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OPTIMIZATION-BASED APPROACH FOR A BETTER CENTRIFUGAL SPREADING

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Abstract: The use of centrifugal spreaders for application of granular fertilizers raises concern about application accuracy. These spreaders enable to distribute almost uniform deposits with regularly spaced parallel tramlines but lead over and under fertilizer application when distances separating the successive tractor trajectories are not constant or if paths are not parallel between them. The aim of this study is to propose an optimization-based method to compute optimal variables for uniform fertilizer application. Mechanical constraints are also introduced so that the calculated parameters can be afterwards used as reference variables for the control of the spreader. The resulting improvements are illustrated by numerical examples for a field with geometrical singularities.

Keywords: Parameter optimization, Reference variables, Mathematical programming, Agriculture, Environmental engineering

1. INTRODUCTION

Fertilization practice is one of the most important operations in agricultural production. Indeed, this task is necessary to apply nutrients satisfying the demands of the plants according to yield objectives. To carry out inputs application, centrifugal spreaders are very popular thanks to their low cost and simplicity of use. Unfortunately, because of an inappropriate strategy, they lead to application errors. These errors involve negative consequences such as fertilizers waste, dramatic drop in production and harmful environmental effects like eutrophication phenomenon for example (Isherwood, 1998). Faced with these statements, american and european governments decided to impose strict rules concerning the fertilization practice. These economic and environmental pres-

sure cause then a significant increase in investigations for better nutrients applications. In this paper, we develop a new approach to improve fertization accuracy. The method is based on an optimization strategy where the cost function is formalized from a spatial distribution model previously studied in (Colin, 1997; Olieslagers, 1997). In order to not untimely solicit actuators and respect mechanical limits of the spreader, we are led to consider an optimization problem subject to constraints. Furthermore, thanks to this approach, optimal parameters should be used as reference variables for the future control of the machine.

This document is organized as follows. The next section deals with centrifugal spreading principles and drawbacks. In section 3, the criterion function

is formalized from the existing fertilizers spatial distribution model. Faced with a large scale problem, a decomposition of this one is proposed in section 4. The last section illustrates simulation results obtained after optimization algorithm execution in the main field body with parallel and non parallel tractor paths.

2. CENTRIFUGAL SPREADING CONTEXT

According to the soil and crops characteristics, agricultural engineers specify precise application rate to permit a correct growth of plants. These recommended doses often take the form of prescribed dose maps. These ones depicts the considered field which is virtually gridded and where each mesh corresponds to the previously specified doses. The operation consisting in applying nutrients with respect to the map represents fertilization itself. To get the distributed doses close to the desired ones, centrifugal spreaders with dual spinning discs are the most used applicators. During the spreading process, while the tractor progresses, fertilizers granulars contained in the hopper pour onto each disc and are ejected by centrifugal effect. With precision farming technologies, in order to apply inputs according to the machine location and a prescribed dose map, tractor-implement combination is equipped with a GPS antenna, a radar speed sensor and an actuator. The two first tools enable to know the tractor position and speed. Concerning the actuator, it permits to control the fertilizer mass flow rate. The actual amount of applied fertilizers currently called spread pattern has an irregular distribution which is often underlined by the transverse distribution curve obtained by summing the amounts along each travel direction. An example of spread pattern and its related transverse distribution for only one disc are illustrated in Fig.1. As depicted in Fig.2, this spatial distribution heterogeneousness lead then the tractor driver to follow outward
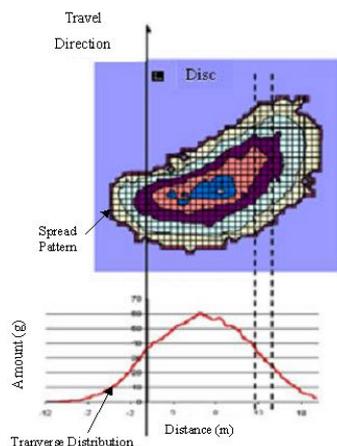
and return paths in order to obtain an uniform deposit from transverse distributions summation for each successive travels within the field. The spreading regularity is achieved when the distance between two consecutive overlapping lines, currently called working width, is equal to the distance separating two successive tramlines. Besides, these overlapping lines correspond to symmetry axis which make two successive paths coincide. So, it is clear that fertilization strategy is today based above all on best overlappings of the transverse distributions with respect to the different tractor paths. Most of the experiments and simulations undertaken in order to assess fertilizer application accuracy or study device settings are only carried out by using this method as specified in test procedures defined by (ASAE, 2001). Some studies relying on these tests are also led to assess performance of applicators like in (Yule et al., 2005). Unfortunately, this reasoning do not take into account the actual phenomenon occurring during the spreading process. Indeed, in fact, the global deposit of nutrients in the farmland is due to spread pattern overlappings and thus is the result of the heterogeneous spatial distribution summation at each position of the spreader. Then, though the application results are correct for parallel paths, some local application errors are unavoidable when geometrical singularities are met in the field like non parallel tramlines, start and end of spreading...Some errors examples are illustrated in Fig.3 where results are obtained by simulating spread patterns overlappings with settings imposed by manufacturers rules. To minimize these issues, a solution could be to look for optimal paths for the machine as in (Dillon et al., 2003). However, this type of solution cannot
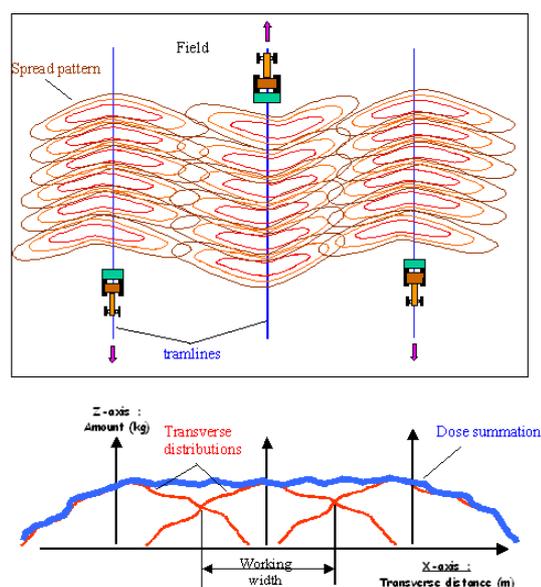

Fig. 1. Spread pattern (spatial distribution) and transverse distribution

Fig. 2. Fertilization strategy based on transverse distribution summation.

(a) Prescribed dose map obtained from agronomical considerations.

(b) Actual dose map obtained by applying the fertilization method based on adjustment of transverse distribution.

(c) Application errors map calculated from the difference between the prescribed dose map and actual dose map.
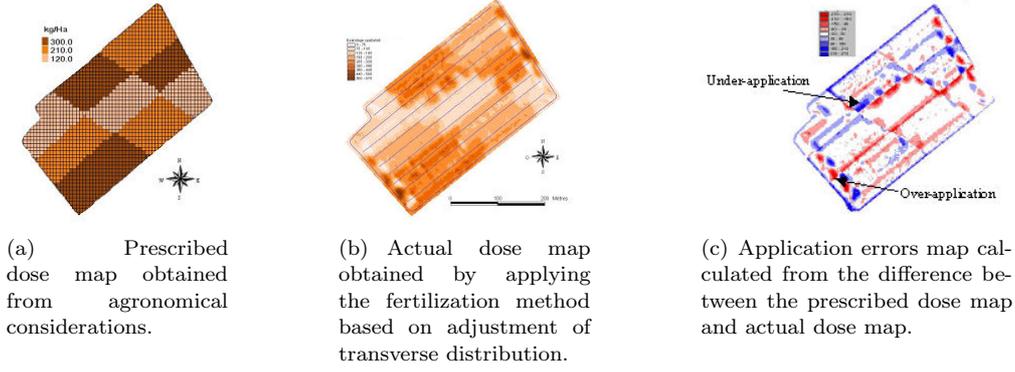
Fig. 3. Application errors resulting from the reasoning based on the best transverse distribution investigation and not on spread patterns overlappings.

be applied when tramlines are already fixed by other agricultural operations like sowing. So, it is obvious that some efforts must be done to achieve better spread patterns arrangement according to geometrical constraints met in farmland during fertilization practice. The computed adjustments should be continuously achieved for each position of applicator by modifying its settings. In this work, we study then a method which permits to calculate optimal parameters to have the best spread patterns arrangement within the arable land in the presence of imposed tractor trajectories.

3. OPTIMIZATION CRITERION

Let us first precise some notations used to define the spread pattern model. This model uses different variables such as the time ($t \in \mathbb{R}$), the spatial domain, in other words the field, ($\Omega \in \mathbb{R}^2$), the path ($s(t) \in \Omega$), the coordinates of points ($x \in \Omega$), the distance between $s(t)$ and x ($r(x,t) \in \mathbb{R}$) and the angle between $\overrightarrow{s(t)x}$ and $s(t)$ ($\theta(x,t) \in \mathbb{R}$). The spread pattern is currently defined by its medium radius and medium angle. The first parameter, varying with the speed of disc, corresponds to the distance between the disc centre and the spread pattern one while the second, modifiable with the fertilizers dropping point on the disc, states the angle between the travel direction and the straight line passing through the disc centre and the spatial distribution one. The respective mass flow rates for the right and left discs are defined by $m(t)$ and $d(t)$. $\rho(t)$ and $\xi(t)$ stand for the medium radius related to the right and left discs respectively. At last, the right and left discs medium angles are defined by $\xi(t)$ and $\psi(t)$. Furthermore, if we assume σ_r and σ_θ to be the respective constant standard deviations for the medium radius and the medium angle, we can calculate the right and left spatial distributions q_r and q_l, according to (Colin, 1997), as:

$$q_r(x, m(t), \rho(t), \varphi(t)) = \tau \cdot \exp(-A(x,t)^2/a)$$
$$\cdot \exp(-B(x,t)^2/b) \quad (1)$$
$$q_l(x, d(t), \xi(t), \psi(t)) = \kappa \cdot \exp(-C(x,t)^2/a)$$
$$\cdot \exp(-D(x,t)^2/b) \quad (2)$$
$$\text{with} \qquad A(x,t) = r(x,t) - \rho(t) \qquad (3)$$
$$B(x,t) = \theta(x,t) - \varphi(t) \qquad (4)$$
$$C(x,t) = r(x,t) - \xi(t) \qquad (5)$$
$$D(x,t) = \theta(x,t) - \psi(t) \qquad (6)$$

and where $a = 2\sigma_r^2$, $b = 2\sigma_\theta^2$, $\tau = m(t)/(2\pi\sigma_r\sigma_\theta)$ and $\kappa = d(t)/(2\pi\sigma_r\sigma_\theta)$. To simplify notations, we define $M(t) = (m(t), d(t)) \in \mathbb{R}^2$, $R(t) = (\rho(t), \xi(t)) \in \mathbb{R}^2$ and $\Phi(t) = (\varphi(t), \psi(t)) \in \mathbb{R}^2$. The global distribution is then equal to the summation of right and left contributions:

$$q_{tot}(x, M(t), R(t), \Phi(t)) = q_r(x, m(t), \rho(t), \varphi(t))$$
$$+ q_l(x, d(t), \xi(t), \psi(t)) \quad (7)$$

Therefore, the actual distributed dose $Q \in \mathbb{R}^2$ during the interval of time $(0, T)$ for single tramline can be evaluated as:

$$Q(x, M, R, \Phi) = \int_0^T q_{tot}(x, M(t), R(t), \Phi(t)) \, dt \quad (8)$$

If $Q^* \in \mathbb{R}^2$ stands for the prescribed dose, to reduce harmful fertilization effects, the following functional must be minimized:

$$F(M, R, \Phi) = \int_\Omega [Q(x, M, R, \Phi) - Q^*]^2 \, dx \quad (9)$$

Given that (9) cannot be calculated in an analytical way, discretization is carried out. Then, Ω is gridded so that Q and Q^* can be computed with bilinear approximations. A temporal discretization is also performed by dividing the interval $(0, T)$ into n elements with equal length $\delta = T/n$. Thus, we can define $t_j = j\delta$ with $j = 0, 1, ..., n$. Consequently, we can assume $M_j = M(t_j)$, $R_j = R(t_j)$ and $\Phi_j = \Phi(t_j)$. The corresponding vectors are defined as $\boldsymbol{M} = [M_0 \cdots M_n]^T$, $\boldsymbol{R} = [R_0 \cdots R_n]^T$ and $\boldsymbol{\Phi} = [\Phi_0 \cdots \Phi_n]^T$. Moreover, me-

chanical limits are taken into account, by assuming the functions M, R and Φ and their time derivative to be subject to bound constraints. The set of solutions is then $S = \left\{(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) \in \mathbb{R}^{6(n+1)}\right\}$ so that:

$$\begin{cases} M_{min} \leq \boldsymbol{M} \leq M_{max} \\ R_{min} \leq \boldsymbol{R} \leq R_{max} \\ \Phi_{min} \leq \boldsymbol{\Phi} \leq \Phi_{max} \\ |M_{i+1} - M_i| \leq \alpha\delta \\ |R_{i+1} - R_i| \leq \beta\delta \\ |\Phi_{i+1} - \Phi_i| \leq \gamma\delta, \end{cases} \quad (10)$$

whith α, β and γ are known parameters fixed according to the spreader characteristics. Consequently, we are led to consider the nonlinear programming problem given by:

$$(\mathcal{P}) \qquad \min_{(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) \in S} F(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{\Phi}) \qquad (11)$$

Given that S is bounded closed, according to the Weierstrass theorem, there exists at least one solution to $(\mathcal{P})$. The field mostly contains several tramlines and thus the actual distributed dose is the result of the summation of the applied dose for each k-indexed tractor trajectory:

$$Q(x, U) = \sum_{k=1}^{w} Q_k(x, U) \qquad (12)$$

$$\text{with } Q_k(x, U) = \int_{t_i^k}^{t_f^k} q_{tot}(x, M(t), R(t), \Phi(t)) dt \, (13)$$

and where $U = (M, R, \Phi)$. w is, here, the number of paths and the trajectories $s^k(t)$ are assumed to be defined in the interval (t_i^k, t_f^k). By considering the definitions of $M_j^k = M(t_j^k)$, $R_j^k = R(t_j^k)$, and $\Phi_j^k = \Phi(t_j^k)$ we can also use the discretization techniques as before. Therefore, from these definitions, optimization in the whole field considering all paths can be carried out by solving the problem $(\mathcal{P})$.

4. PROBLEM DECOMPOSITION

Like for the main prescribed dose map depicted in Fig.3(a), farmland are virtually 1 m-gridded. By applying the discretization method previously detailed, we are led to consider a large scale problem. Indeed,to lose informations as little as possible, 2 samples of parameters per elementary mesh are computed. If we assume only 3 paths 100 m long in the field, the number of parameters reaches then 3600. So, it is clear that this large scale problem cannot be solved directly by an optimization algorithm in view of the high computational time which would be involved. Consequently, a problem decomposition is necessary. First of all, let us define the following notations:

$$K_1 = \{k \in \mathbb{N} | \, 1 \leq k \leq w\},$$
$$K_2 = \{k \in \mathbb{N} | \, 1 \leq k \leq w - 1\},$$
$$K_3 = \{k \in \mathbb{N} | \, 2 \leq k \leq w\},$$

$$L_1 = \{l \in \mathbb{N} | \, \forall z \geq 2 \in \mathbb{N}, \, 1 \leq l \leq z\},$$
$$L_2 = \{l \in \mathbb{N} | \, \forall z \geq 2 \in \mathbb{N}, \, 2 \leq l \leq z\},$$
$$\Omega = \bigcup_{k \in K_1} \Omega^k, \qquad \Omega^k = \bigcup_{l \in L_1} \Omega_l^k,$$

where $\Omega^k \in \mathbb{R}^2$ the kth subdomain of Ω, and $\Omega_l^k \in \mathbb{R}^2$ the lth subdomain of Ω^k. To deal with each path $s^k(t)$ apart from the others, the subdomains Ω^k are defined so that:

$$\partial\Omega^k \cap \Omega^{k+1} = s^{k+1}(t), \forall (k,t) \in K_2 \times (t_i^{k+1}, t_f^{k+1})$$
$$\partial\Omega^k \cap \Omega^{k-1} = s^{k-1}(t), \forall (k,t) \in K_3 \times (t_i^{k-1}, t_f^{k-1})$$

In order to make easier to understand the spatial decomposition, Fig.4 illustrates the example of three parallel tramlines in a domain Ω with a rectangular geometry. Furthermore, by assuming the vectors $\boldsymbol{M}_l^k$, $\boldsymbol{R}_l^k$, $\boldsymbol{\Phi}_l^k$ to be the respective restrictions of $\boldsymbol{M}$, $\boldsymbol{R}$ and $\boldsymbol{\Phi}$ in the subdomain Ω_l^k, we can also define the set S_l^k as the restriction of S in the same subdomain. By considering the symmetries conditions exposed in the first section, $(\mathcal{P})$ is naturally decomposed in the following way:

$$(\mathcal{P}') \begin{cases} \min \sum_{l=1}^{z} \sum_{k=1}^{w} J_l^k(x, \boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \\ (\boldsymbol{m}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \in S_l^k, \, (l,k) \in L_1 \times K_1 \end{cases} \quad (14)$$

whith

$$J_l^k(x, \boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) = \int_{\Omega_l^k} [Q_l^k(x) - Q^*]^2 dx \quad (15)$$

where $Q_l^k(x)$ is the actual applied dose within Ω_l^k taking into account not only the amounts already distributed in Ω_l^{k-1} and Ω_{l-1}^k but also the future ejected dose along the path $s^{k+1}(t)$ which is predicted so that it respects the symmetries properties previously underlined. The problem $(\mathcal{P}')$ is an optimization problem subject to inequality constraints. For $(l,k) \in L_1 \times K_1$, to minimize the functional J_l^k, the problem $(\mathcal{P}_{ineq})$ is considered and defined as:

$$(\mathcal{P}_{ineq}) \begin{cases} \min J_l^k(\boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \\ u_j \leq h_j(\boldsymbol{M}_l^k, \boldsymbol{R}_l^k, \boldsymbol{\Phi}_l^k) \leq v_j, \\ j = 1, 2, \dots dim(\boldsymbol{M}_l^k) \end{cases} \quad (16)$$
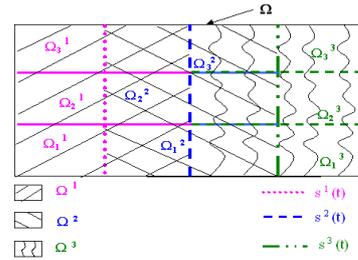

Fig. 4. Rectangular domain Ω divided into 9 subdomains Ω_l^k, $1 \leq l \leq 3$, $1 \leq k \leq 3$.

where h_j stands for the jth double inequality, u_j and v_j its lower and upper bound. To obtain an acceptable solution after algorithm execution and avoid solving the problem which consists in determining saturated constraints, we decide to apply an augmented lagrangian algorithm (Bertsekas, 1982) which severely penalizes unacceptable parameters. Moreover, we also use a l-bfgs technique shown to be efficient with large scale optimization problem (Byrd et al., 1994).

5. NUMERICAL RESULTS

In our case, we are only interested in application optimization in the main field body. The prescribed dose is fixed at 100 Kh/Ha. Indeed, nowadays, even in the case of fixed desired dose, farmers are unable to obtain satisfying results. The speed of tractor is also constant and equal to 10 Km/h. The studied field is illustrated in Fig.5. The paths 2 to 6 are parallel. The default working width is defined as being 24 m. However, the distance between the 4th and 5th tramlines is equal to 25 m. Besides, the 6th path is 19 m from the 5th one. At last, the first path includes a travel direction change causing a bottleneck at the headland. Nowadays, for this kind of farmland, the settings imposed by manufacturers do not change along the process and correspond to the rules established for a 24 m working width. As shown in Fig.6, these settings lead to over-application between the 5th and 6th tramlines and also at the end of the first trajectory. An under-dosage zone appears slightly below the break-point marking the beginning of the travel direction change for the first path. Everywhere else, error is included between -8% and +6% and are then acceptable. Before applying our optimization algorithm, mechanical constraints are imposed by taking into account the characteristics of the most used applicators. From these constraints, after algorithm execution, we succeed in reducing fertilization errors between -8.7% and +6.2% as illustrated in Fig.7 which is very satisfying. We obtain little
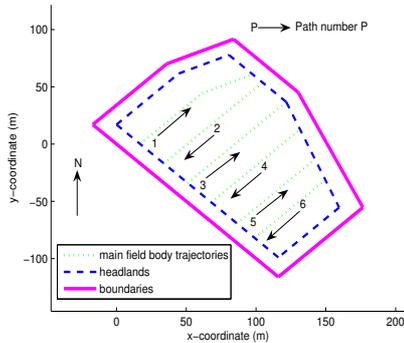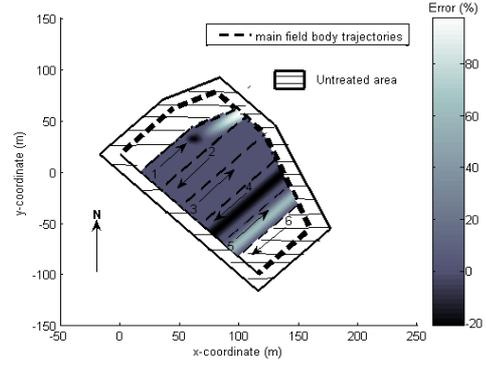

Fig. 6. Application errors obtained with the manufacturers settings

application errors for the first and second travels. Here for successive parallel tramlines, optimal parameters are computed so that they are time independent. Thus for the trajectories 3 to 6, these ones are gathered in Table 1. As expected, for the path 5 the mass flow rate for the right disc slumps because of the narrow pass occurring in this case. The corresponding medium radius and angle are also slightly modified to obtain uniform overlappings. Unlike the parallel tramlines, the optimal parameters are time dependent for the two first trajectories. The optimal variables for the first path are shown in Fig.8. For the left disc, after the travel direction shift, the medium radius slumps. Concerning the medium angle and mass flow rate, slight variations occur around the break-point marking the travel direction shift. Regarding the right disc, the medium angle increases before the break-point and drops straight away after this one. The mass flow rate displays the same trends for this path. Considering the optimal decision variables during the second path in Fig.9, we can note that all parameters increase at the beginning and stabilize after a while. This phenomenon is inherent to the narrowing existing when the tractor comes in the arable land and the pass widening occurring after some distance. Here, for the different studied cases, the computed parameters, respecting the overall specified con-
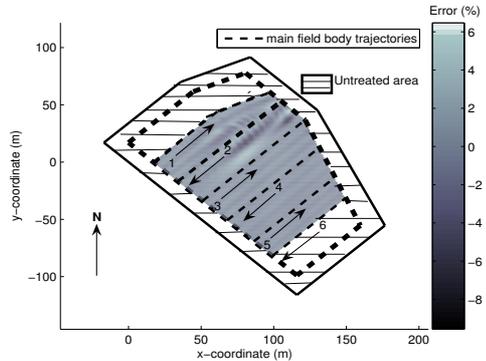

Fig. 5. Field with parallel and non parallel tramlines

Fig. 7. Application errors obtained after optimization

Table 1. Optimal values for successive parallel tramlines (M_f: Mass Flow Rate (Kg/min); R_m: Medium Radius; θ_m: Medium Angle (°))

Left Disc				
	Path 3	Path 4	Path 5	Path 6
M_f	20.17	22.99	24.94	21.11
R_m	15.33	16.45	16.63	15.11
θ_m	-19.68	-19.13	-17.28	-19.48
Right Disc				
	Path 3	Path 4	Path 5	Path 6
M_f	19.84	18.67	11.74	13.89
R_m	15.24	15.18	12.51	12.72
θ_m	20.12	21	18.74	17.22

straints, can then used as reference variables in a future control strategy. Thus, the machine settings (speed of discs, fertilizers dropping point position and mass flow rate) can be continuously modified along each tramline as advised by (Olieslagers *et al.*, 1997).

6. CONCLUSION

A new method relying on optimization techniques has been proposed in order to fulfill the economic and environmental requirements inherent to fertilization practice. From the spread pattern analytical model and the mechanical characteristics of the spreader, a cost function subject to constraints has been formalized. Faced with a large scale problem after discretization, a decomposition of this one has been proposed. In order to deal with the constraints and the large size of the problem, an augmented lagrangian method associated with a l-bfgs technique have been implemented. Thanks to this strategy, fertilization errors have been significantly reduced and the optimal parameters should be considered as reference values to control the machine. In order to achieve a spatial optimization within the whole farmland, works are necessary to deal with the boundaries
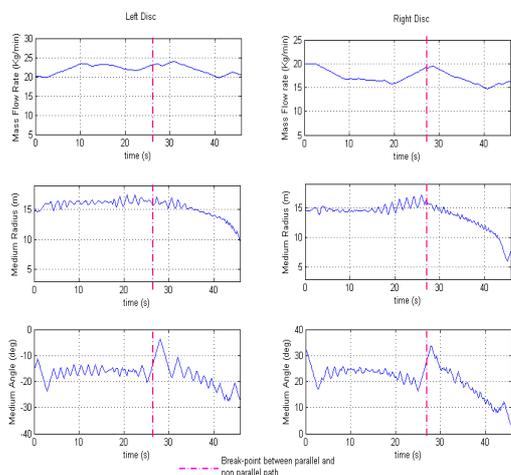

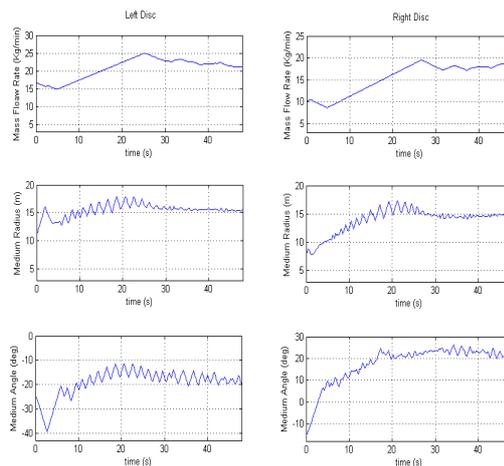Fig. 8. Optimal parameters for the first path

Fig. 9. Optimal parameters for the second path

where modified spread patterns are often applied. Combined with the use of information technology and recent advances in agricultural vehicle guidance, these studies should efficiently improve fertilization application for the future.

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