

# Reduced-order model of the BGK equation based on POD and optimal transport

Florian Bernard, Angelo Iollo, Sébastien Riffaud

► **To cite this version:**

Florian Bernard, Angelo Iollo, Sébastien Riffaud. Reduced-order model of the BGK equation based on POD and optimal transport. 6th European Conference on Computational Mechanics (ECCM 6) - 7th European Conference on Computational Fluid Dynamics (ECFD 7), Jun 2018, Glasgow, United Kingdom. hal-02427327

**HAL Id: hal-02427327**

**<https://hal.inria.fr/hal-02427327>**

Submitted on 6 Jan 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Reduced-order model of the BGK equation based on Proper Orthogonal Decomposition and Optimal Transport

F. Bernard<sup>1,2</sup>, A. Iollo<sup>1,2</sup> and S. Riffaud<sup>1,2,\*</sup>

<sup>1</sup> Institut de Mathématiques de Bordeaux, Université de Bordeaux, 33400 Talence, France

<sup>2</sup> Memphis team, Inria Bordeaux Sud-Ouest, 33400 Talence, France

**Keywords:** BGK equation, Proper Orthogonal Decomposition, Optimal Transport, Galerkin method.

A reduced-order model of the BGK equation leading to fast and accurate computations is presented. The BGK model describes the dynamics of a gas flow in both hydrodynamic and rarefied regimes:

$$\frac{\partial f}{\partial t}(\mathbf{x}, \xi, t) + \xi \cdot \nabla_{\mathbf{x}} f(\mathbf{x}, \xi, t) = \frac{M_f(\mathbf{x}, \xi, t) - f(\mathbf{x}, \xi, t)}{\tau(\mathbf{x}, t)} \quad (1)$$

where  $f$  is the density distribution function representing the density of gas particles at point  $\mathbf{x} \in \mathbb{R}^3$ , velocity  $\xi \in \mathbb{R}^3$  and time  $t \in \mathbb{R}$ , and  $M_f$  is the Maxwellian distribution function. In the reduced-order model, the distribution functions are approximated in velocity space by a small number of basis functions  $\Phi_n$  computed offline:

$$\tilde{f}(\mathbf{x}, \xi, t) = \sum_{n=1}^N a_n^f(\mathbf{x}, t) \Phi_n(\xi) \quad \text{and} \quad \tilde{M}_f(\mathbf{x}, \xi, t) = \sum_{n=1}^N a_n^M(\mathbf{x}, t) \Phi_n(\xi) \quad (2)$$

In the offline phase, the BGK equation is sampled in order to collect information on the distribution functions. Then, optimal transportation provides additional samples by interpolating the distribution functions to complete the sampling. Finally, the basis functions are built by Proper Orthogonal Decomposition. During the online phase, the offline knowledge is used to compute approximations of the density distribution function at low cost. The BGK equation is projected onto the basis functions, leading to an hyperbolic system of partial differential equations. Moreover, the projection is modified to conserve mass, momentum and energy of the gas. The resulting system is then solved by an IMEX Runge-Kutta scheme in time and a finite-volume scheme in space to compute the coefficients  $a_n^f$  and  $a_n^M$ . The results show the significant reduction of the computational cost and the accuracy of the reduced-order model:



Figure 1: Vertical plate in a flow at  $\text{Kn} = 0.0345$ . Streamlines and horizontal velocity component (left panel). Prediction error (right panel). The database is created for a Mach number of 0.63 and the predicted solutions are for Mach 0.23,0.33,0.43,0.53,0.63. The number of modes is either 15 or 20. Computational time is divided by 45.

## References

- [1] F. Bernard. *Efficient asymptotic preserving schemes for BGK and ES-BGK models on cartesian grids*. PhD thesis, Bordeaux, 2015.
- [2] L. Sirovich. Turbulence and the dynamics of coherent structures. i. coherent structures. *Quarterly of applied mathematics*, 45(3):561?571, 1987.
- [3] J. Solomon, F. De Goes, G. Peyré, M. Cuturi, A. Butscher, A. Nguyen, T. Du, and L. Guibas. Convolutional wasserstein distances: Efficient optimal transportation on geometric domains. *ACM Transactions on Graphics (TOG)*, 34(4):66, 2015.