State and parameter estimation in linear systems with delays

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Consider systems in the form of

$$\dot{x}(t) = A(t)x(t) + B(t)x(t-\tau) + \Phi(t)\theta \tag{1a}$$
$$y(t) = C(t)x(t) + D(t)x(t-\tau) \tag{1b}$$

where θ is an *unknown* constant vector. The matrices $A(t), B(t), C(t), D(t), \Phi(t)$ are known, bounded and piecewise continuous. The delay τ is known.

Remark: $\Phi(t)\theta$ can be generalized to $\Phi_1(t)\theta_1 + \Phi_2(t-\tau)\theta_2$: let $\Phi(t) = [\Phi_1(t)\ \Phi_2(t-\tau)]$ and $\theta^T = [\theta_1^T\ \theta_2^T]$ so that $\Phi(t)\theta = \Phi_1(t)\theta_1 + \Phi_2(t-\tau)\theta_2$. More generally, it is possible to consider

$$\Phi(t)\theta = \Phi_1(t-\tau_1)\theta_1 + \cdots + \Phi_m(t-\tau_m)\theta_m,$$

provided the delays are all known.

Assumption 1. In the particular case $\Phi(t)\theta \equiv 0$, a state observer is available in the form of

$$\dot{\hat{x}}_0(t) = A(t)\hat{x}_0(t) + B(t)\hat{x}_0(t-\tau) + L(t)[y(t) - C(t)\hat{x}_0(t) - D(t)\hat{x}_0(t-\tau)], \tag{2}$$

or in other words, the error dynamics

$$\dot{\eta}(t) = [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t-\tau), \tag{3}$$

is such that

$$\lim_{t\to+\infty} \eta(t) = 0 \ \text{ exponentially.} \tag{4}$$

□

The adaptive observer for system (1):

$$\dot{\Upsilon}(t) = [A(t) - L(t)C(t)]\Upsilon(t) + [B(t) - L(t)D(t)]\Upsilon(t-\tau) + \Phi(t) \tag{5a}$$
$$\begin{aligned}\dot{\hat{x}}(t) = {} & A(t)\hat{x}(t) + B(t)\hat{x}(t-\tau) + \Phi(t)\hat{\theta}(t) + L(t)[y(t) - C(t)\hat{x}(t) + D(t)\hat{x}(t-\tau)] \\ & + \Upsilon(t)\dot{\hat{\theta}}(t) + [B(t) - L(t)D(t)]\Upsilon(t-\tau)[\hat{\theta}(t) - \hat{\theta}(t-\tau)]\end{aligned} \tag{5b}$$
$$\dot{\hat{\theta}}(t) = \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]^T \left\{ y(t) - C(t)\hat{x}(t) + D(t)\hat{x}(t-\tau) - D(t)\Upsilon(t-\tau)[\hat{\theta}(t) - \hat{\theta}(t-\tau)] \right\} \tag{5c}$$

where Γ is either a constant positive definite matrix or a recursively computed time varying matrix.

The red terms are unusual. They will be helpful for error dynamics analysis.

Define the estimation errors:

$$\tilde{x}(t) \triangleq x(t) - \hat{x}(t) \tag{6}$$
$$\tilde{\theta}(t) \triangleq \theta - \hat{\theta}(t). \tag{7}$$

Then (recall that θ is a constant vector)

$$\dot{\tilde{x}}(t) = A(t)\tilde{x}(t) + B(t)\tilde{x}(t-\tau) + \Phi(t)\tilde{\theta} - L(t)C(t)\tilde{x}(t) - L(t)D(t)\tilde{x}(t-\tau) \\ + \Upsilon(t)\dot{\tilde{\theta}}(t) + [B(t) - L(t)D(t)]\Upsilon(t-\tau)[\tilde{\theta}(t) - \tilde{\theta}(t-\tau)]. \quad (8)$$

Now define

$$\eta(t) \triangleq \tilde{x}(t) - \Upsilon(t)\tilde{\theta}(t), \quad (9)$$

then

$$\begin{aligned}\dot{\eta}(t) = {} & [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t-\tau) \\ & + [A(t) - L(t)C(t)]\Upsilon(t)\tilde{\theta}(t) + [B(t) - L(t)D(t)]\Upsilon(t-\tau)\tilde{\theta}(t-\tau) \\ & + \Phi(t)\tilde{\theta}(t) + \Upsilon(t)\dot{\tilde{\theta}}(t) \\ & + [B(t) - L(t)D(t)]\Upsilon(t-\tau)[\tilde{\theta}(t) - \tilde{\theta}(t-\tau)] \\ & - \dot{\Upsilon}(t)\tilde{\theta}(t) - \Upsilon(t)\dot{\tilde{\theta}}(t). \end{aligned} \quad (10)$$

In this last equation, the first occurence of $[B(t) - L(t)D(t)]\Upsilon(t-\tau)\tilde{\theta}(t-\tau)$ cancels out with a later term, so does $\Upsilon(t)\dot{\tilde{\theta}}(t)$. Hence

$$\begin{aligned}\dot{\eta}(t) = {} & [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t-\tau) \\ & + \{[A(t) - L(t)C(t)]\Upsilon(t) + [B(t) - L(t)D(t)]\Upsilon(t-\tau) + \Phi(t) - \dot{\Upsilon}(t)\}\tilde{\theta}(t), \end{aligned} \quad (11)$$

which is then simplified, by taking into account (5a), to

$$\dot{\eta}(t) = [A(t) - L(t)C(t)]\eta(t) + [B(t) - L(t)D(t)]\eta(t-\tau). \quad (12)$$

According to Assumption 1, $\eta(t) \to 0$.

Now consider the error dynamics $\tilde{\theta}(t)$. It follows from (5c) that

$$\dot{\tilde{\theta}}(t) = -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]^T \left\{ C(t)\tilde{x}(t) + D(t)\tilde{x}(t-\tau) - D(t)\Upsilon(t-\tau)[\hat{\theta}(t) - \hat{\theta}(t-\tau)] \right\} \quad (13)$$

$$\begin{aligned} & = -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]^T[C(t)\eta(t) + D(t)\eta(t-\tau)] \\ & \quad - \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]^T \left\{ C(t)\Upsilon(t)\tilde{\theta}(t) + D(t)\Upsilon(t-\tau)\tilde{\theta}(t-\tau) + D(t)\Upsilon(t-\tau)[\tilde{\theta}(t) - \tilde{\theta}(t-\tau)] \right\} \\ & = -\Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]^T[C(t)\eta(t) + D(t)\eta(t-\tau)] \\ & \quad - \Gamma[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]^T[C(t)\Upsilon(t) + D(t)\Upsilon(t-\tau)]\tilde{\theta}(t). \end{aligned} \quad (14)$$

The homogeneous part of this error dynamics (corresponding to $\eta(t) \equiv 0$) is stable under the following assumption.

Assumption 2. There exist $T > 0$ and $\alpha > 0$ such that, for all $t \geq t_0$,

$$\int_t^{t+T} [C(s)\Upsilon(s) + D(s)\Upsilon(s-\tau)][C(s)\Upsilon(s) + D(s)\Upsilon(s-\tau)]^T ds \geq \alpha I. \quad (15)$$

□

The remaining analysis is then similar to already published cases.