quality of service for the customers. Historically, the neutrality of the access to the Internet has characterized the first steps of the development of the Internet and much of the industrial activity that uses the Internet. Alternative non-neutral approaches have been recently pushed forward by Internet Service Providers (ISPs), content providers (CPs) and by equipment providers (EPs). Deviating from its original neutral character may have far-reaching consequences on the whole e-economy and on the society in which the Internet has become so central. This has pushed many countries to take regulation actions to determinate the future characteristics of the Internet and some of which have been followed by legislation on the matter. A key step in the policy making has been the launching of public consultations in various countries (USA, UK, France and others) as well as at the EU level. All society sectors have been invited to answer these consultations. The global objectives in these consultations are related to this net neutrality debate and have the following two major goals:

- Contribute to the network neutrality debate in proposing a new economic model to compare the expected outcome of non-neutral approaches with neutral ones in terms of the quality of service offered.

- Study competition aspects related to network neutrality and particularly, the exclusive contracts between CPs with ISPs.

Nowadays, the users have access to several CPs that provide the same content. For example, CPs like Netflix®, Hulu®, M-GO, etc. provide video content and CPs like Deezer®, iTunes® and Spotify® provide music content. Also, the CPs have different pricing policies such as flat-rate or pay-as-you-go, or options like buy a song or a full Long Play (LP). Our goal in this paper is to analyze the impact of non-neutral pricing policies introduced through agreements betweens the ISPs and the CPs on the network economic system. In fact, the decision about from which CP the users request contents is based on two main features: **price** and **quality**.

In the Internet, the access price and the content price are set by the ISPs and CPs, respectively, and access prices are set independently of the content price. In our framework, CPs competitively set content price to maximize their profit. Recently, CPs and ISPs introduced a new pricing policy in order to play a more important role in this techno-economical market. The pricing policy is based on the exclusive contracts between the ISPs and the CPs. In such contracts, the subscribers get exclusive preferential access to the content of a CP that is in agreement with the ISP which provides them last mile connectivity. Such pricing policies induces a non-neutral Internet, as the end users have different access charges for the same content from the same CP; i.e., such pricing schemes lead to identity based discrimination. A particular example of such non-neutral pricing behavior happened recently in Europe. Orange® is a French ISP, and Deezer® is a French music streaming service provider (a CP). According to the Financial Times[1]: "As part of the deal with Deezer®, Orange® will make available a special mobile-only tariff for pay-monthly customers, to avoid the $9.99 standalone cost of Deezers top package." Therefore, customers with an Internet subscription with Orange will have preferential offer for listening music on the website Deezer®. This type of non-neutral collusion between a CP and an ISP arises in streaming services offered over the Internet, which accounts for half of the global internet traffic (see Figure 2). Another example, in the USA, is related to Apps/softwares where several CPs (if we see an App as a content) are also competing over the Internet. In fact, Google® has an agreement with Verizon®, a wireless ISP. This partnership is expressed in the form of three free user accounts for Google Apps[2] to the Verizon® customers.

Apart from the revenues from selling content, the CPs earn revenue also through advertisements, and the amount of advertisement revenue depends on the level of activity or the demand for their contents. The latter case is related to the quality of the content provided by a CP, which is assumed to be related to the number of subscribers asking/downloading contents. We consider this interaction between subscribers as a routing game framework. Therefore, each Internet user decides the way to split their demand among several content providers. Evaluating the quality of service (or experience) in communication networks is not an easy task. Many papers have focused on this objective, and evaluation of the quality perceived by customers in a network neutrality model is non-trivial. Our approach considers a routing game framework for modeling the

[1]Tim Bradshaw, *Deezer takes on Spotify with Orange deal*, ft.com, September 7, 2011

[2]Google Apps for Verizon: *Google Apps for Business now available for Verizon customers*, 01/24/2011 posted by Monte Beck, Vice President of Small Business Marketing, Verizon.

Figure 15.1: The large share of data flow to internet users from CP streaming service like Netflix and YouTube [2].

interactions between the subscribers downstream flows. Routing games provide a natural framework to model interactions among the users and characterize the quality of service perceived by them at different CPs as a function of the content price.

We model this complex interacting system within a two-sided market framework as a multistage game model composed of a congestion game at the lower level and a noncooperative pricing game at the upper level. It is natural to expect that exclusive contracts between the CPs and the ISPs can have different impacts on the equilibrium of the multistage game and therefore on the costs and profits of the different players. We would like to analyze these aspects of non-neutrality through a rigorous mathematical framework. We would also like to analyze the collusion decision for a CP (to collude or to stay independent) and also take into account the revenue generated by advertisements.

In this paper, we study the impact of a particular feature of the non-neutrality debate that arises in the relations between service and content providers, i.e. the possibility that an ISP or CP give preference, in terms of access price or content price, to certain subscribers. In some industries, laws against such kind of vertical monopolies are enforced. In some cases, companies are obliged to split their activity into separate specialized companies; this was the case of railways companies in Europe which were obliged to separate their rail infrastructure and the service part of the activity which concerns public transportation by trains. In contrast, in the telecom market, the same company may propose both the networking and content services, or similarly an ISP and a CP can make a *collusion.* This paper studies the implication of such economic relationships between providers on the Internet ecosystem. Specifically, we will address the question: Is a *Pricing agreement/collusion* between an ISP and a CP good for subscribers? We suggest here a novel point of view of a *pricing agreement/collusion*: an issue actively discussed in the ongoing net neutrality debate. Usually in the network neutrality debate the problem of agreement or disagreement between ISP and CP is a vertical foreclosure (Degradation of traffic) [74]. This type of problem has been observed in France between Free (a French ISP) and Google[3]. In our framework, if a CP and an ISP have a *pricing agreement*, then a subscriber of the ISP mentioned above does not have to pay for the access to this CP's content.

Our main contribution in this paper is to model and analyze the effect of new pricing policies proposed by some of the ISPs and the CPs that enter into exclusive contracts. We first consider ISPs as passive players that connect CPs to end users in a two-sided market model, and study the behavior of CPs and the end users in the new pricing regime using a multi-stage game framework. The subscribers interact through their downstream traffic carrying their content. Their behavior is influenced by the congestion that occurs at the CP-side in the access link between CPs and ISPs. The CPs decisions are influenced by the "price war" as most of them sell the same kind of contents (multimedia contents as movies, LPs, popular TV shows, etc.), and also to attract more users as this will increase their advertisements revenues. A non-neutral aspect of the access to contents results from the exclusive contracts and therefore we are interested to study the impact of such contracts between CPs and ISPs on the equilibrium performances of the market.

The main contributions of this work are:

1. In the two-sided market framework, we consider a hierarchical game in which the higher level is a normal form non-cooperative game between the CPs (content price), and at the lower level, a non-cooperative (non-atomic) routing game between the subscribers (user set demand for content).

2. In the hierarchical game, we first study the sub-game perfect equilibria (SPE) of the routing game between subscribers. Depending on the pricing policies of the CPs and the perceived congestions levels at each CPs, the subscribers compete to minimize the overall delay (download time) and the

[3] "An ad-block shock France vs Google ", The Economist, January 12, 2013

total cost for the content. Results have been obtained and are described in Section 4. We look for existence of a symmetric SPE by studying best-response functions.

3. Based on the the SPE of users demands, we study a non-cooperative game between the CPs and determine their content price at equilibrium. In our multi-stage game framework, the CPs compete through their content prices and aim to maximize their revenues from content and advertisements.

4. Assuming that each CP is in collusion with an ISP (inducing a preferential tariff for his content to some specific subscribers), we study the new equilibrium of the multi-stage game. The analysis process is based on backward induction: we first determine the SPE for the subscriber game, and then the Nash equilibrium for the CPs game.

5. Finally, we study another decision step for the CPs– whether or not to collude with an ISP. CPs decide first whether to collude with an ISP or not, and then set the content price. Such a decision by a CP impacts other CPs' decisions to collude or not and also their content prices.

15.2 Related works

Several recent papers in the literature deal with game-theoretic models for network neutrality analysis. The survey article [2] provides summary of various issues discussed in the network neutrality debate. The authors describe the models used to analyze various issues in the net neutrality debate and compare the results. In the following section, we discuss the literature relevant to the proposed research.

The two-sided model proposed in [94] investigates the effect of network neutrality regulation in both monopoly and duopoly setting between ISPs. The authors show that neutrality increases social welfare in the duopoly case. However, investment decisions and congestion effects are not taken into account into the model. Our two-sided market model is similar to [193] by considering fixed number of CPs and ISPs. The authors compare the return on investment under one-sided or two-sided pricing, and they show that this amount is comparable. The congestion effect is not considered in this paper. Investment mechanisms in a two-sided market has been proposed in [200]. The model consists of two interconnected ISPs represented as profit maximizing firms that choose quality investment levels and then compete in prices for both CPs and consumers. The authors consider a large number of CPs and consumers. The revenue of the CPs come from advertising only. In our framework, we also consider the revenue generated form content selling, which is important for multimedia streaming CP. The game-theoretic model proposed is a 6-stage game in which several competitions occur at each level between CPs, ISPs and consumers.

The effect of investments on the quality of services (QoS) is studied in [200]. The authors consider a model where the interactions between CPs and also between consumers come from their choice of ISP, that impacts also their quality of service (QoS). The latter depends on the congestion at each ISP, at the CP side, which is a function of the mass of consumers and CPs connected to it. The investment decision of each ISP determines the QoS in a deterministic manner. The ISP receives payment from both sides– CPs and consumers. The authors consider two scenarios: neutral and non-neutral. They show that investments are higher in the non-neutral regime because it is easier to extract revenue through appropriate CP pricing. Interestingly, the congestion effect is taken into account at the CP level. Specifically, the authors compute the value of the content by dividing the quality level associated to each CP by the mass of CPs that connect to the same platform in order to incorporate congestion effects: more CPs in a platform generate more congestion, reducing value. Then, from this point of view, this paper deals also with a congestion feature that impacts the subscriber behavior as in our framework. But, the important difference from our setting is about the interaction model between subscribers which is based on a routing game.

Models from queuing theory have been used to analyze the effect of investments in [59]. The congestion effect is taken into account as the average sojourn time in an M/M/1 queue, and the investment decision of the ISP affects directly the service rate of this queue. In [10], the authors study the discrimination effect at the service level. Particularly, the authors look at the effects of net neutrality regulation on the investment incentives of ISPs and CPs. The QoS based on a congestion model is also expressed by an M/M/1 queue. Their results are ambiguous, but key effects are exposed. The difficulty is that the average sojourn time in an M/M/1 queue is not linear and then closed-form solutions are usually difficult to obtain. Another model, proposed in [167], is based on a queueing congestion model. The model is related to content analysis, and the authors show that strategic quality degradation and non-neutral ISP reduces content variety. Routing game based model is used in

[17] to study a non-cooperative game between subscribers. Subscribers play a non atomic selfish routing game and CPs control both flows and prices, while in our proposal CPs control only prices and subscribers determine the source of the traffic they wish to download from. Another preliminary work [158] is related to exploration of the effects of content-specific pricing, including multiple CPs providing different types of content. But the competition between providers is not considered. The authors have analyzed various theoretical aspect of collusions in routing games [25] and in nonneutral networks [24] [**?**]. In [25], the authors studied the effect of collusions in routing games and extended the performance metric *price of collusion* [139] to evaluate the effect of collusion on both the colluding and non-colluidng players.

15.3 Multistage game for two-sided market framework

The general mathematical framework is a multistage game composed of several non-cooperative games at each level. The task description provides technical details on the type of solution concept that will be studied and how to get it. We also describe the role of each player in the multi-stage game coupled with their actions and utilities.

15.3.1 Model

We consider a general economic model of content distribution over the Internet as a two-sided market with competition [29]. In fact, the ISPs provides a platform connecting end-users or subscribers to the Internet Content Providers (CPs). We assume that several CPs are able to distribute the same global content over the different ISPs. If we think about music streaming, most of the same artists are on the different CP like iTunes, Deezer, Spotify, etc. The same remark applies to movies streaming CPs.

We consider a fixed number M of CPs. The streaming traffic is carried through the network by high level ISPs which have direct links to all CP. In practice, when a subscriber requests a specific content from a CP, this streaming flow needs to traverse several different networks. This complex transit of traffic between networks is governed by a variety of agreements between different tier providers. We consider the local monopolistic ISP that provides the last mile connection service to CPs. This type of model assumption is usually considered in two-sided market models like [10] and [94].

Finally, those ISPs are connected to subscribers (as a mass). We consider in our system N different ISPs. Each subscriber is connected to a single ISP and cannot access directly any CP. As each subscriber is connected to a unique ISP, we make the abuse of identifying a subscriber with its ISP. Then we talk about "subscriber n" instead of "subscriber connected to ISP n". We consider in our study the download traffic; contents on the Internet transit mostly from servers to clients (video-on-demand, movie broadcast, etc). The source of each content flow is a CP and the corresponding destination is a subscriber.

Figure 15.2: Traffic flows in our two-sided market framework. Congestion occurs at the CP-side level in the last-mile connection service to ISP.

Requests about contents are generated from each subscriber n (i.e. subscriber connected to ISP n and reached all CP m. At each CP m, any request from a subscriber n induces a traffic rate (music or videos streaming traffic, downloading, etc.) $x_n^m > 0$, which is therefore aggregated and sent to the subscriber mass connected to ISP n. We assume that the total traffic flow from ISP n to its subscriber is $\phi_n = \sum_{m=1}^{M} x_n^m$. This total traffic called also total demand ϕ_n, is an average value of the total amount of traffic downloading to the mass of subscribers connected to ISP n. The subscriber decision about requesting content from one CP to the other depends on the quality of service, which is expressed here in terms of CP-side congestion. Indeed, on the subscriber side, the ISP can dimension correctly the downstream (from ISP to subscribers) network as the total traffic is fixed. But, at the CP side, the total upstream (from CP to ISP) traffic is not fixed, it is equal to $\sum_{n=1}^{N} x_n^m$ and depends on the number of requests to this CP. Then, we model the CP-side congestion effect coming from the interaction of the demand of all the subscribers to this CP as a non-cooperative routing game. In fact, the congestion level in a link depends on the total traffic trough this link. For example on Figure **??**, we observed that the link between CP M and ISP M is more congested compared to the access links of other CPs. In fact, all subscribers connected to ISP N request content to CP M and not any other CP. The traffic rate that goes trough this link is important. Whereas the access link between CP 1 and ISP 1 is less congested as its traffic is composed only from a small proportion of the subscriber 1 total traffic ϕ_1. Then, the first modeling technique that will be used in this work is ***routing game*** which is related to algorithmic game theory [**?**] and Wardrop equilibrium concept [285]. Indeed, we consider a noncooperative routing game where the decision of subscriber connected to ISP n, is how to split his download traffic ϕ from all the CP, i.e. the decision variables for subscriber connected to ISP n is the vector $x_n = (x_n^1, x_n^2, \ldots, x_n^M)$.

Subscriber costs: We denote by p^m the charge, per unit of traffic, that a subscriber has to pay in order to download traffic from CP m. Then, for a traffic quantity x_n^m, the subscriber n has to pay $x_n^m p^m$ to the CP m. We consider that a subscriber prefers to download his content from the less crowded CP, due to congestion cost effect at the CP link. This congestion cost depends on the total download traffic generated at each CP m, that is $\sum_n x_n^m$. Let $D^m : \mathbb{R}^+ \mapsto \mathbb{R}^+$ be the congestion cost function at CP m (exactly the congestion is suffered on the access link between CP m and its ISP) which we assume to be convex and increasing. The congestion cost perceived by a subscriber n who downloads a traffic rate x_n^m from CP m is equal to $x_n^m \cdot D^m(\sum_n x_n^m)$. Then, the total cost (content price + congestion cost) for a subscriber connected to ISP n is given by:

$$C_n(x_n, x_{-n}, p) = \sum_m x_n^m \left[D^m(\sum_n x_n^m) + p^m \right], \tag{15.1}$$

where $x_n = \{x_n^1, \ldots, x_n^M\}$ is the decision vector for the subscriber connected to ISP n, x_{-n} is the decision vectors of all the other subscribers connected to other ISPs and $p = \{p^1, \ldots, p^M\}$ is the content price vector of all the CP. This latter price is expressed per unit rate. Each subscriber n will minimize his cost function under his demand constraint:

$$\min_{x_n} C_n(x_n, x_{-n}, p) \quad \text{such that} \quad \sum_{m=1}^{N} x_n^m = \phi_n.$$

CP Profits: The revenue of CP m is defined by:

$$\Pi(x, p^m, p^{-m}) = p^m \sum_n x_n^m,$$

where x_n^m is the traffic flow from CP m to ISP n, p^m is the price charged by CP m, p^{-m} is the price vector of all the other CP. Each CP determines his content price p^m in order to maximize his profit, taking into account the traffic flow (demand) requested by subscribers. Then, this problem can be solved by considering a ***multistage game***.

Multistage game: The players in our two-sided framework are the CPs and the subscribers. The multistage game we study consists of the following stages:

1. Content prices decisions: All CP determine simultaneously their content price p_m.

2. Demand splitting decisions: All subscriber determine simultaneously their downloading streaming rate x_n^m from each CP m.

We can see that for the moment the ISP plays the role of a platform between the CPs and the subscribers, and that they are inactive in the economic market studied. We plan to introduce active ISP as decision makers in the next step of this proposal. We solve this multistage game by considering sub game perfect Nash equilibrium and we use a backward induction technique. We assume that the CPs update/change their prices at a slower timescale compared to the decision request of subscribers. Then, the routing game between the subscribers is solved by considering the content prices as fixed. That is why we consider the previous stages ordering. Moreover, the multistage game is closely related to an Equilibrium Problem with Equilibrium Constraints (EPEC) [252] and can be written as follows:

$$\forall m \in \{1, \ldots, M\}, \quad \max_{p^m} \; p^m \sum_n \underline{x}_n^m(\mathbf{p}), \quad \text{(Leader)}$$

such that

$$\forall n \in \{1, \ldots, N\}, \quad \underline{x}_n^m(\mathbf{p}) \in \arg\min_{x_n} \; C_n(x_n, x_{-n}, p). \quad \text{(Follower)}$$

Generally speaking, this type of multistage game is not trivial to study but closed-form solutions can be obtained when the game is symmetric. We thus assume that the decision variable of each CP m is in an interval, i.e., $p^m \in [0, p_{\max}]$. The system is totally symmetric, in the sense that the quantity of traffic ϕ is the same for all subscribers n, and congestion cost functions do not depend on m. In all our mathematical analysis we consider a linear congestion cost function $D(x) = ax$ as in [17]. Based on this symmetry property of the game, we can use results from [3] and assume the existence of a symmetric equilibrium for our multistage game. First, this symmetric assumption can be justified by the fact that in a large network, we can approximate the behavior of many subscribers with only one subscriber which has the average characteristics of all the subscribers. Secondly, this assumption allows us to obtain explicit formulations of the equilibrium of our multistage game with a routing game as constraint.

15.3.2 Neutral scenario

The neutral scenario denotes the case in which each CP decides independently its content price. In other words, ISPs do not influence the content price decision of the CP. Therefore, there is no price discrimination between subscribers in order for them to access to the content of different CPs. In this case, we have the following preliminary result.

Theorem 31. *For all CP m, there exists a unique symmetric equilibrium $(x_n^m, p^m) = (\underline{x}, \underline{p})$ of our multistage game, given by $\forall (n, m) \in \{1, \ldots, N\} \times \{1, \ldots, M\}$:*

$$\underline{x}_n^m = \frac{\phi}{M} \quad \text{and} \quad \underline{p} = (N-1)\phi a.$$

We observe that due to the competition between the subscribers and also between the CPs, the total downstream rate of each subscriber is equally splitte between all the CPs. Also, based on this result, we are able to determine the cost of each subscriber and the revenue of each CP at the equilibrium situation of our multistage game. In fact, this proposition gives the equilibrium prices and the value of the total traffic generated by each CP at equilibrium. Note that we have uniqueness of this total traffic at equilibrium:

$$\forall m, \quad \sum_{n=1}^{N} \underline{x}_n^m = \frac{N\phi}{M}.$$

The cost for a subscriber connected to CP n at the equilibrium is given by:

$$C_n(\underline{x}, \underline{p}) = \phi^2 a(N - 1 + \frac{N}{M}).$$

The revenue for any CP m is:

$$\Pi_m(\underline{x}, \underline{p}) = \frac{(N-1)N}{M}\phi^2 a.$$

15.3.3 Non-neutral scenario

We consider in this second scenario that each ISP n makes an agreement with a CP. Then, we assume that the number of ISP is equal to the number of CP, i.e. $M = N$. In order to simplify the notations, n is the index of the CP which has an agreement with ISP n. These agreements or collusions, imply that the content price p^n is equal to 0 for the traffic generated from the CP n to the ISP n. Then, the total cost for the subscriber connected to ISP n becomes:

$$C_n^v(x_n, x_{-n}, p) = \sum_{m \neq n} x_n^m \left[D(\sum_n x_n^m) + p^m \right] + x_n^n D(\sum_n x_n^n),$$

where $\mathbf{p}$ is the vector (size $N-1$) of the content prices for all CP except n. The revenue of the CP m is now:

$$\Pi^m(p^m, p^{-m}) = p^m \sum_{n \neq m} \underline{x}_n^m(p).$$

Then, we have now a similar multistage game in which the cost function of the subscribers is slightly modified. The sub game perfect equilibrium solution of the routing game between the subscribers is no more symmetric and we define the following variables. Let y_n^n be the quantity of traffic requested by a subscriber connected to an ISP n from the CP n associated to its ISP, and y_n^m the download traffic requested by a subscriber connected to an ISP n from CP m, with $m \neq n$.

Theorem 32. *For all CP m, there exists for all $(i, n, m) \in \{1, \ldots, I\} \times \{1, \ldots, N\}^2$ a symmetric equilibrium $(y_n^m, y_n^n, p^m) = (\underline{z}, \underline{y}, \underline{q})$, which is given by:*

$$\underline{q} = a\phi \frac{(N+1)}{3N-1}, \quad \underline{z} = \frac{\phi}{N}(\frac{2N-2}{3N-1}) \quad \textit{and}$$

$$\underline{y} = \frac{\phi}{N}(1 + \frac{(N-1)(N+1)}{3N-1}).$$

We observe that a subscriber does not download all its content from the CP which has an agreement with its ISP. In fact, part of its downstream traffic will be from other CPs. We are now able to express the cost for the subscriber connected to ISP n, at the equilibrium, is given by:

$$C_n^v(\underline{y}, \underline{z}, \underline{q}) = \phi^2 a + 2a\phi^2 (\frac{N-1}{3N-1})^2 (\frac{N+1}{N}).$$

The reward for CP m at the equilibrium is:

$$\Pi_m^v(\underline{q}, \underline{y}, \underline{z}) = 2a\phi^2 (\frac{N-1}{3N-1})^2 (\frac{N+1}{N}).$$

The first remark is that in our context of interaction between subscribers, their optimal decision implies that each subscriber downloads part of his demand from other CPs than the the privileged one, even if they have to pay for. Another remark is that the download traffic from the privileged CP, $\underline{y}$, has a bounded limit of $\frac{\phi}{3}$ when the number of provider N tends to infinite. In the neutral context, all the download rates converge to 0. Thus, it means that by making agreement or collusion with an ISP, each CP has a minimum quantity guarantee of traffic to send. It is an important result for dimensioning CP network infrastructure and also when considering the advertising incomes. This part will be considered in the future works section.

15.3.4 Comparison between neutral and non-neutral

The previous mathematical results help us to study the effects of the agreement between CPs and ISPs. We are able to determine which scenario (the neutral or non-neutral one) induces lower costs for subscribers. The next theorem proves that even if the neutral scenario implies symmetry and free will for the subscribers about the CP, as they pay the same price for accessing the content, the non-neutral scenario is even better for the subscribers.

Theorem 33. *Let us assume that $M = N$. At equilibrium, the agreement between service and content providers is costless for the subscribers, i.e., for all subscribers connected to ISP n we have:*

$$C_n^v(\underline{y}, \underline{z}, \underline{q}) < C_n(\underline{x}, \underline{p}). \tag{15.2}$$

This result seems counterintuitive, in fact, agreements between companies are usually not allowed by government of several countries in order to protect consumers. However, according to our theorem, in our setting, such agreements between CPs and ISPs are not harmful for the subscribers and are even better in terms of costs.

15.3.5 A competition over agreement between CPs

In the past section we have imposed economic agreements between ISP and CP. The scenario was the following, CPs set prices that subscribers have to pay, but not the economic topology of the network (in other words if there are pricing agreements or not between CPs and ISPs). We propose now to let each CP decides by himself to make an agreement (strategy A) with an ISP or not (strategy NA). We prove that the strategy vector $(NA, \ldots, NA)$ is a pure Nash equilibrium. In order to prove this result, we first assume that all the subscribers except one, called n', play NA. We will prove that n' has no interest in not playing NA. However to do so, we need to compute the utility of this CP n' if he plays A. The next proposition gives his utility when he plays A and the others play NA.

Proposition 27. *The utility of the CP n' when he makes an agreement and all the other CP do not, is given by:*

$$\Pi_n'(\underline{p}_{n'}, \underline{p}_{-n'}, \underline{x}_{m'}, \underline{x}_m) = \frac{a\phi(N+1)\left(2N^2+N-1\right)}{(4\,N^3-5\,N^2+2\,N-1)}$$
$$+\frac{a\phi(N-1)\left(2\,N^2+N-1\right)}{(N+1)(4\,N^3-5\,N^2+2\,N-1)} \times \left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1} - \frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)$$

With the help of the previous proposition, we have the next theorem:

Theorem 34. *If $\phi > 1$ the utility of the player m is lower when he plays NA than when he plays A, i.e.*

$$\Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) < \Pi_m(\underline{p}, \underline{x}).$$

This first result implies that $(NA, \ldots, NA)$ is a nash equilibrium.

This last theorem reduces the importance of the agreement. Indeed, we recall that according to theorem 33, agreements are beneficial for subscribers. However, the last theorem 35 proves that CPs are not interested in making such agreements in a selfish situation. Therefore, policy makers may want to force agreements by, for example, designing new rules and then reduce the cost for the subscribers.

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Proposition 28. *The utility of the CP n' when he makes an agreement and all the other CP do not, is given by:*

$$\Pi'_n(\underline{p}_{n'}, \underline{p}_{-n'}, \underline{x}_{m'}, \underline{x}_m) = \frac{a\phi(N+1)\left(2N^2+N-1\right)}{(4\,N^3-5\,N^2+2\,N-1)}$$
$$+\frac{a\phi(N-1)\left(2\,N^2+N-1\right)}{(N+1)(4\,N^3-5\,N^2+2\,N-1)} \times \left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1} - \frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)$$

With the help of the previous proposition, we have the next theorem:

Theorem 35. *If $\phi > 1$ the utility of the player m is lower when he plays NA than when he plays A.*

$$\Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) < \Pi_m(\underline{p}, \underline{x})$$

This first result implies that $(NA, \ldots, NA)$ is a nash equilibrium.

The previous theorem minimize the importance of aggrement. Indeed, we recall that according to theorem 3, aggrements is more interting for subscribers that a neutral situation. However, theorem 4 prove that aggrements it is not a good scenario for the CP and so finally it emeges that the neutral situation will emerge.

15.5 Conclusions and perspectives

From the research that has been carried out in this chapter, it is possible to conclude that agreement between CPs and IPSs is not an harmful situation for the subscribers. To reach this conclusion, we propose a theoretical model that captures two scenarios (depending on the parameters), a neutral and a non-neutral one. We were able to find mathematical closed form expression of the Nash equilibrium in each scenario and we prove mathematically that an agreement between an ISP and a CP induces a lower cost for subscribers than a neutral situation when such agreements are not possible. The second outcome of our chapter is that CPs will not make agreements selfishly, and so incentives from the government is necessary in order to improve the welfare of the subscribers. We prove that the situation where each CP does not make an agreement is a pure Nash Equilibrium. The findings of the chapter suggest that our approach and results could be useful for government and policy makers for the design of new regulation rules in the telecom market. More research into the generalization of our model is still necessary before obtaining a definitive answer to whether or not agreements between CPs and IPSs are beneficial for the subscribers. A first generalization could be to consider more general topology and asymmetric scenarios.

APPENDIX

Proof of theorem 31

First let

$$L_n(\mathbf{x_n}, \mathbf{x_{-n}}, \mathbf{p}, \lambda_n) =$$

$$\sum_m x_n^m \left[D^m(\sum_n x_n^m) + p^m \right] - \lambda_n(\sum_m x_n^m - \phi)$$

the Lagrangian function associated to the cost function $C_n(\cdot)$. We look for a symmetric equilibrium between the CPs, i.e. for the noncooperative pricing game at the upper layer. Then we assume that CPs $m' \in \{1, \ldots, M\} - \{m\}$ play q and one CP, say m, plays p^m. We want to find some q where the best reply of CP m against q is q. First, we have to determine the equilibrium flows between the subscribers, depending on those prices,i.e. $\underline{\mathbf{x}}(p^m, q)$ for all p^m and q. We look for $\underline{\mathbf{x}}(p^m, q)$, a solution of the following system:

$$\begin{cases} \dfrac{\partial L_n}{\partial x_n^m}(\mathbf{x_n}, \mathbf{x_{,-n}}, p^m, q, \lambda_n) = 0 \\ \sum_m x_n^m = \phi, \ \forall (n, m) \end{cases} .$$

This game has a strong symmetric property as all subscribers are interchangeable. Then, we can restrict ourselves to two strategies x and y where x is a request for CP m' and y is for CP m. This induces a great simplification in the analysis of our complex hierarchical game. Thanks to [3], the previous system is equivalent to the following one:

$$\begin{cases} x\dfrac{\partial D}{\partial x}(Nx) + D(Nx) + q - \lambda = 0 \\ \\ y\dfrac{\partial D}{\partial y}(Ny) + D(Ny) + p^m - \lambda = 0 \\ \\ (M-1)x + y = \phi. \end{cases}$$

We denote by $\underline{x}$ and $\underline{y}$ a solution of this system. We consider the linear cost function $D(x) = ax$ and thus we have to solve the following linear system:

$$\begin{cases} ax + a(Nx) + q - \lambda = 0 \\ ay + a(Ny) + p^m - \lambda = 0 \\ (M-1)x + y = \phi. \end{cases}$$

If $\underline{x} < 0$ (or $\underline{y} < 0$) then $\underline{x} = 0$ (or $\underline{y} = 0$). And if $\underline{x} > \phi$ (or $\underline{y} > \phi$) then $\underline{x} = \phi$ (or $\underline{y} = \phi$).

Let's now consider the CP m and how it's going to optimize its revenue, which is the function $R^m(p^m, q) = p^m \underline{y} N$. Its best reply against all other CPs that play q is given by p^m solution of $\frac{\partial R^m}{\partial p^m}(p^m, q) = 0$. We have to find a certain p which is a solution of $\frac{\partial R^m}{\partial p^m}(p, p) = 0$. We denote this equilibrium by $\underline{p}$. Considering the linear cost function, we obtain:

$$\underline{p} = (N-1)\phi a.$$

If $\underline{p} > p_{max}$ then $\underline{p} = p_{max}$. We have a particular interest in the case where $p_{max} > (N-1)\phi a$. Then the equilibrium flow from CP n to ISP m is $\underline{x}_n^m = \frac{\phi}{M}$.

Proof of theorem 32
In order to compute an equilibrium for the game with agreements, we can use the method described previously.
We consider the following Lagrangian function:

$$L_n^v(\mathbf{x_n}, \mathbf{x_{-n}}, \mathbf{p}, \lambda_n) =$$

$$\sum_{m \neq n} x_n^m \left[D^m(\sum_n x_n^m) + p^m \right] + x_n^n [D(\underset{n}{n}) x_n^n] - \lambda_n (\sum_m x_n^m - \phi).$$

As previously, we assume that CPs $m' \in \{1, \ldots, N\} - \{m\}$ play q and the CP m plays p^m. Now again there are several symmetries: we can see that there are two types of subscribers. Each subscriber of each type are interchangeable. Type 1 is subscribers with an agreement with CP m. Type 2 is subscribers without an agreement with CP m. The variables of Type 1 subscribers are:

- x is the flow from CP m',
- y is the flow from CP m.

The variables of Type 2 subscribers are:

- u is the flow from CP m,
- v is the flow from the CP with has an agreement,
- w is the flow from all the other CP except CP m and CP with the agreement.

Due to symmetry, the system is equivalent to the following one with 5 variables (x, y, u, v, w):

$$\begin{cases} x\dfrac{\partial D}{\partial x}(x+v+(N-2)w) + D(x+v+(N-2)w) + q - \lambda_1 = 0 \\ v\dfrac{\partial D}{\partial v}(x+v+(N-2)w) + D(x+v+(N-2)w) - \lambda_2 = 0 \\ w\dfrac{\partial D}{\partial w}(x+v+(N-2)w) + D(x+v+(N-2)w) + q - \lambda_2 = 0 \\ y\dfrac{\partial D}{\partial y}(y+(N-1)u) + D(y+(N-1)u) - \lambda_1 = 0 \\ u\dfrac{\partial D}{\partial u}(y+(N-1)u) + D(y+(N-1)u) + p^m - \lambda_2 = 0 \\ (M-1)x + y = \phi. \\ (M-2)w + u + v = \phi \end{cases}$$

Considering the linear cost function $D(x) = ax$. We have to solve the linear system that are given below:

$$\begin{cases} ax + a(x + v + (N-2)w) + q - \lambda_1 = 0 \\ av + a(x + v + (N-2)w) - \lambda_2 = 0 \\ aw + a(x + v + (N-2)w) + q - \lambda_2 = 0 \\ ay + a(y + (N-1)u) - \lambda_1 = 0 \\ au + a(y + (N-1)u) + p^m - \lambda_2 = 0 \\ (M-1)x + y = \phi. \\ (M-2)w + u + v = \phi \end{cases}$$

We denote $\underline{x}$, $\underline{y}$, $\underline{u}$, $\underline{v}$, $\underline{w}$ the solution of the previous system. The revenue of CP m is $R_v^m(p^m, q) = p^m \times (N-1)\underline{u}$. To compute the equilibrium price we need to find p which solves $\frac{\partial R_v^m}{\partial p^m}(p,p) = 0$. If $\underline{q} > p_{max}$ then $\underline{q} = p_{max}$. We have a particular interest in the case where $p_{max} > a\phi\frac{(N+1)}{3N-1}$. Equilibrium price is given by:

$$\underline{q} = a\phi\frac{(N+1)}{3N-1}.$$

Then the equilibrium flow from CP n to ISP m, $n \neq m$ is $\underline{z} = \frac{\phi}{N}(\frac{2N-2}{3N-1})$, and the equilibrium flow from CP n to ISP n is

$$\underline{y} = \frac{\phi}{N}(1 + \frac{(N-1)(N+1)}{3N-1}).$$

Proof of Theorem 33

We compare the expressions of the subscriber cost in each case. When an agreement exists between service and content providers, the cost for any subscriber connected to CP n is given by:

$$C_n^v(\underline{y}, \underline{z}, \underline{q}) = \phi^2 a + 2a\phi^2(\frac{N-1}{3N-1})^2(\frac{N+1}{N}).$$

We have to compare this expression with the following one, which is the cost for any subscriber also connected to CP n but in the case where no agreements are possible between service and content providers, that is:

$$C_n(\underline{x}, \underline{p}) = \phi^2 a(N - 1 + 1).$$

comparing these expressions, we get:

$$C_n^v(\underline{y}, \underline{z}, \underline{q}) < C_n(\underline{x}, \underline{p}).$$

Proof of proposition 28

We have the same methodology as in the other proofs. We assume that CPs $m'' \in \{1, \ldots, N\} - \{m', m\}$ plays q, CP m, plays p^m, and CP m' plays r. CP m' is the only one with an agreement with the ISP m'.

In order to compute the equilibrium between the subscribers, for all (p^m, q, r), we have to define all the strategy of all subscribers . Let x the request of a subscriber of ISP n for CP m, y the request of a subscriber of ISP n for CP m'' and z the request of a subscriber of ISP n for CP m' for all $n \in \{1, \ldots, N\} - \{m'\}$. And let x' the request of a subscribers of ISP $n' = m'$ for CP m, y' the request for CP m'' and z' the request for CP m' with $n \in \{1, \ldots, N\} - \{m'\}$. We have now to solve:

$$
\begin{cases}
x\dfrac{\partial D}{\partial x}((N-1)x+x') + D((N-1)x+x') + q - \lambda_1 = 0 \\
x'\dfrac{\partial D}{\partial x'}((N-1)x+x') + D((N-1)x+x') + q - \lambda_2 = 0 \\
y\dfrac{\partial D}{\partial y}((N-1)y+y') + D((N-1)y+y') + p^m - \lambda_1 = 0 \\
y'\dfrac{\partial D}{\partial y'}((N-1)y+y') + D((N-1)y+y') + p^m - \lambda_2 = 0 \\
z\dfrac{\partial D}{\partial z}((N-1)z+z') + D((N-1)z+z') + r - \lambda_1 = 0 \\
z'\dfrac{\partial D}{\partial z'}((N-1)z+z') + D((N-1)z+z') + r - \lambda_2 = 0 \\
(N-2)x + y + z = \phi \\
(N-2)x' + y' + z' = \phi
\end{cases}
$$

We denote by $\underline{x}$, $\underline{y}$, $\underline{z}$, $\underline{x'}$, $\underline{y'}$, $\underline{z'}$ a solution of this system. We consider the linear cost function $D(x) = ax$ and thus we have to solve the following linear system:

$$\begin{cases} ax + a((N-1)x + x') + q - \lambda_1 = 0 \\ ax' + a((N-1)x + x') + q - \lambda_2 = 0 \\ ay + a((N-1)y + y') + p^m - \lambda_1 = 0 \\ ay' + a((N-1) + y') + p^m - \lambda_2 = 0 \\ az + a((N-1)z + z') + r - \lambda_1 = 0 \\ az' + a((N-1)z + z') + r - \lambda_2 = 0 \\ (N-2)x + y + z = \phi \\ (N-2)x' + y' + z' = \phi \end{cases}$$

The solution of this system is:

$$x = \frac{\phi a(N+1) + p - 2\,q + 2\,r}{Na(N+1)}$$

$$y = \frac{\phi a(N+1) - (N-1)p + (N-2)q + 2\,r}{Na(N+1)}$$

$$z = \frac{\phi a(N+1) + p + (N-2)\,q - 2(N-1)\,r}{Na(N+1)}$$

$$x' = \frac{\phi a(N+1) + p - 2\,q - (N-1)r}{Na(N+1)}$$

$$y' = \frac{\phi a(N+1) - (N-1)p + (N-2)\,q - (N-1)r}{Na(N+1)}$$

$$z' = \frac{\phi a(N+1) + p + (N-2)q + (N-1)^2 r}{Na(N+1)}$$

And after some computation, in order to compute the price competition between CP we finally obtain the next utility function at equilibrium of CP m':

$$\Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) = \frac{a\phi(N+1)\left(2N^2+N-1\right)}{(4\,N^3-5\,N^2+2\,N-1)}$$
$$+\frac{a\phi(N-1)\left(2\,N^2+N-1\right)\left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1}-\frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)}{(N+1)(4\,N^3-5\,N^2+2\,N-1)}$$

Proof of theorem 35

First we have to simplify the expression of Π'_m:

$$\Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) = \frac{a\phi(N+1)\left(2N^2+N-1\right)}{(4\,N^3-5\,N^2+2\,N-1)}$$
$$+\frac{a\phi(N-1)\left(2\,N^2+N-1\right)\left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1}-\frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)}{(N+1)(4\,N^3-5\,N^2+2\,N-1)}$$

$$\Leftrightarrow \Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) = \frac{a\phi(N+1)(N+1)(N-\frac{1}{2})}{(4\,N^3-5\,N^2+2\,N-1)}$$
$$+\frac{a\phi(N-1)(N+1)(N-\frac{1}{2})\left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1}-\frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)}{(N+1)(4\,N^3-5\,N^2+2\,N-1)}$$

$$\Leftrightarrow \Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) = \frac{a\phi(N+1)^2(N-\frac{1}{2})}{(4\,N^3-5\,N^2+2\,N-1)}$$
$$+\frac{a\phi(N-1)(N-\frac{1}{2})\left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1}-\frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)}{(4\,N^3-5\,N^2+2\,N-1)}$$

$$\Leftrightarrow \Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) = \left[\frac{a\phi(N-\frac{1}{2})}{(4\,N^3-5\,N^2+2\,N-1)}\right]$$
$$\times\left[(N+1)^2+(N-1)\left(\frac{\left(4\,N^2+5\,N+1\right)}{4\,N^2-N+1}-\frac{2\left(2\,N^3+N^2-N\right)}{4\,N^3-5\,N^2+2\,N-1}\right)\right].$$

Now we can compare Pi'_m and Pi_m. We can see that if we have $P(N) < (N-1)$ with:

$$P(N) = \frac{\Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'})}{a\phi}$$

$$\times \left[(N+1)^2 + (N-1) \left(\frac{\left(4\,N^2 + 5\,N + 1\right)}{4\,N^2 - N + 1} - \frac{2\left(2\,N^3 + N^2 - N\right)}{4\,N^3 - 5\,N^2 + 2\,N - 1} \right) \right]$$

which means that:

$$\Pi'_m(\underline{p}_{m'}, \underline{p}_m, \underline{x}_{m'}, \underline{p}_{m'}) < \Pi_m(\underline{p}, \underline{x}).$$

After some algebras, we obtain that $P(N) < (N-1)$ which proves the proposition. Then, the decision vector where each CP does not make an agreement is a Nash equilibrium.