Preparing Symmetric Crypto for the Quantum World

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Preliminaries...

No quantum knowledge needed for following this talk

Outline

- Introduction

  Motivation, scenarios and evolution
- Useful quantum tools
- Presentation of some results
  - Building new useful quantum tool: collision and k-xor algorithms
  - Some quantum attacks (Simon +)
- Final conclusion and Open problems

Motivation

Cryptanalysis: Foundation of Confidence

- Ideal security defined by generic attacks ($2^{|K|}$).
  Does real security meet this ideal security?
  Need of continuous security evaluation.

> Any attack better than the generic one is considered a "break".

- We are often left with an empirical measure of the security: cryptanalysis.

Very Important Notion: Security Margin

If no attack is found on a given cipher, what can we say about its robustness?

The security of a cipher is not a 1-bit information:

- Round-reduced attacks.
- Analysis of components.

$\Rightarrow$ determine and adapt the security margin.

Very Important Notion: Security Margin

- Best attacks determine the security margin
  $\Rightarrow$ Possibly with high complexities: find the highest number of rounds reached.

- Allows to compare primitives.

- The estimates of security margin need to be precise and correct in order to be meaningful.

Post-Quantum Cryptography

Asymmetric (e.g. RSA):

Shor's algorithm: Factorization in polynomial time

$\Rightarrow$ current systems not secure!

Solutions: lattice-based, code-based cryptography..

Symmetric (e.g. AES):

Grover's algorithm: Exhaustive search $2^{|K|} \longrightarrow 2^{|K|/2}$

Double key length for equivalent ideal security.

Much to learn about cryptanalysis when having quantum computing available.

Post-Quantum Cryptography

Problem for present existing long-term secrets.
$\Rightarrow$ start using quantum-safe primitives NOW.

Important tasks:

- Conceive the cryptanalysis algorithms for evaluating the security of symmetric primitives in the P-Q world.
- Use them to evaluate and design symmetric primitives for the P-Q world.

On Quantum Attacks

- Compare to best generic attack,
- generic attack is accelerated, so
- broken classical primitive might be unbroken in a quantum setting:

e.g. a primitive might not have 256-bits security against a classical adversary but might have 128-bit security against a quantum one.

Scenarios and Models

Considered Scenarios

- **Model Q_0**
  classical attacks with classical computers.
- **Model Q_1**
  Q_0 + access to a quantum computer.
- **Model Q_2**
  Q_1 + superposition queries to a quantum cryptographic oracle (QCO).
- **Model Q_3**
  Q_1 + superposition queries with the differences of a secret key in a QCO.

Model Q_0

Nothing new here.

Model Q_1

- So far, the best we have obtained is a quadratic speed-up, but it can be smaller:
  - If a primitive is safe in Q_0, it will also be in Q_1.
- Does this mean that (so far) the Q_1 scenario/results are not interesting?

No!

safe = no attack better than generic attack

Model Q_1

In a post-quantum future:

- Classical or quantum surnames will disappear:
  Expected security given by their best generic attack (*e.g.* Grover).
  And security margin? $\rightarrow$ determined by the highest number of rounds cryptanalyzed with any attack more performant than generic.
- Q_1 results: important information needed for determining the unique and future security margin.

Model Q_2

Very powerful, BUT...

Many good reasons to study security in this scenario:

- Simple: used in security proofs.
- Non-trivial: Many constructions still seem resistant.
- Inclusive of all intermediate scenarios: protocols, obfuscation, hybrid machines, incompetent users...

Model Q_2

Defined and used in many results:

[Zhandry12], [Boneh-Zhandry13], [Damgård-Funder-Nielsen-Salvail13], [Mossayebi-Schack16], [Song-Yun17], Simon's attacks, FX, AEZ...

An attack in this model $\Rightarrow$ we need to be extra careful when implementing the primitive in a quantum computer.

Model Q_3

Super strong model:

Everything is broken [Roetteler-Steinwandt 15]

Too strong model!

Another scenario classification

Scenario A) With big quantum memory or

Scenario B) quantum memory limited to $poly(n)$

The first one: interesting from a theoretical point of view and for considering trade-offs,

The second one: more "realistic" scenario.

Evolution

First Results

Quantum Symmetric Cryptanalysis:

- Quantum analysis of CubeHash [Leurent 10]
- Simon on 3-round Feistel [Kuwakado Morii 10]
- Simon on Even-Mansour [Kuwakado Morii 12]
- Quantum MITM iterated ciphers [Kaplan14]
- Quantum Related-Key [Roetteler-Steinwandt15]

Quantum Symmetric Cryptanalysis

- In 2015/2016:

[Kaplan-Leurent-Leverrier-NP16] Simon on modes/slide attacks.

[Kaplan-Leurent-Leverrier-NP16b] Diff/linear.

Many new results since: FX [Leander-May17], parallel multi-preim. [Banegas-Bernstein17], Multicollision [Hosoyamada-Sasaki-Xagawa17], Mitm Q1 [Hosoyamada Sasaki 18], DS Mitm Feistel [Hosoyamada Sasaki 18], Miss-in-the-middle [Xie, Yang 18], Feistel key-recovery [Dong, Wang 18], CCA on Feistel [Ito et al 19]...

Recent activity from QUASYModo

- Efficient Collisions [Chailloux NP Schrottenloher Asiacrypt17],
- Quantum cryptanalysis of AEZ [Bonnetain SAC17]
- On modular additions [Bonnetain NP Asiacrypt 2018]
- k-xor problem [Grassi NP Schrottenloher Asiacrypt2018]
- AES quantum evaluation [Bonnetain NP Schrottenloher 18]
- On quantum slide attacks [Bonnetain NP Schrottenloher 18]
- Quantum security analysis of CSIDH[Bonnetain Schrottenloher18]
- *Optimal merging the k-xor problem [NP Schrottenloher 19]*
- *Improved low-qubit hidden shift algorithms [Bonnetain 19]*

Some Useful Quantum Tools

Some Quantum Tools...

...that have been useful so far.

- Amplitude Amplification (AA) /Grover
- Quantum Counting
- Quantum Collisions
- Simon
- Kuperberg

Amplitude Amplification (Grover's generalization)

Exhaustive search:
Given $f : \{0,1\}^n \to \{0,1\}$, find one element $x \in \{0,1\}^n$ such that $f(x) = 1$.

- Classical complexity: $\Omega(\frac{2^n}{|\mathsf{supp}(f)|})$.
- Quantum complexity [Brassard-Hoyer 97]: $\Omega(\sqrt{\frac{2^n}{|\mathsf{supp}(f)|}})$.

In detail, we will see later: $\mathcal{O}\left(\sqrt{\frac{2^n}{|\mathsf{supp}(f)|}}(s_T + f_T)\right)$.

Quantum Counting Algorithm

Distinguish a biased distribution:
Given a Bernouilli distribution, determine with high probability whether it has a parameter $1/2$ or $1/2 + \varepsilon$.

- Classical complexity: $\mathcal{O}\left(\frac{1}{\varepsilon^2}\right)$.
- Quantum complexity:
  [Brassard-Hoyer-Tapp 98] $\mathcal{O}\left(\frac{1}{\varepsilon}\right)$.

Quantum Collision Algorithms

Collision problem: Given a random function $H : \{0,1\}^n \to \{0,1\}^n$, find $x, y \in \{0,1\}^n$ with $x \neq y$ such that $H(x) = H(y)$.

- Classical complexity: $\Omega(2^{n/2})$.
- Quantum complexity: [Brassard-Hoyer-Tapp 97] $\mathcal{O}\left(2^{n/3}\right)$ in queries, in time and in quantum memory $\to$ scenario A. (Scenario B later)

Simon's algorithm

Simon's problem:

Given $f : \{0,1\}^n \to \{0,1\}^n$ such that

$\exists s \mid f(x) = f(y) \iff [x = y \text{ or } x \oplus y = s]$,

find s.

- Classical complexity: $\Omega(2^{n/2})$.
- Quantum complexity [Simon 94]: $\widetilde{\mathcal{O}}(n)$.

Kuperberg's algorithm

Hidden Shift Problem with modular addition:
Let f, g be two injective functions, $(\mathbb{G}, +)$ a group. Given the promise that there exists $s \in \mathbb{G}$ such that, for all x, $f(x) = g(x + s)$, retrieve s.

- Classical complexity: $\Omega(2^{n/2})$.
- Quantum complexity:
  [Kuperberg 05] $2^{\widetilde{\mathcal{O}}(\sqrt{n})}$.

Some new Results
New useful Quantum Tools

Some New Useful Quantum Tools

- New Quantum Collision Algorithm
- Quantum K-xor Algorithms
- Multicollisions
- Grover-meets-Simon
- Simon-meets-Kuperberg
- Framework for quantizing classical attacks
- Quantumly efficient DDT equivalent
- Miss-in-the-middle search

Collision Search

with A. Chailloux, A. Schrottenloher

Collision Search Problem

Given a random function $H : \{0,1\}^n \rightarrow \{0,1\}^n$, find $x, y \in \{0,1\}^n$ with $x \neq y$ such that $H(x) = H(y)$.

Many applications: *e.g.* generic attacks on hash functions.

(Multi-target preimage search can be seen as a particular case).

Best known algorithms

	Time	Queries	Qubits	Classical Memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$O(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$O(n)$	0
BHT	$2^{2n/3}$*	$2^{n/3}$	$O(n)$*	$2^{n/3}$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	0

Considered Model

- The same one as in the previous collision quantum algorithms BUT we limit the amout of quantum memory available to a small amount $\mathcal{O}(n)$: scenario B instead of A.
- Available small quantum computers seem like the most plausible scenario.
- We are interested in the theoretical algorithm and we did not take into account yet implementation aspects.

Starting Point: BHT Algorithm

- Optimal number of queries,
- $\mathcal{O}(n)$ qubits (scenario B),
- But time?

BHT: Summarized procedure

- Build a list L of $2^{n/3}$ elements (classical memory),
- Exhaustive search for finding one element that collides: With AA, the number of iterations is: $(\frac{2^n}{2^{n/3}})^{1/2} = 2^{n/3}$.
- Testing the membership with L for the superposition of states costs $2^{n/3}$ with n qbits:

Time: $2^{n/3} + 2^{n/3}(1 + 2^{n/3}) \approx 2^{2n/3}$

Can we improve this?

Let's build the list L with distinguished points
e.g. $H(x_i) = 0^u || z$, for $z \in \{0,1\}^{n-u}$.

The cost of building the list is bigger: $2^{n/3+u/2}$.
The setup of AA is bigger: $2^{u/2}$
The membership test stays the same: $|L| = 2^{n/3}$
BUT The number of iterations is smaller: $2^{n/3-u/2}$

Time: $2^{n/3+u/2} + 2^{n/3-u/2}(2^{u/2} + 2^{n/3}) \approx 2^{2n/3-u/2} + 2^{n/3+u/2}$

With optimal parameters

The cost will be optimized for a certain size of L: $2^v \neq 2^{n/3}$.

Time: $2^{v+u/2} + 2^{\frac{n-v-u}{2}}(2^{u/2} + 2^v)$
For $v = n/5$, $u = 2n/5$: Time: $\widetilde{\mathcal{O}}\left(2^{2n/5}\right)$

Comparison

	Time	Queries	Qubits	Classical Memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$O(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$O(n)$	0
BHT	$2^{2n/3}$	$2^{n/3}$	$O(n)$	$2^{n/3}$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	0
New algorithm	$2^{2n/5}$	$2^{2n/5}$	$O(n)$	$2^{n/5}$

Example of Applications

- Hash functions: Collision and Multi-preimages time from $2^{n/2}$ to $2^{2n/5}$ and $2^{3n/7}$ (Q1).

  Ex.- time and queries for $n = 128$:

  Pollard rho$= 2^{64}$

  vs

  Ours $= 2^{51}$ with less than 1GB classical.

- Multi-user setting.
- Operation modes.
- Bricks for Cryptanalysis.

About Parallelization

- What about comparison with parallel rho?
  This algo provides new trade-offs.
  For comparison, previous example $n = 128$:
  Parallel rho$= 2^{51}$ with 2^{13} processors
  vs
  Ours $= 2^{51}$ with less than 1GB classical.
- When both parallelized: up to $2^{n/3}$ processors this algorithm is more time-efficient than parallel rho.

Conclusion - Collision

New efficient collision search algorithm with small quantum memory (nothing *scary*, new trade-offs):

First algorithm with less than $2^{n/2}$ computations in scenario B.

Many applications in symmetric cryptography.

Open question: is it possible to meet the optimal $2^{n/3}$ in time with small quantum memory?

Quantum Efficient Algorithms for the k-xor Problem (and Update)

with L. Grassi, A. Schrottenloher

k-xor problem with random functions

Given query access to a random function $H : \{0,1\}^n \to \{0,1\}^n$, find $x_1, \ldots, x_k$ such that $H(x_1) \oplus \ldots \oplus H(x_k) = 0$.

For us, equivalent to the case with k different random functions.

Many applications (with k-sum, similar algorithms apply), ex.: attacks on FSB, XLS, SWIFFT; correlation attacks.

The 3-xor problem

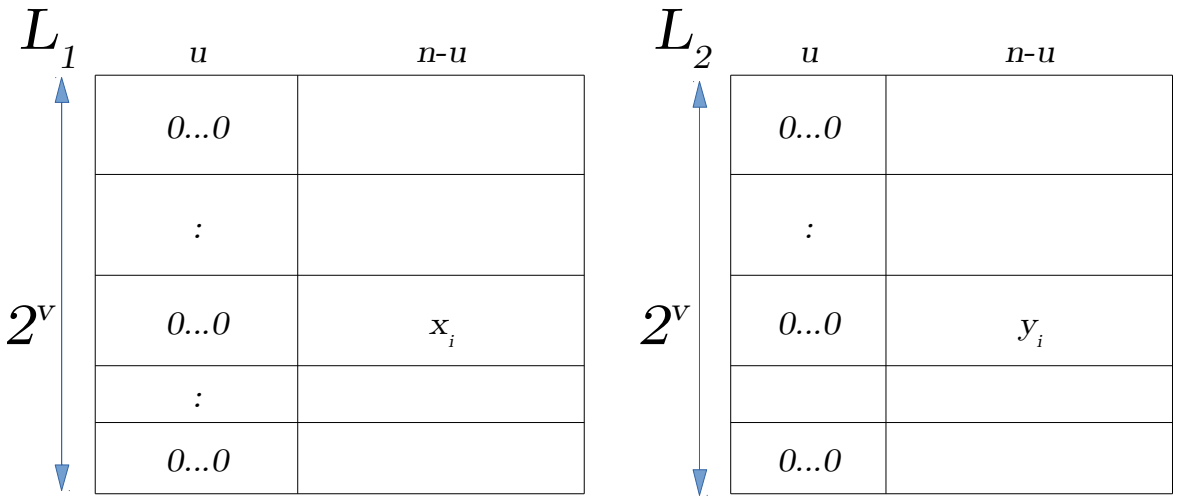
Find 3 elements that xor to 0: not much better than collision in classical setting.

Classically, no exponential time acceleration, only logarithmic:

Complexity of $\widetilde{\mathcal{O}}\left(2\right)^{n/2}$.

3-xor: Scenario B Algorithm

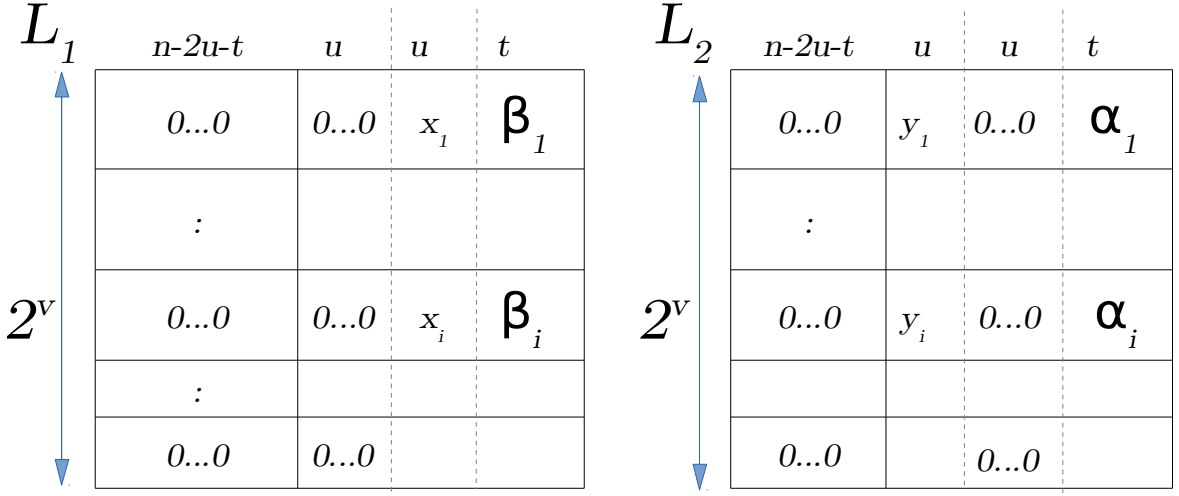
- 1st approach, distinguished point: $2^v = 2^{n/8}$, $T = 2^{3n/8}$



- Intuition: With a memory of $2^v + 2^v$ we obtain 2^{2v} potential collisions.

3-xor: Scenario B Algorithm

- 1st approach: $2^v = 2^{n/8}$, $T = 2^{3n/8}$
- 2nd approach, techniques linked to "list merging":



Improved time$= 2^{5n/14}$, with $2^v = 2^{n/7}$.

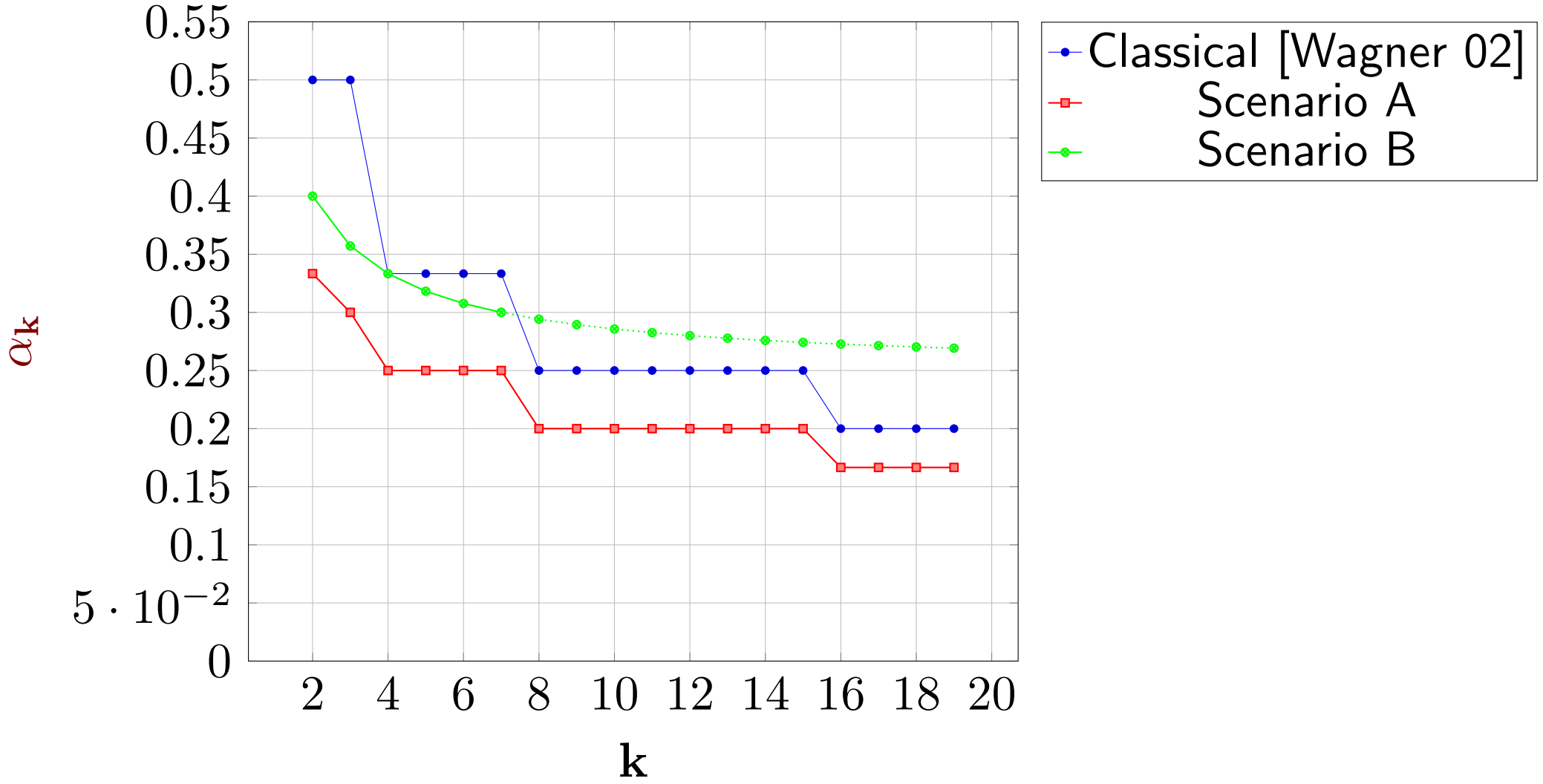
- Exponentially better than collision, contrary to classical!

3-xor: Scenario A Algorithm

- Same technique as before, but no need for a common prefix of zeroes.

- This gives
  QM$= 2^{n/5}$ and Time$= 2^{3n/10}$.

The k-xor algorithms



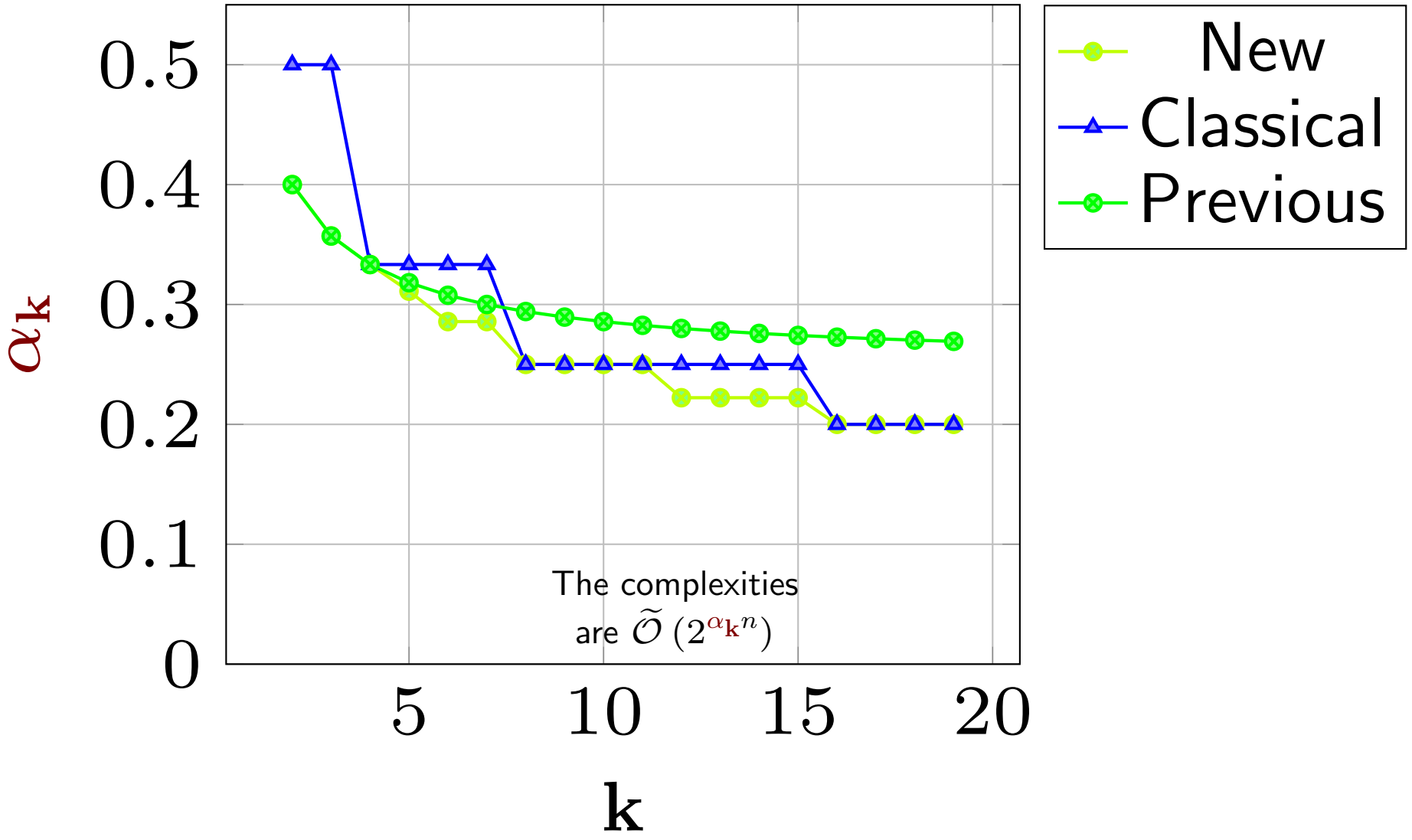
The time complexities are $\widetilde{\mathcal{O}}\left(2^{\alpha_{\mathbf{k}} n}\right)$

k-xor algorithms: Very Recent results

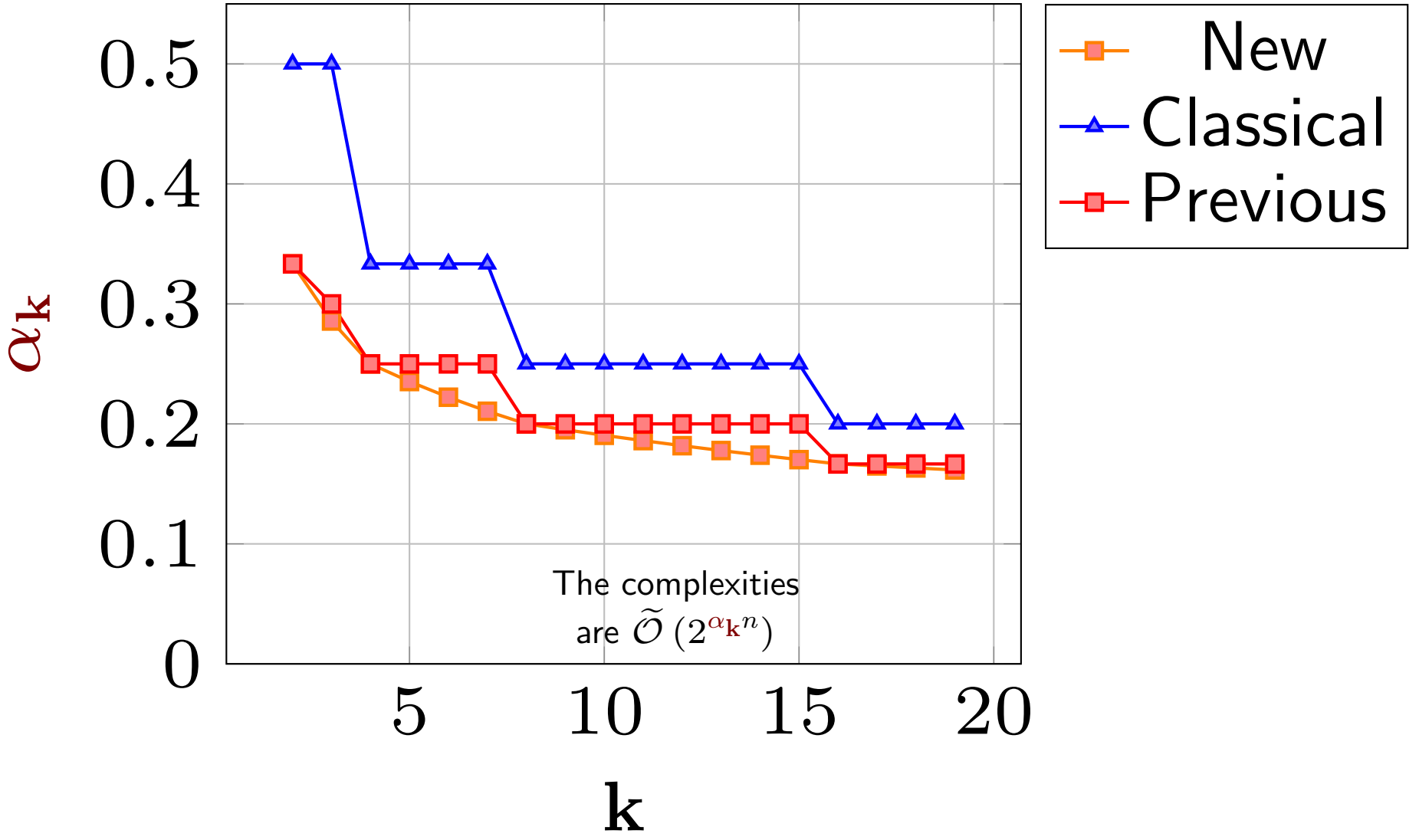
- Related to dissection: partial solutions to subproblems $n' < n$, $k' < k$ and combining them.
- When optimal? Not intuitive at all! $\Rightarrow$ Recursive MILP program for optimality in both scenarios.

Can we reach better complexities than $\widetilde{\mathcal{O}}\left(2^{n/(\lfloor \log_2(k) \rfloor + 2)}\right)$ when k is not a power of 2 in scenario A? Can we obtain time complexities better than classical for $k \geq 8$ in scenario B?

New Results: scenario B



New Results: scenario A



Conclusion - k-xor - Optimal Merging

- The quantum 3-xor problem is exponentially easier than the quantum collision problem (in both settings), contrary to classical.
- The time for solving the 3-xor problem in scenario A beats the lower bound for quantum collision of $2^{n/3}$
- For generic k, scenario B improves Wagner for half the values, and scenario A improves for all k (interpolated curve).

Some Results on Quantum Attacks

New Quantum Attacks

- Differential/Linear
- Simon-based
- Kuperberg-based
- Slide attacks
- DS-MITM

And dedicated analysis:

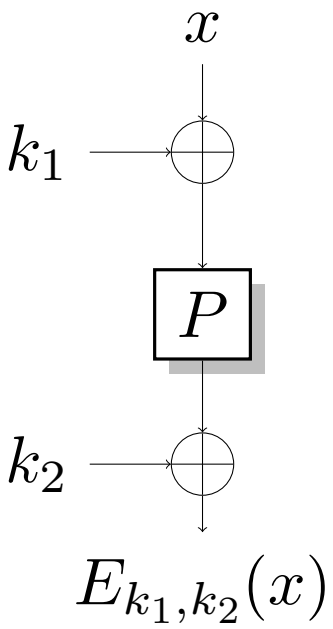
- FX and Feistel constructions
- Q_2 attack on AEZ
- Q_2 attack on Poly-1305
- AES Quantum analysis

Simon and Kuperberg Attacks

with X. Bonnetain, M. Kaplan, G. Leurent, A. Leverrier

Simon on Even-Mansour [Kuwakado Morii 12]

- [Even-Mansour 97] cipher: $DT > 2^n$



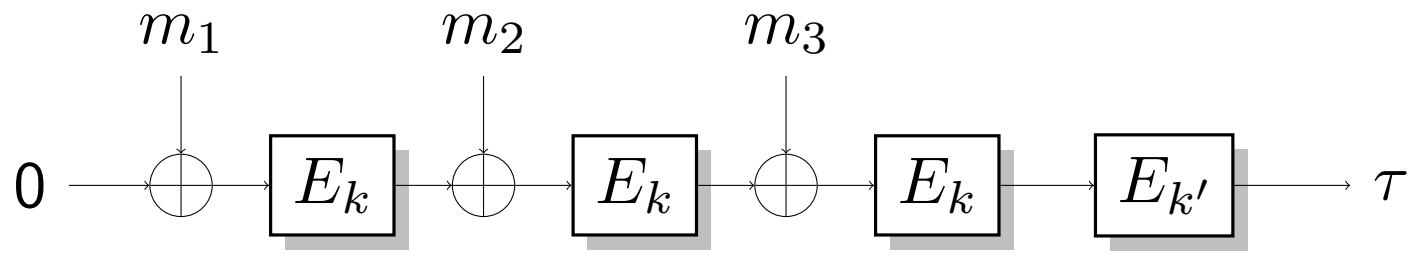
$$f(x) = E_K(x) \oplus P(x) \rightarrow f(x) = f(x \oplus k_1)$$

Simon's algo on $f \Rightarrow k_1$ in $\mathcal{O}(n)$

[Kaplan-Leurent-Leverrier-NP 16]

Simon on most authentication modes + slide attacks

- For example encrypt-last-block CBC-MAC:



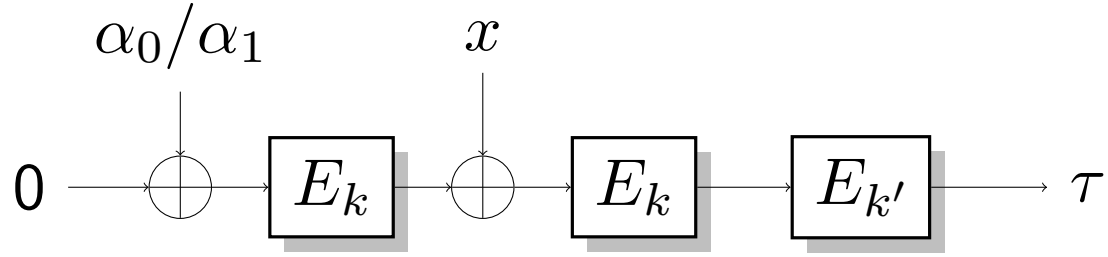
$$f : \{0,1\} \times \{0,1\}^n \to \{0,1\}^n$$
$$b, x \quad \mapsto \mathsf{CBCMAC}(\alpha_b \| x) = E_{k'}\left(E_k\left(x \oplus E_k(\alpha_b)\right)\right).$$

$$\mathsf{CBCMAC}(\alpha_1 \| x \oplus E_k(\alpha_0) \oplus E_k(\alpha_1)) =$$
$$E_{k'}\left(E_k\left((x \oplus E_k(\alpha_0) \oplus E_k(\alpha_1)) \oplus E_k(\alpha_1)\right)\right) = \mathsf{CBCMAC}(\alpha_0 \| x)$$
$$s = 1 \| E_k(\alpha_0) \oplus E_k(\alpha_1)$$

[Kaplan-Leurent-Leverrier-NP 16]

Simon on most authentication modes + slide attacks

- For example encrypt-last-block CBC-MAC:



$$f : \{0,1\} \times \{0,1\}^n \to \{0,1\}^n$$
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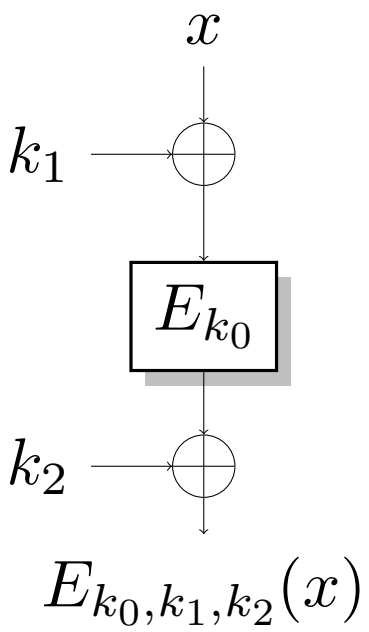
$$s = 1 \| E_k(\alpha_0) \oplus E_k(\alpha_1)$$

Simon and Grover on FX construction

The FX construction is a natural construction for extending the key-length $n \Rightarrow 2n$.



[Leander May 17] Combined Simon with Grover:
$\rightarrow$ broken in $O(2n2^{n/2})$

Tweaking to resist Simon's algo.?

- In [Alagic Russell 17] several proposals. Most efficient: replace xor by modular additions.
- Hidden shift problem in $\mathbb{Z}/(N)$.
- No algorithm in polynomial time: Kuperberg in $2^{O(\sqrt{n})}$
- Up to what point do primitives resist?

Motivation and results [Bonnetain-NP18]

- 4. Dimension symmetric primitives
- 1. More precise evaluation of Kuperberg's algorithm complexity+improvement
- 2. What about parallel modular additions?
- 3. New Quantum attacks (Feistel's slide, FX)

Improvement and Simulation

Our improvement: all the bits with one iteration.
$O(n^2 2^{\sqrt{2\log_2(3)n}}) \Rightarrow O(n 2^{\sqrt{2\log_2(3)n}})$

Our simulations: $0.7 \times 2^{1.8\sqrt{n}}$ for recovering full s.
Code available: ask Xavier Bonnetain if interested.
`xavier.bonnetain@inria.fr`

Results - Conclusion

- Improved Kuperberg's algorithm and new algorithm for parallel modular additions.
- State size needed for 128-bit security.
  at least 5200 bits (but for FX)
  $\Rightarrow$ not very realistic.
- Might be better to just avoid vulnerable constructions, or try different patches.
- Recently: concrete security of some Isogeny-based primitives [Bonnetain-Schrottenloher]

Final Conclusion

General Conclusion (for now) 1/2

- No reason to panic, symmetric crypto seems to be holding on well
- Bigger internal states?
- Ideas from quantum analysis might improve classical analysis
- Many things yet to do to precisely evaluate security, to find best attacks, to adjust parameters...

General Conclusion (for now) 2/2

- What about Q2? No consensus:
  Surprising-scary results vs useless model?

- IMHO: Very strong model but when possible, better to avoid Q2 attacks: symmetric modus operandi works well in part because we are never too paranoid: (attacks on 2^{200} declare ciphers broken,...)

- At least: information worth knowing.

Aristotle?

Open problems

- Propose an efficient AE mode Q_2-safe
- New quantum attacks: QFT ?
- Quantum security evaluation of primitives(LW)
- Generic key-length extensions?
- Design of primitives with bigger state
- Time-memo Trade-offs for k-xor algorithms
- Evaluating quantum implementation of algorithms
- ...

Quantum-Safe Symmetric Primitives

Lots of things to do !

- And what about quantum asymmetric cryptanalysis??

  Necessary to evaluate the concrete security of proposed primitives.

  Possible links between both.

Many thanks to André Schrottenloher, Xavier Bonnetain, Anne Canteaut, Gaetan Leurent, Anthony Leverrier...

ERC QUASYModo

https://project.inria.fr/quasymodo/

- 1 PhD position
- 1 PostDoc position