Discussion on 'Dam break in rectangular channels with different upstream-downstream widths'

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1 Governing equations - conservation vs. invariance

We start by observing that the hydraulic head is not necessarily the most appropriate variable for writing compatibility conditions across a channel section. Indeed, it is not a conserved variable. It obeys Bernoulli's theorem, obtained by combining the mass and momentum conservation equations:

$$\partial_t \mathbf{u} + \nabla E + \frac{1}{g}(\nabla \times \mathbf{u}) \times \mathbf{u} = -\mathbf{s}_f \tag{1}$$

where g is the gravitational acceleration, $E = h + z_b + \frac{||\mathbf{u}||^2}{2g}$ is the hydraulic head (called the specific energy in [3]), $\mathbf{s}_f$ is the friction slope and $\mathbf{u}$ is the flow velocity. Straightforward algebraic manipulations yield the following governing equations for E and the flow energy $e = \left(E - \frac{h}{2}\right) h$:

$$\partial_t E + \mathbf{u}.\nabla E + \nabla.\mathbf{q} = -\mathbf{u}.\mathbf{s}_f \tag{2a}$$

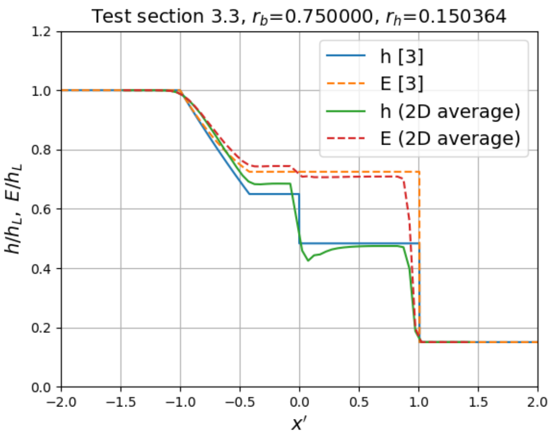
$$\partial_t e + \nabla.(\mathbf{q}E) = -\mathbf{q}.\mathbf{s}_f \tag{2b}$$

where $\mathbf{q} = h\mathbf{u}$ is the unit discharge vector. The hydraulic head E is not a conserved variable in the strict sense in that its governing equation cannot be recast in conservation form. However, it is an invariant along streamlines under the assumptions of a divergence free flow and no energy dissipation. Consequently, it seems more appropriate to state the *invariance* of E rather than its conservation. In contrast, averaging equation (2b) across a channel section is meaningful because e is a conserved variable.

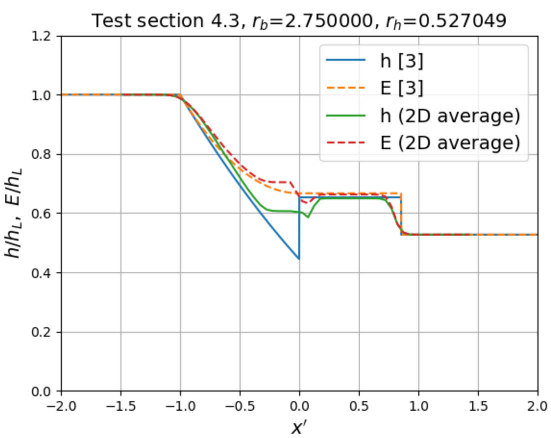
Besides, several procedures can be proposed for averaging E over a section. For instance, E can be computed at each point from the local value of $||\mathbf{u}||^2/2g$, then averaged into $< E(\mathbf{u}) >$. Another possibility is to average the velocity vector $\mathbf{u}$ into $< \mathbf{u} >$ over the section, then compute the square of its norm and $E(|| < \mathbf{u} > ||^2)$. In most of the cases reported hereafter, all procedures tested give similar results and only the average $< E(\mathbf{u}) >$ is presented. At any rate, neither E nor e can be assumed invariant across a sudden narrowing or widening in the general case because of the presence of head losses. This is the main point of this discussion.

2 2D simulation results

We performed numerical simulations of the 2D Shallow Water equations with flat bottom and a discontinuous channel's width, reproducing test configurations presented by Valiani and Caleffi in sections 3.3 (channel contraction, see [3, Fig. 4] and Fig. 1a in the following) and 4.3 (channel expansion, see [3, Fig. 7] and Fig. 1b in the following). For the sake of brevity, we do not reproduce all the configurations depicted in [3, Fig. 12] in the paper, but our conclusions remain valid for all of them. The two-dimensional model for Initial Value Problem (IVP) 3.3 is 3 m long. Its wide and narrow sections are 5cm and 3.75cm wide respectively. The model for IVP 4.3 has the same length, with 1 cm and 2.75 cm wide sections respectively. In both models, the narrower section is meshed using 10 cells across. For the sake of clarity, the graphs in Fig. 1 present the dimensionless averaged hydraulic head $< E >$. The dimensionless water depth is also plotted in the Figures. These averages are compared with those obtained from the analytical solutions presented in [3]. We can observe that the hydraulic head E/h_L is not invariant across the width discontinuity, contrarily to the assumption made in [3]. As a consequence the water depths and the heads computed by the approach [3] differ from those in the averaged 2D solution. In particular, a decrease is observed at the discontinuity due to the head loss in the 2D simulation. Moreover, in IVP 4.3 (Figure 1b), we observe an important difference in the behaviour of the upstream solution. In the solution [3], the rarefaction wave



(a) IVP 3.3



(b) IVP 4.3

Figure 1: Numerical simulations of the 2D Shallow Water equations reproducing some of the Valiani and Calefi's tests' configurations. Full and dashed lines represent respectively the water depth and the hydraulic head, as functions of the dimensionless coordinate $x' = (x/t)/(\sqrt{gh_L})$. Blue and orange lines are the analytic solution derived in [3]; green and red lines are the averaged results of the 2D simulations.

connects the left state of the IVP to the width discontinuity, while an intermediate region of constant state is observed next to the discontinuity in the 2D solution.

As an additional test case, we considered a configuration with a small depth ratio ($h_R/h_L = 0.1$) and a very large width ratio ($b_R/b_L = 10$), which is similar to that presented in [1]. According to [3, Fig. 12], this configuration corresponds to the one described in [3, Section 4.4] (expansion, small r_h). Similarly to Fig. 1, we plotted in

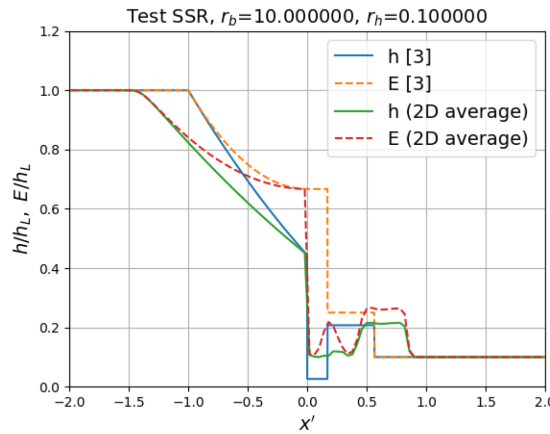


Figure 2: Test case with $h_R/h_L = 0.1$ and $b_R/b_L = 10$. Full and dashed lines represent respectively the water depth and the hydraulic head, as functions of the dimensionless coordinate $x' = (x/t)/(\sqrt{gh_L})$. Blue and orange lines are the analytic solution derived in [3]; green and red lines are the averaged results of the 2D simulations.

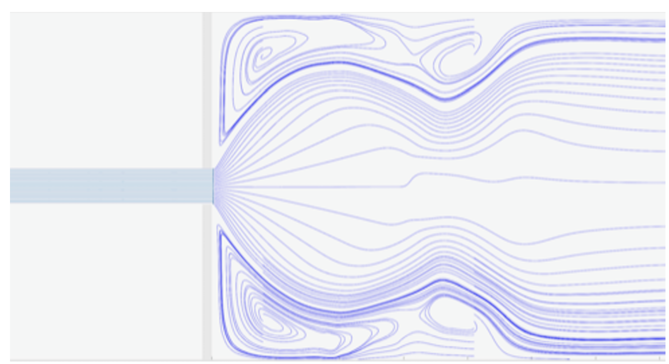


Figure 3: Streamlines for the testcase with $h_R/h_L = 0.1$ and $b_R/b_L = 10$. Vortices illustrating the head loss appear in the top and bottom corners, right after the expansion of the channel.

Fig 2 the water depth and the hydraulic head both for the 2D model and for the analytical solution described in [3].

The dissipation of the hydraulic head is due to the formation of vortices right after the sudden contraction or expansion of the channel. Fig. 2 present the streamlines of the 2D solution for the expansion case (corresponding to the same simulation presented in Fig. 2), as an illustration of this physical phenomenon.

To conclude, the above simulations confirm Ostapenko's statement [2] that the validity of compatibility conditions for IVPs involving channel width discontinuities should be checked against experiments, be they numerical or laboratory-based.

References

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