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解　説

Model Predictive Control for Biped Walking Motion Generation

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1. Introduction

One of the major difficulties in making a robot walk is keeping its balance. Not considering other important questions such as energy efficiency, keeping the balance of the robot will be the only focus of this article: where should the robot place its feet, how should it move its body in order to move safely in a given direction, even in case of strong perturbations?

This major difficulty comes from the contact forces with the ground, which are needed for walking, but unfortunately constrained in direction and amplitude. As stated in a key review paper in control theory [20]: "The prevalence of hard constraints is accompanied by a dearth of control methods for handling them (...) Model Predictive Control is one of few suitable methods (...)". We are going to review therefore in this article how Model Predictive Control (MPC) is used to generate stable dynamic walking motions. For a general introduction to MPC, please refer to Ref. [1] in this Journal issue.

An early observation, extremely efficient and far-reaching, is that not all the motion of the robot is constrained. As a result, the idea of *artificial synergy synthesis* [30] is to assign some degrees of freedom of the robot to take care of these ground contact forces constraints, so that the rest of the motion of the robot can be realized almost independently. The original proposition was to use trunk rotations to ensure that the ground contact forces follow a pre-defined pattern, more precisely, a pre-defined trajectory of the Center of Pressure (CoP), also called the Zero Moment Point (ZMP), an approach successfully used for example on the WL-12RV robot in Waseda University [35].

But it has been argued that predefining the evolution of the CoP is not necessary nor even desirable [15] [33], and trunk rotations have been shown to have a relatively weak influence on balance [6]. As a result, the prevailing option has been to handle the contact force constraints directly through the motion of the Center of Mass (CoM) of the robot. For this reason, the focus of this paper will be on the motion of the CoM with respect to the contact points with the ground. Angular momentum will also play a role, but all the rest of the motion of the robot will be considered to be tackled independently, either with standard Inverse Kinematics methods, or more advanced approaches such as discussed in [8] in this Journal issue.

2. The dynamics of walking, Viability and Capturability

As for all robots which have the capacity to move in their environment, the dynamics of walking robots can be separated between joint dynamics, and displacement dynamics, the latter involving their angular momentum L and linear momentum $P = m\dot{c}$, where m is the whole mass of the robot, and c the position of its CoM. In order to control these linear and angular momentums, the robot needs external forces, ground contact forces in the case of walking [32].

The constraints on the ground contact forces are usually handled by focusing on the CoP p, which, in case of unilateral contact, is bound to stay in the support polygon $\mathcal{S}_i$. The CoP is related to the linear and angular momentums in the following way:

$$p^{x,y} = c^{x,y} - \frac{mc^z(\ddot{c}^{x,y} + g^{x,y}) + R\dot{L}^{x,y}}{m(\ddot{c}^z + g^z)} \in \mathcal{S}_i, \tag{1}$$

where g is the gravity vector, R a simple $\pi/2$ rotation, and the coordinates x and y follow the contact surface

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while z is naturally orthogonal. In case the walking surface is horizontal, $g^{x,y}=0$, and a standard simplification is to consider that the effect of vertical motions of the CoM can be neglected, as well as the effect of variations $\dot{L}^{x,y}$ of the angular momentum, what leads to the simplified relation

$$p^{x,y}=c^{x,y}-\frac{c^z}{g^z}\ddot{c}^{x,y}\in\mathcal{S}_i, \tag{2}$$

which makes clear how the motion of the CoM of the robot is related to the ground contact forces constraints. As mentioned in the Introduction, the joint dynamics will be considered to be tackled independently.

A key aspect of this constrained dynamics is that the robot can be in a state where it hasn't fallen yet, but it has already lost balance completely, so it is bound to fall inexorably, as imposed by the constraints [34]. Formalazing this commonplace observation requires mathematical tools which are slightly unusual in standard control theory, mostly developed in the Viability theory [4], and introduced for the analysis of walking robots in [31].

Such states where the robot is bound to fall are called *non-viable*, and avoiding them requires a capacity to anticipate consequences of actions well ahead of time. This is where Model Predictive Control proves to be helpful. But we still need a way to discriminate viable states from non-viable ones, and in the general case, this is not possible without unreasonably lengthy computations.

Some specific cases can be identified as viable, such as cyclic motions, which have been the focus of limit cycle walking robots [13]. Note that biped walking is obviously a rythmic motion, alternating left and right steps, but in the general case, it may not be strictly cyclic at all: we need a more general approach. One of the most effective options then is to focus on so-called *capturable* states, defined by the ability to stop safely in 0, 1 or any given number of steps. This definition encompasses most of the viable states of interest for walking robots [18].

Fortunately, capturable states can be identified analytically with the help of the compound variable

$$\xi=c+\frac{1}{\omega}\dot{c}, \tag{3}$$

introduced independently as the *eXtrapolated Center of Mass* [14], the *Capture Point* [26] or the *divergent component* of the dynamics [28]. Three denominations that correspond to three key properties of this variable: it lies ahead of the CoM, and if it lies within the support polygon, it can be kept fixed, so the CoM converges to it and comes to a stop.

Another option is to observe that falling motions are motions with very high dynamics, diverging exponentially in the case of the linear dynamics (2) [34]. As a result, looking to continuously minimize the integral of the norm of any n^{th} derivative of the motion of the CoM

$$\int_{t_0}^{\infty}\left\|c^{(n)}(t)\right\|dt \tag{4}$$

appears to naturally avoid falling motions when possible, maintaining the viability of the robot when possible.

We are going to see now that the walking motion generation schemes that allow most of the great humanoid robots of today and yesterday to walk (and run) appear to be a form of MPC, although rarely stated that way, involving one of the two approaches we have just seen: capturability, or optimal control of the CoM.

3. Model Predictive Control

There are two fundamental ingredients to prove stability in MPC theory: integral cost functions that have to be minimized in the future, and *terminal constraints* that the system is imposed to satisfy in the near future [17]. Many variants have been proposed [20], but two extremes are particularly significant in the case of walking robots, either considering only an integral cost function, without any terminal constraint [2], or the opposite, only a terminal constraint, without any cost function [3]. These two control designs can be proven to lead to stable behaviors under mild technical conditions. In the case of walking robots, cost functions will naturally be related to the integral (4), while terminal constraints will naturally be capturability constraints, either constraining the whole state of the CoM, or only the divergent component (3) of its dynamics.

The earliest approach to walking motion generation was to first compute footprints, depending on the environment of the robot and its goal, and compute the CoM and feet motion only once these footprints have been fixed. In order to do so, the simplified dynamics (2) is successfully used in most cases, but the more complete dynamics (1) has also been considered, either with the whole dynamics of the robot, or through more simple N point mass models.

Taking into account the support polygon as a strict constraint in (1) or (2) has usually been regarded as too complex numerically, and therefore rarely considered this way (we will discuss this point in the next Section, which focuses on numerical considerations). As a result, the most frequent approaches are either to simply minimize the distance between the CoP and the center of the support polygon, or to predefine its trajectory within the support polygon with very little possible variations.

In most approaches, it is imposed then that the CoM comes to a stop at the end of the next two steps. It won't really stop there, since this constraint is continuously receding away in the future, always kept at the end of the next two steps: following the theory of MPC, this is a *terminal constraint*, that only imposes the capturability, and therefore the viability of the state of the robot. This approach has been successfully used in the recent humanoid robots of Waseda University [19], in the Johnny robot of Munich University [5], the H7 robot of Tokyo University [24], the Toyota Partner robots [27], the Sony QRIO robot [22], and also the Kawada HRP-2 robot [21]. The Honda Asimo robot has been the first to constrain only the divergent component (3) of its dynamics [28].

The standard walking motion generation scheme of the Kawada HRP-2 robot has been the first one to implement the other approach, without terminal constraints, and based instead on the optimal control of the CoM [16]. This allows immediate adaptation of the motion of the CoM in case of perturbations [23], and has been further refined by considering the support polygon in (2) as a strict constraint [33], what is successfully used in Aldebaran Nao robots [10].

It is only recently that the footprints have been considered as additional parameters within the MPC schemes, allowing much more reactive walking motions, especially in case of strong perturbations. This has been tested very successfully with a terminal contraint on the divergent component of the dynamics in Ref. [29], and proposed also with the optimal control of the CoM approach in Ref. [11]. Note that if the footprints are decided online, their geometric feasibility needs to be checked online as well. A simple but effective option to do so is to consider a polygonal approximation of the reachable volume of the CoM with respect to each foot on the ground [12].

4. Numerical considerations

When having to find the solution to a mathematical problem, what we usually need in robotics is an efficient numerical algorithm, to compute this solution online on an embedded computer. In this regard, concentrating on analytical solutions can be misleading since analytical formulas may turn out very complex to compute, and the other way around, efficient algorithms may exist to compute mathematical objects very complex to describe analytically. This issue is particularly important for the MPC schemes discussed here, which take the form of moderately complex optimization problems, but can be solved in fact very efficiently if approached in a proper way, even in the absence of analytical formulas.

The MPC schemes discussed here turn out to be Quadratic Programs (QP) in most cases. When unconstrained, these can be expressed as

$$\min_{x} \ \frac{1}{2}x^T Q x + g^T x, \tag{5}$$

and require simply solving a linear system

$$Qx + g = 0. \tag{6}$$

In our case, the matrix Q is often predictable and structured in such a way that solving this system can be done with just a matrix-vector multiplication, or even less [9].

In case there are linear equality constraints

$$Ax + b = 0, \tag{7}$$

such as a terminal constraint, or constraints on the CoP, the linear system that has to be solved can be more complex:

$$\begin{pmatrix} Q & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} x \\ -\lambda \end{pmatrix} + \begin{pmatrix} g \\ b \end{pmatrix} = 0. \tag{8}$$

But if these constraints happen to be trivial and directly fix some variables of the problem,

$$\{x_i = b_i, \ldots\}, \tag{9}$$

the constrained problem will in fact be faster to solve than the unconstrained one, since it will obviously involve less free variables. In this case, the more the problem is constrained, the faster it is to solve!

As a result, it can have a huge impact on computational efficiency to express our optimization problem with respect to a variable x which is directly the physical value that may be constrained. In our case, this physical value is naturally the CoP [7].

In case there are linear inequality constraints

$$Ax + b \leq 0, \tag{10}$$

a few iterations may be required [25]. In our case though, we have to solve a new QP at each sampling time, which is very close to the previous one. As a result, very few iterations are necessary if you already know the solution to the previous QP, and with a careful design and implementation, it is even possible to do without any iterations at all [9].

What we observe here is that a seemingly complex inequality constrained QP can be solved in fact very efficiently, and should be regarded therefore as a very sensible and effective option when considering advanced controllers such as the MPC schemes discussed here for biped walking robots. The conclusion is that roboticists shouldn't be afraid of seemingly complex control designs, considering for example the support polygon in (2) as a strict constraint, since this may in fact be very efficient to compute.

5. Conclusion

We have seen that the walking motion generation schemes that allow most of the great humanoid robots of today and yesterday to walk (and run) appear to be a form of Model Predictive Control, although rarely stated that way. All these schemes anticipate the motion of the robot during the next two steps in order to ensure viability, either through a capturability terminal constraint or through optimal control of the CoM, which are the two fundamental ingredients used to prove stability in MPC theory.

In all cases, the artificial synergy synthesis approach is adopted, focusing on the motion of the CoM of the robot with respect to contact points, considering that the rest of the motion can be handled more or less independently. All these motion generation schemes, as diverse as they may look, appear to share in fact the same general design: Model Predictive Control of the CoM of the robot with respect to contact points. The rest is details.

This reveals the possibility to crossbreed all these approaches, retaining the best features of each: more precise multiple mass models, for generating both walking and running motions, with online footprint selection and adaptive timing, sensor feedback, all in order to obtain in the end the ultimate robust and versatile online motion generation scheme. These are the next obvious steps.

Some of the best legged robots, the Boston Dynamics and Schaft biped and quadruped robots, are missing from this analysis. They are without a doubt the legged robots exhibiting the most impressive dynamic motions today, robust and versatile. Exact details about their control algorithms are scarce, but various clues suggest that they might share the same design discussed here.

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