On deciding stability of high frequency amplifiers

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Motivation

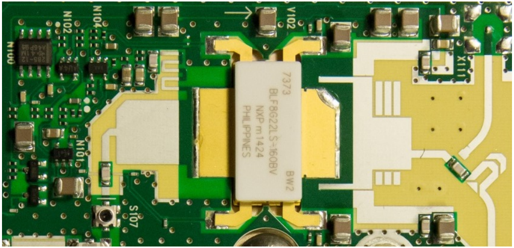
- Amplifiers at high frequency are ubiquitous (Cell phones, relays...). They need to be quick to design and produced in large quantites.
- Computer-assisted design (CAD) and simulation before production.
- Powerful "frequency simulation" tools give a reliable prediction of the response, but that response might be unstable.
- **Need** for a tool to predict stability/unstability in the frequency domain.

Inside

An amplifier is made of interconnected

- resistors, inductors, capacitors,
- diodes/transistors,
- lossless transmission lines wich cannot be neglected at high frequency inducing delays.

Forcing periodic signal ▶▶ periodic solution in the amplifier ▶▶ amplified signal.



Harmonic Balance

The Harmonic Balance method, through Fourier development, Laplace transform and fixed point methods permits to :

- approximate the periodic solution of the circuit,
- linearize the circuit arround the periodic solution,
- give a frequency response to a periodic signal wich disturbs the linearized circuit.

Our focus : Structure of the harmonic transfer function, its singularities, links with stability.

Summary

General system, T-periodic :

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^{N} B_{1,i}(t)y(t-\tau_i) \\ y(t) = \sum_{i=1}^{N} B_{2,i}(t)y(t-\tau_i) + A_2(t)x(t),\ \ t \geq s, \end{cases}$$

- $\mathcal{L}^2 := \mathbb{R}^n \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t,s) : \mathcal{L}^2 \to \mathcal{L}^2$
- Monodromy operator $U(T,0)$
-

$$\left.\begin{array}{c} L^2 \text{ exponential} \\ \textit{stability} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} Sp(U(T,0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array}\right.$$

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$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^{N} B_{1,i}(t)y(t-\tau_i) \\ \boxed{y(t) = \sum_{i=1}^{N} B_{2,i}(t)y(t-\tau_i)} + A_2(t)x(t), \ t \geq s, \end{cases}$$

- $L^2 := \mathbb{R}^n \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $U(t,s) : L^2 \to L^2$
- Monodromy operator $U(T,0)$
- $$\left.\begin{matrix} L^2 \text{ exponential} \\ \textit{stability} \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} Sp(U(T,0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{matrix}\right.$$

Behaviour at high frequency

High frequency system :

$$\begin{cases} x(t) = 0 \\ y(t) = \sum_{i=1}^{N} B_{2,i}(t) y(t-\tau_i), \ t \geq s, \end{cases}$$

- $\tilde{L}^2 := \{0_n\} \times L^2([-\tau_N, 0], \mathbb{R}^k)$.
- Solution operator $V(t,s) : \tilde{L}^2 \to \tilde{L}^2$.
- Monodromy operator $V(T,0)$.
- $$\left.\begin{array}{c} L^2 \text{ exponential} \\ \textit{stability} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} Sp(V(T,0)) \text{ included in} \\ \text{disc of radius } r < 1 \end{array}\right.$$

Compact perturbation

Lemma

We have :

$$U(t,s) = V(t,s)P + K(t,s), \ \ t \geq s$$

with $K(t,s)$ compact operator $L^2 \to L^2$ for all t, s and P the canonical projection $L^2 \to \tilde{L}^2$.

Theorem

If the high frequency system is exponentially stable then the monodromy operator $U(T,0)$ possesses at most a finite number of eigenvalues $\zeta_1, \cdots, \zeta_n$ outside a disk of a radius strictly less than 1.

L^2 stability equivalent to C^0 stability

Proposition (Chitour and al 2016, BFLP 2019)

A periodic delay system is L^2 exponentially stable if and only if it is C^0 exponentially stable.

Theorem

Assuming the system at high frequency is L^2 exponentially stable. Then the general system is L^2 exponentially stable if and only if it is C^0 exponentially stable.

Assumption : High frequency system exponentially stable $\Leftarrow$ always true for "realistic" circuit.

Input-Output system

$$\begin{cases} \frac{dx(t)}{dt} = A_1(t)x(t) + \sum_{i=0}^{N} B_i^1(t)y(t-\tau_i) + C_1(t)u(t) \\ y(t) = \sum_{i=1}^{N} B_i^2(t)y(t-\tau_i) + A_2(t)x(t) + C_2(t)u(t) \\ z(t) = \sum_{i=1}^{N} B_i^3(t)y(t-\tau_i) + A_3(t)x(t) + C_3(t)u(t), \;\; t \geq 0, \end{cases}$$

- $x(t), y(t), z(t) = 0$ for $t < 0$,
- Input $u \in L^2_{loc}([0, +\infty), \mathbb{R})$ current perturbation, output z the voltage,
- All coefficients are $T - periodic$.

- $X(t,\alpha)$ response at time t to an impulse at time α
  $z(t) = \int_0^t X(t,\alpha)u(\alpha)d\alpha$
- $G(s,t) = \int_0^{+\infty} X(t,t-\alpha)e^{-s\alpha}d\alpha$: Laplace Transform
- $G_k(s) = \frac{1}{T}\int_0^T G(s,t)e^{ik\omega_0 t}dt$ with $\omega_0 := \frac{2\pi}{T}$

Definition (Harmonic Transfer Function HTF)

The infinite matrix $H(s)$ defined by $H_{m,n}(s) := G_{n-m}(s + \frac{2i\pi m}{T})$ for $s \in \mathbb{C}$ is called the harmonic transfer function.

$$Z(s) := \int_0^{+\infty} z(t)e^{-st}dt \text{ and } U(s) := \int_0^{+\infty} u(t)e^{-st}dt.$$

$$\begin{pmatrix} \vdots \\ Z(s+i\omega_0) \\ Z(s) \\ Z(s-i\omega_0) \\ \vdots \end{pmatrix} = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \cdots & G_2(s-i\omega_0) & G_1(s) & G_0(s+i\omega_0) & \cdots \\ \cdots & G_1(s-i\omega_0) & G_0(s) & G_{-1}(s+i\omega_0) & \cdots \\ \cdots & G_0(s-i\omega_0) & G_{-1}(s) & G_{-2}(s+i\omega_0) & \cdots \\ \iddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ U(s+i\omega_0) \\ U(s) \\ U(s-i\omega_0) \\ \vdots \end{pmatrix}$$

- HTF is an operator valued analytic map (values: continuous ops $l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$)
- its entries $\{G_n\}$ are complex valued analytic maps

Structure of the Harmonic Transfer Function

Define $z_{j,k} = \frac{ln(\zeta_j)+2ik\pi}{T}$ for j in $\{1...n\}$, k in $\mathbb{Z}$.

Theorem

In $\{s \in \mathbb{C}, \Re(s) \geq \gamma\}$ for some $\gamma < 0$,

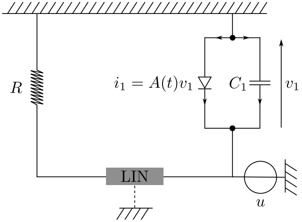
- *H is a meromorphic operator $l^2(\mathbb{Z}) \rightarrow l^2(\mathbb{Z})$ wich possibly poles at $\{z_{j,k}, j \in \{1...n\}, k \in \mathbb{Z}\}$.*

Under observability/controllability assumptions,

- *for all j, there is at leat a k such that $z_{j,k}$ is a pole of H, and also a pole of one G_n.*

If no pole in right half plane, exponential C^0 stability.

Example : Brayton 1976



Theorem

If $T/\tau_1 \notin \mathbb{Q}$, all points of a certain vertical line in the left-half plane are essential singularities of the HTF, as an operator valued analytic map.

But are they singularities for some G_n ?

Contribution

- Math. foundation of HF amplifiers stability decision in CAD
- Projection on the unstable part and rationnal approximation to find the poles
- Advances in stability of periodic delay systems

Open questions

- For fixed j, which $z_{j,k}$ is a pole of which G_n?
  (In practice, few G_n are computed.)
- Bound on the number of unstable poles?
- May the (stable) singularities of the G_n's be other than poles ?