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Adaptive stabilization by delay with biased measurements

Denis Efimov * **Stanislav Aranovskiy** ** **Emilia Fridman** ***
Dimitry Sokolov **** **Jian Wang** † **Alexey A. Bobtsov** ‡

* *Inria, Univ. Lille, CNRS, UMR 9189 - CRIStAL, F-59000 Lille, France*
** *CentaleSupélec –IETR, Avenue de la Boulaie, 35576 Cesson-Sévigné, France*
*** *School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel*
**** *Université de Lorraine, Inria, CNRS, LORIA, F-54000 Nancy, France*
† *Hangzhou Dianzi University, Hangzhou, China*
‡ *ITMO University, 197101 Saint Petersburg, Russia*

Abstract: The problem of output robust adaptive stabilization for a class of Lipschitz nonlinear systems is studied under assumption that the measurements are available with a constant bias. The state reconstruction is avoided by using delayed values of the output in the feedback and adaptation laws. The control and adaptation gains can be selected as a solution of the proposed linear matrix inequalities (LMIs). The efficiency of the presented approach is demonstrated on a nonlinear pendulum through simulations.

1. INTRODUCTION

Design of identification algorithms, estimators and regulators for dynamical systems are fundamental and complex problems studied in the control theory. In many cases, due to information transmission in the input-output channels, delays appear in the dynamics of the controlled plant. Influence of a delay on the system stability is vital in many cases (Gu et al., 2003; Fridman, 2014), and it usually leads to degradation of the performances of regulation or estimation (Fridman, 2014). However, in some cases introduction of a delay may result in an improvement of the system transients (see (Fridman and Shaikhet, 2016, 2017; Efimov et al., 2018) and the references therein). The idea of these papers is that unmeasured components of the state can be calculated using delayed values of the measured variables, which allows a design of observer to be passed by.

The goal of this note is to extend the results obtained in (Fridman and Shaikhet, 2016, 2017) for linear systems to adaptive stabilization of a class of nonlinear systems, which contain a globally Lipschitz nonlinearity, and have a part of the measurements available with a constant bias, which is induced by a sensor error. Since for embedded control and estimation solutions, the amount of computations needed for realization is a critical resource (less important than the used memory in some scenarios), in this note we avoid to design an (reduced order) observer for the state, but introduce delayed measurements in the feedback and adaptation algorithm. The closed-loop system becomes time-delayed, then stability analysis of the regulation error is based on the Lyapunov-Krasovskii functional proposed in (Fridman and Shaikhet, 2016, 2017).

It is important to note that there exist papers devoted to adaptive control of time-delay systems as, for example, (Mirkin and Gutman, 2010; Pepe, 2004) (the uncertain parameters appear in the state equation only), or papers dealt with adaptive control for systems with (multiplicative) uncertain parameters in the output equation (Zhang and Lin, 2019a,b) (without presence of time delays), but to the best of our knowledge there is no theory on the intersection of these approaches. This work and the considered problem statement is motivated by a pendulum control application, which is finally used for illustration.

The outline of this work is as follows. The preliminaries are given in Section 2. The problem statement and the adaptive control design are presented in sections 3 and 4, respectively. A nonlinear pendulum application is considered in Section 5.

2. PRELIMINARIES

Denote by $\mathbb{R}$ the set of real numbers and $\mathbb{R}_+ = \{s \in \mathbb{R} : s \geq 0\}$.

For a Lebesgue measurable function of time $d : [a, b] \to \mathbb{R}^m$, where $-\infty \leq a < b \leq +\infty$, define the norm $\|d\|_{[a,b)} = \text{ess}\sup_{t\in[a,b)} |d(t)|$, where $|\cdot|$ is the standard Euclidean norm in $\mathbb{R}^m$, then $||d||_\infty = \|d\|_{[0,+\infty)}$ and the space of d with $\|d\|_{[a,b)} < +\infty$ ($||d||_\infty < +\infty$) we further denote as $\mathcal{L}^m_{[a,b]}$ ($\mathcal{L}^m_\infty$).

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Denote by $C^n_{[a,b]}$, $a,b \in \mathbb{R}$ the Banach space of continuous functions $\phi : [a,b] \to \mathbb{R}^n$ with the uniform norm $\|\phi\|_{[a,b]} = \sup_{a\le s\le b}|\phi(s)|$; and by $\mathbb{W}^{1,\infty}_{[a,b]}$ the Sobolev space of absolutely continuous functions $\phi : [a,b] \to \mathbb{R}^n$ with the norm $\|\phi\|_{\mathbb{W}} = \|\phi\|_{[a,b)} + ||\dot{\phi}||_{[a,b)} < +\infty$, where $\dot{\phi}(s) = \frac{\partial\phi(s)}{\partial s}$ is a Lebesgue measurable essentially bounded function, *i.e.* $\dot{\phi} \in \mathcal{L}^n_{[a,b]}$.

A continuous function $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\sigma(0) = 0$; it belongs to class $\mathcal{K}_\infty$ if it is also radially unbounded. A continuous function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{KL}$ if $\beta(\cdot,r) \in \mathcal{K}$ and $\beta(r,\cdot)$ is decreasing to zero for any fixed $r > 0$.

The symbol $\overline{1,m}$ is used to denote a sequence of integers $1,...,m$. For a symmetric matrix $P \in \mathbb{R}^{n\times n}$, the minimum and the maximum eigenvalues are denoted as $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$, respectively. For a matrix $A \in \mathbb{R}^{n\times m}$, $|A| = \sqrt{\lambda_{\max}(A^\top A)}$ is the induced norm. The identity matrix of dimension $n \times n$ is denoted by I_n.

2.1 Neutral time-delay systems

Consider an autonomous functional differential equation of neutral type with inputs (Kolmanovsky and Nosov, 1986):

$$\dot{x}(t) = f(x_t, \dot{x}_t, d(t)) \quad (1)$$

for almost all $t \ge 0$, where $x(t) \in \mathbb{R}^n$ and $x_t \in \mathbb{W}^{1,\infty}_{[-\tau,0]}$ is the state function, $x_t(s) = x(t+s)$, $-\tau \le s \le 0$, with $\dot{x}_t \in \mathcal{L}^n_{[-\tau,0]}$; $d(t) \in \mathbb{R}^m$ is the external input, $d \in \mathcal{L}^m_\infty$; $f : \mathbb{W}^{1,\infty}_{[-\tau,0]} \times \mathcal{L}^n_{[-\tau,0]} \times \mathbb{R}^m \to \mathbb{R}^n$ is a continuous function, that is globally Lipschitz in the second variable with a constant smaller than 1, ensuring forward uniqueness and existence of the system solutions (Kolmanovsky and Nosov, 1986). We assume $f(0,0,0) = 0$. For the initial function $x_0 \in \mathbb{W}^{1,\infty}_{[-\tau,0]}$ and disturbance $d \in \mathcal{L}^m_\infty$ denote a unique solution of the system (1) by $x(t,x_0,d)$, which is an absolutely continuous function of time defined on some maximal interval $[-\tau,T)$ for $T > 0$, then $x_t(x_0,d) \in \mathbb{W}^{1,\infty}_{[-\tau,0]}$ represents the corresponding state function with $x_t(s,x_0,d) = x(t+s,x_0,d)$ for $-\tau \le s \le 0$.

Given a locally Lipschitz continuous functional $V : \mathbb{R} \times \mathbb{W}^{1,\infty}_{[-\tau,0]} \times \mathcal{L}^n_{[-\tau,0]} \to \mathbb{R}_+$ define its derivative in the Driver's form:

$$D^+V(t,\phi,d) = \lim_{h\to 0^+}\sup \frac{1}{h}[V(t+h, x_h(\phi,\tilde{d}), \dot{x}_h(\phi,\tilde{d})) - V(t,\phi,\dot{\phi})],$$

where $x_h(\phi,\tilde{d})$ is a solution of the system (1) for $\phi \in \mathbb{W}^{1,\infty}_{[-\tau,0]}$ and $\tilde{d}(t) = d$ for all $t \ge 0$ and some $d \in \mathbb{R}^m$.

2.2 ISS of time delay systems

The input-to-state stability (ISS) property is an extension of the conventional stability paradigm to the systems with external inputs (Teel, 1998; Pepe and Jiang, 2006; Fridman et al., 2008).

Definition 1. (Pepe and Jiang, 2006; Fridman et al., 2008) The system (1) is called practical ISS, if for all $x_0 \in \mathbb{W}^{1,\infty}_{[-\tau,0]}$ and $d \in \mathcal{L}^m_\infty$ there exist $q \ge 0$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$|x(t,x_0,d)| \le \beta(\|x_0\|_{\mathbb{W}},t) + \gamma(||d||_\infty) + q \quad \forall t \ge 0.$$

If $q = 0$ then (1) is called ISS.

For establishment of this stability property, the Lyapunov-Krasovskii theory can be used (Pepe and Jiang, 2006; Fridman et al., 2008; Efimov et al., 2018).

Definition 2. A locally Lipschitz continuous functional $V : \mathbb{R}_+ \times \mathbb{W}^{1,\infty}_{[-\tau,0]} \times \mathcal{L}^n_{[-\tau,0]} \to \mathbb{R}_+$ (*i.e.*, $V(t,\phi,\dot{\phi})$) is called simple if $D^+V(t,\phi,d)$ is independent on $\ddot{\phi}$.

For instance, a locally Lipschitz functional $V : \mathbb{R}_+ \times \mathbb{W}^{1,\infty}_{[-\tau,0]} \to \mathbb{R}_+$ is simple, another example of a simple functional is given in Theorem 6 below.

Definition 3. A locally Lipschitz continuous functional $V : \mathbb{R}_+ \times \mathbb{W}^{1,\infty}_{[-\tau,0]} \times \mathcal{L}^n_{[-\tau,0]} \to \mathbb{R}_+$ is called practical ISS Lyapunov-Krasovskii functional for the system (1) if it is simple and there exist $r \ge 0$, $\alpha_1,\alpha_2 \in \mathcal{K}_\infty$ and $\alpha,\chi \in \mathcal{K}$ such that for all $t \in \mathbb{R}_+$, $\phi \in \mathbb{W}^{1,\infty}_{[-\tau,0]}$ and $d \in \mathbb{R}^m$:

$$\alpha_1(|\phi(0)|) \le V(t,\phi,\dot{\phi}) \le \alpha_2(\|\phi\|_{\mathbb{W}}),$$
$$V(t,\phi,\dot{\phi}) \ge \max\{r,\chi(|d|)\} \Longrightarrow D^+V(t,\phi,d) \le -\alpha(V(t,\phi,\dot{\phi})).$$

If $r = 0$ then V is an ISS Lyapunov-Krasovskii functional.

Theorem 4. (Fridman et al., 2008) If there exists a (practical) ISS Lyapunov-Krasovskii functional for the system (1), then it is (practical) ISS with $\gamma = \alpha_1^{-1} \circ \chi$.

Converse results for Theorem 4 can be found in (Pepe et al., 2017; Efimov and Fridman, 2019).

3. ROBUST OUTPUT ADAPTIVE REGULATION WITH BIASED MEASUREMENTS

Consider a nonlinear system for the time $t \ge 0$:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t),\\
\dot{x}_2(t) &= A_{21}x_1(t) + A_{22}x_2(t) + A_{23}x_3(t)\\
&\quad + B_1(u(t) + \Omega(t)\theta_2) + L_1\phi(x(t)) + d_1(t),\\
\dot{x}_3(t) &= A_{31}x_1(t) + A_{32}x_2(t) + A_{33}x_3(t)\\
&\quad + B_2(u(t) + \Omega(t)\theta_2) + L_2\phi(x(t)) + d_2(t),\\
y_1(t) &= x_1(t) + \theta_1, \quad y_2(t) = x_3(t),
\end{aligned} \quad (2)$$

where $x_1(t) \in \mathbb{R}^n$ and $x_2(t) \in \mathbb{R}^n$ are the position and velocity, respectively, $x_3(t) \in \mathbb{R}^p$ is an additional state, $x(t) = [x_1^\top(t)\ x_2^\top(t)\ x_3^\top(t)]^\top \in \mathbb{R}^{2n+p}$ is the total state vector of (2), the initial conditions $x(0) = x_0 \in \mathbb{R}^{2n+p}$ are unknown; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) = [y_1^\top(t)\ y_2^\top(t)] \in \mathbb{R}^{n+p}$ is the output available for measurements, $d(t) = [d_1^\top(t)\ d_2^\top(t)] \in \mathbb{R}^{n+p}$ is the disturbance with $d \in \mathcal{L}^{n+p}_\infty$; $\theta_1 \in \mathbb{R}^n$ is the vector of biases in the measurements of the position $x_1(t)$, $\theta_2 \in \mathbb{R}^r$ is the vector of uncertain parameters in the state dynamics, $\theta = [\theta_1^\top\ \theta_2^\top] \in \mathbb{R}^{n+r}$, the regressor $\Omega : \mathbb{R}_+ \to \mathbb{R}^{m\times r}$ is a known continuous matrix function; the nonlinearity ϕ can be partitioned as

$$\phi(x) = \begin{bmatrix}\phi_1(x)\\ \phi_2(x_3)\end{bmatrix},$$

where $\phi_1 : \mathbb{R}^{2n+p} \to \mathbb{R}^{s_1}$ and $\phi_2 : \mathbb{R}^p \to \mathbb{R}^{s_2}$, then we can decompose

$$L_1 = [L_{11}\ L_{12}],\ L_2 = [L_{21}\ L_{22}],$$

and $\phi(x)$ is assumed to be Lipschitz continuous; all the matrices are constant and known having the corresponding dimensions.

Therefore, the considered system is subject to unknown inputs $d(t)$, it contains unknown parameters θ (part of them corrupt the measurements), and the equations of (2) are nonlinear and time-varying due to the presence of ϕ and Ω, respectively. The goal is to design an (dynamical) output control input $u(t) = u(y_1(t), y_2(t), \Omega(t))$ ensuring a practical ISS property of the closed loop system for all $x_0 \in \mathbb{R}^{2n+p}$ and all $d \in \mathcal{L}_\infty^{n+p}$ under the restriction to minimize the computational complexity of the algorithm (in order to be able to use the proposed solution as a component of an embedded system).

We will use the following hypothesis about the properties of (2):

Assumption 1. For the regressor function $\Omega(t)$ there is a known upper bound $\bar{\Omega} \geq 0$ such that $\sup_{t\geq 0} |\Omega(t)| \leq \bar{\Omega}$.

The function ϕ admits a global Lipschitz constant $\gamma \geq 0$: $|\phi(x) - \phi(x')| \leq \gamma |x - x'|$ for all $x, x' \in \mathbb{R}^{2n+p}$. The function $\phi_2 : \mathbb{R}^p \to \mathbb{R}^{s_2}$ is known.

Under the introduced restriction on ϕ the system (2) has well-defined solutions for all $t \geq 0$ for any $x_0 \in \mathbb{R}^{2n+p}$ and any $d \in \mathcal{L}_\infty^{n+p}$ (Khalil, 2015).

4. MAIN RESULTS

Due to a rather complicated structure of the considered system and introduced uncertainty, clearly, for realization of a robustly stabilizing control it is necessary to use the full state $x(t)$ information. Consequently, it is required to design an estimator for $x_1(t)$, which is measured with an unknown bias θ_1, and for its velocity $x_2(t)$. Facing all uncertain terms presented in (2), *i.e.* θ_2 and $d(t)$, such an observation problem becomes rather intriguing, and a corresponding observer solving these issues will be also complex and nonlinear. In (Fridman and Shaikhet, 2016, 2017) an approach is presented for design of a linear delayed output static control for a linear system, which avoids a state observer design by introducing the estimates of $x(t)$ through delayed output $y(t)$ values. Hence, such a method has a low computational capacity (since for delay operation only memory is needed). In this work we will follow the same approach.

Defining $x_1(t-h) = x_1(0)$ for $t \in [0, h]$, where $h > 0$ is the delay, the control algorithm proposed in this paper is

$$\begin{aligned} u(t) = &-(K_1 + K_2)y_1(t) + K_2 y_1(t-h) \\ &-K_3 y_2(t) - K_4 \phi_2(y_2(t)) + K_1 \hat{\theta}_1(t) \\ &-\Omega(t)\hat{\theta}_2(t), \end{aligned} \tag{3}$$

where $\hat{\theta}_1(t) \in \mathbb{R}^n$ and $\hat{\theta}_2(t) \in \mathbb{R}^r$ are the estimates of θ_1 and θ_2, respectively; $K_i \in \mathbb{R}^{m\times n}$ for $i = 1, 2$, $K_3 \in \mathbb{R}^{m\times p}$ and $K_4 \in \mathbb{R}^{m\times s_2}$ are the control gains to be derived. Similarly, an adaptive law for $\hat{\theta}_1(t)$ can be synthesized:

$$\begin{aligned} \dot{\hat{\theta}}_1(t) = &(F_1 + F_2)y_1(t) - F_2 y_1(t-h) \\ &+F_3 y_2(t) + F_4 \phi_2(y_2(t)) - F_1 \hat{\theta}_1(t), \end{aligned} \tag{4}$$

where $F_i \in \mathbb{R}^{n\times n}$ for $i = 1, 2$, $F_3 \in \mathbb{R}^{n\times p}$ and $F_4 \in \mathbb{R}^{n\times s_2}$ are the adaptation gains which will be defined later. An adaptive law for $\hat{\theta}_2(t)$ is more sophisticated, and such a choice of the structure will become clear from the stability analysis given next:

$$\begin{aligned} \dot{\hat{\theta}}_2(t) = &\Omega^\top(t)[(S_1 + S_2)y_1(t) - S_2 y_1(t-h) \\ &+S_3 y_2(t) + S_4 \phi_2(y_2(t)) - S_1 \hat{\theta}_1(t)] \\ &-S_5 \hat{\theta}_2(t), \end{aligned} \tag{5}$$

where $S_i \in \mathbb{R}^{m\times n}$ for $i = 1, 2$, $S_3 \in \mathbb{R}^{m\times p}$, $S_4 \in \mathbb{R}^{m\times s_2}$ and $S_5 \in \mathbb{R}^{r\times r}$ are also the adaptation gains.

Remark 5. There is also an algebraic way to solve the problem of estimation of unknown values θ_1, θ_2 and signals $x_1(t)$, $x_2(t)$ (in the framework of indirect adaptive control), which is based on some structural restrictions and auxiliary filtering. Indeed, let $p = n$ and

$$J_1 L_{11} = J_2 L_{21}$$

for some matrices J_1 and J_2, then define $\zeta(t) = J_1 x_2(t) - J_2 x_3(t)$ with

$$\begin{aligned} \dot{\zeta}(t) = &Y_1(y_1(t) - \theta_1) + Y_2 \dot{y}_1(t) \\ &+Y_3 y_2(t) + Y_4(u(t) + \Omega(t)\theta_2) \\ &+Y_5 \phi_2(y_2(t)) + J_1 d_1(t) - J_2 d_2(t), \end{aligned}$$

where

$$\begin{aligned} Y_1 = J_1 A_{21} - J_2 A_{31},\ Y_2 = J_1 A_{22} - J_2 A_{32}, \\ Y_3 = J_1 A_{23} - J_2 A_{33}, \\ Y_4 = J_1 B_1 - J_2 B_2,\ Y_5 = J_1 L_{12} - J_2 L_{22}. \end{aligned}$$

Let also for brevity

$$d_1(t) = 0,\ d_2(t) = 0,\ Y_1 \neq 0.$$

Note that by construction:

$$\dot{\zeta}(t) = J_1 \ddot{y}_1(t) - J_2 \dot{y}_2(t),$$

and equating the expressions for $\dot{\zeta}(t)$ we obtain:

$$\tilde{\zeta}(t) = -Y_1\theta_1 + Y_4 \tilde{\Omega}(t)\theta_2,$$

which is a linear regressor model with respect to unknown parameters θ_1 and θ_2 that can be used for their identification, where $\tilde{\zeta}(t)$ and $\tilde{\Omega}(t)$ variables calculated as

$$\tilde{\Omega}(t) = \frac{\lambda^2}{(s+\lambda)^2}\Omega(t),$$

$$\begin{aligned} \tilde{\zeta}(t) = &J_1 \frac{\lambda^2 s^2}{(s+\lambda)^2} y_1(t) - Y_2 \frac{\lambda^2 s}{(s+\lambda)^2} y_1(t) \\ &-J_2 \frac{\lambda^2 s}{(s+\lambda)^2} y_2(t) - \frac{\lambda^2}{(s+\lambda)^2}[Y_1 y_1(t) + Y_3 y_2(t) \\ &+Y_4 u(t) + Y_5 \phi_2(y_2(t))] \end{aligned}$$

with s being the differentiating operator and $\lambda > 0$ is a tuning parameter of the filters. Inversely, if

$$d_1(t) = 0,\ d_2(t) = 0,\ Y_1 = 0,\ \theta_2 = 0,$$

then

$$\psi(t) = \frac{1}{s+1}\zeta(t) = J_1 \frac{s}{s+1} y_1(t) - J_2 \frac{1}{s+1} y_2(t)$$

is a variable that we can calculate, and

$$
\begin{aligned}
\psi(t) &= \zeta(t) - \frac{1}{s+1}\dot{\zeta}(t) \\
&= \zeta(t) - \frac{1}{s+1}[Y_2\dot{y}_1(t) + Y_3 y_2(t) \\
&\quad + Y_4 u(t) + Y_5\phi_2(y_2(t))].
\end{aligned}
$$

Hence, for a nonsingular J_1 we obtain:

$$
x_2(t) = J_1^{-1}\tilde{\psi}(t),
$$

where

$$
\begin{aligned}
\tilde{\psi}(t) &= \psi(t) + Y_2\frac{s}{s+1}y_1(t) + \frac{1}{s+1}[Y_3 y_2(t) \\
&\quad + Y_4 u(t) + Y_5\phi_2(y_2(t))] + J_2 y_2(t)
\end{aligned}
$$

can be calculated using filters. If $d_1(t) \neq 0$ and $d_2(t) \neq 0$, then these approaches lead to a reconstruction of unknown parameters and variables corrupted by noises, and robust estimation tools should be applied. Next, a control design has to be performed. A drawback of such solutions is also their computational complexity comparing to (3), (4), (5), where just additional adaptation algorithms (observers or filters) are introduced to calculate $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ in order to compensate the influence of θ_1 and θ_2.

It is worth noting that a complexity of (3), (4), (5) comes from another side. Initially the system (2) is delay-free (the state $x(t) \in \mathbb{R}^{2n+p}$), then (3), (4), (5) introduces the delay h and transforms the system into the retarded type time-delay dynamics (Fridman, 2014) (the state function is from $C^{2n+p}_{[-h,0]}$), while for the stability analysis we will perform below an additional transformation of the closed-loop system to the neutral type (with the state from $\mathbb{W}^{1,\infty}_{[-h,0]}$). Therefore, (3), (4), (5) needs a rather sophisticated analysis, but it allows a simple realization.

The restrictions on selection of the control and adaptation gains, and the conditions to check, are given in the following theorem.

Theorem 6. Let Assumption 1 be satisfied, if for given K_i, F_i with $i = \overline{1,4}$ and S_i with $i = \overline{1,5}$ the system of linear matrix inequalities

$$
\begin{gathered}
Q \leq 0,\ P = P^\top > 0,\ \alpha > 0,\ \beta > 0,\ \delta > 0,\ \eta > 0, \qquad (6)\\
\mathcal{M}^\top\mathcal{M} \leq \rho I_{s_1+s_2},\ 4\frac{e^{-\varpi h}}{h^2}I_n \geq \alpha\mathcal{B}^\top\mathcal{B} \\
\mathcal{S} = \mathcal{G}^\top P + qh^2\mathcal{G}^\top\Gamma^\top\Gamma\mathcal{A}, \\
S_2 = -qh^2\mathcal{G}^\top\Gamma^\top\Gamma\mathcal{B},\ \mathcal{S}_4 = qh^2\mathcal{G}^\top\Gamma^\top\Gamma\mathcal{M} \\
Q = \begin{bmatrix}
Q_{11} & N & N & 0 & N & 0 \\
N^\top & M - \alpha q I_{3n+p} & M & 0 & M & 0 \\
N^\top & M & M - \beta I_{3n+p} & 0 & M & 0 \\
0 & 0 & 0 & Q_{44} & 0 & -S_5 \\
N^\top & M & M & 0 & M - (\eta - qh^2)I_{3n+p} & 0 \\
0 & 0 & 0 & -S_5^\top & 0 & -\delta I_r
\end{bmatrix}, \\
Q_{11} = P\mathcal{A} + \mathcal{A}^\top P + qh^2\mathcal{A}^\top\Gamma^\top\Gamma\mathcal{A} + \rho\beta\gamma^2 C^\top C + \varpi P, \\
Q_{44} = -2S_5 + (qh^2|B_1|^2\bar{\Omega}^2 + \varpi)I_r, \\
N = P + qh^2\mathcal{A}^\top\Gamma^\top\Gamma,\ M = qh^2\Gamma^\top\Gamma,
\end{gathered}
$$

is feasible with respect to P, α, β, δ and η for some $q > 0$, $\varpi > 0$ and $\rho > 0$, where

$$
\mathcal{A} = \begin{bmatrix}
0 & I_n & 0 & 0 \\
A_{21} - B_1K_1 & A_{22} - hB_1K_2 & A_{23} - B_1K_3 & B_1K_1 \\
A_{31} - B_2K_1 & A_{32} - hB_2K_2 & A_{33} - B_2K_3 & B_2K_1 \\
F_1 & hF_2 & F_3 & -F_1
\end{bmatrix},
$$

$$
\mathcal{B} = \begin{bmatrix} 0 \\ B_1K_2 \\ B_2K_2 \\ -F_2 \end{bmatrix},\ \mathcal{G} = \begin{bmatrix} 0 \\ B_1 \\ B_2 \\ 0 \end{bmatrix},\ \mathcal{D} = \begin{bmatrix} 0 \\ I_n \\ I_p \\ 0 \end{bmatrix},
$$

$$
\mathcal{M} = \begin{bmatrix}
0 & 0 \\
L_{11} & L_{12} - B_1K_4 \\
L_{21} & L_{22} - B_2K_4 \\
0 & F_4
\end{bmatrix},\ C = \mathbf{diag}\begin{bmatrix} I_n & I_n & I_p & 0 \end{bmatrix},
$$

$$
\mathcal{S} = \begin{bmatrix} S_1 & hS_2 & S_3 & -S_1 \end{bmatrix},\ \mathcal{S}_4 = \begin{bmatrix} 0 & S_4 \end{bmatrix},
$$

$$
\Gamma = \begin{bmatrix} 0 & I_n & 0 & 0 \end{bmatrix}.
$$

Then the system (2) with the control (3) and adaptive laws (4), (5) is practically ISS.

All proofs are omitted due to space limitations.

The conditions of the theorem connect the control parameters to be tuned (the gains K_i, F_i, S_i and the admissible delay h), the auxiliary constants ($q > 0$, $\varpi > 0$, $\rho > 0$) and the variables of linear matrix inequalities ($Q \leq 0$, $P > 0$, $\alpha > 0$, $\beta > 0$, $\delta > 0$ and $\eta > 0$), which are obtained applying numerical solvers to (6). It is worth noting that the inequality $Q \leq 0$ in (6) is always satisfied for a sufficiently small value of h.

Remark 7. The presentation above is given by factorizing the LMIs in the briefest way, which, however, may be more conservative. For example, taking

$$
\xi(t) = \left[z^\top(t)\ R^\top(t)\ \phi^\top(Cz(t))\ \tilde{\theta}_2^\top(t)\ d^\top(t)\ \theta_2^\top\right]^\top
$$

we obtain that if for given K_i, F_i with $i = \overline{1,4}$ and S_i with $i = \overline{1,5}$ the system of linear matrix inequalities

$$
\begin{gathered}
Q \leq 0,\ P = P^\top > 0,\ \alpha > 0,\ \beta > 0,\ \delta > 0,\ \eta > 0, \qquad (7)\\
\mathcal{S} = \mathcal{G}^\top P + qh^2\mathcal{G}^\top\Gamma^\top\Gamma\mathcal{A}, \\
S_2 = -qh^2\mathcal{G}^\top\Gamma^\top\Gamma\mathcal{B},\ \mathcal{S}_4 = qh^2\mathcal{G}^\top\Gamma^\top\Gamma\mathcal{M} \\
Q = \begin{bmatrix}
Q_{11} & N\mathcal{B} & N\mathcal{M} & 0 & N\mathcal{D} & 0 \\
\mathcal{B}^\top N^\top & \mathcal{B}^\top M\mathcal{B} - 4q\frac{e^{-\varpi h}}{h^2}I_n & \mathcal{B}^\top M\mathcal{M} & 0 & \mathcal{B}^\top M\mathcal{D} & 0 \\
\mathcal{M}^\top N^\top & \mathcal{M}^\top M\mathcal{B} & \mathcal{M}^\top M\mathcal{M} - \beta I_{s_1+s_2} & 0 & \mathcal{M}^\top M\mathcal{D} & 0 \\
0 & 0 & 0 & Q_{44} & 0 & -S_5 \\
\mathcal{D}^\top N^\top & \mathcal{D}^\top M\mathcal{B} & \mathcal{D}^\top M\mathcal{M} & 0 & \mathcal{D}^\top M\mathcal{D} - (\eta - qh^2)I_{n+p} & 0 \\
0 & 0 & 0 & -S_5^\top & 0 & -\delta I_r
\end{bmatrix}, \\
Q_{11} = P\mathcal{A} + \mathcal{A}^\top P + qh^2\mathcal{A}^\top\Gamma^\top\Gamma\mathcal{A} + \beta\gamma^2 C^\top C + \varpi P,
\end{gathered}
$$

is feasible with respect to P, α, β, δ and η for some $q > 0$ and $\varpi > 0$ (where the meaning of other variables is the same as in the formulation of Theorem 6), then the system (2) with the control (3) and adaptive laws (4), (5) is also practically ISS. However, it is worth to stress a more nonlinear nature of (7) comparing with (6).

Introducing additional mild restrictions we can reformulate the conditions of Theorem 6 by considering the control and adaptation gains K_i, F_i, S_i as solutions of LMIs:

Corollary 8. Let Assumption 1 be satisfied, if the system of linear matrix inequalities

$$
\tilde{Q} \le 0,\ P^{-1} = P^{-\top} \ge 0,\ \alpha > 0,\ \beta > 0,\ \delta > 0,\ \eta > 0, \tag{8}
$$
$$
\begin{bmatrix} \rho I_{s_1+s_2} & \mathcal{M}^\top \\ \mathcal{M} & I_{3n+p} \end{bmatrix} \ge 0,\ \begin{bmatrix} I_{3n+p} & P^{-1}C^\top \\ CP^{-1} & \frac{1}{\rho\gamma^2} I_n \end{bmatrix} \ge 0,
$$
$$
\begin{bmatrix} 2P^{-1} - \frac{\alpha}{4} e^{\varpi h} \Delta^{-1} & \mathcal{U}^\top \mathcal{G}^\top - \mathcal{W}^\top \mathcal{I}^\top \\ \mathcal{G}\mathcal{U} - \mathcal{I}\mathcal{W} & I_{3n+p} \end{bmatrix} \ge 0,
$$
$$
\Delta = c \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & c^{-1} I_n & 0 & 0 \\ 0 & 0 & I_p & 0 \\ 0 & 0 & 0 & I_n \end{bmatrix},
$$
$$
\Sigma = \mathcal{G}^\top + qh^2 B_1^\top (\Gamma \mathcal{A}_0 P^{-1} - B_1 \mathcal{U}),
$$
$$
\Sigma_2 = -qh^2 B_1^\top B_1 \mathcal{U}_2,\ \begin{bmatrix} 0 & S_4 \end{bmatrix} = qh^2 B_1^\top \begin{bmatrix} L_{11} & L_{12} - B_1 K_4 \end{bmatrix}
$$
$$
\tilde{Q} = \begin{bmatrix} \tilde{Q}_{11} & I_{3n+p} & I_{3n+p} & 0 & I_{3n+p} & 0 & \tilde{Q}_{17} \\ I_{3n+p} & -\alpha q I_{3n+p} & 0 & 0 & 0 & 0 & -\Gamma^\top \\ I_{3n+p} & 0 & -\beta I_{3n+p} & 0 & 0 & 0 & -\Gamma^\top \\ 0 & 0 & 0 & Q_{44} & 0 & -S_5 & 0 \\ I_{3n+p} & 0 & 0 & 0 & Q_{55} & 0 & -\Gamma^\top \\ 0 & 0 & 0 & -S_5^\top & 0 & -\delta I_r & 0 \\ \tilde{Q}_{17}^\top & -\Gamma & -\Gamma & 0 & -\Gamma & 0 & -\frac{1}{qh^2} I_n \end{bmatrix},
$$
$$
\tilde{Q}_{11} = \mathcal{A}_0 P^{-1} + P^{-1} \mathcal{A}_0^\top - \mathcal{G}\mathcal{U} - \mathcal{U}^\top \mathcal{G}^\top + \mathcal{I}\mathcal{W}
$$
$$
+\mathcal{W}^\top \mathcal{I}^\top + \beta I_{3n+p} + \varpi P^{-1},\ \tilde{Q}_{17} = -P^{-1} \mathcal{A}_0^\top \Gamma^\top + \mathcal{U}^\top B_1^\top,
$$
$$
Q_{44} = -2S_5 + (qh^2 |B_1|^2 \bar{\Omega}^2 + \varpi) I_r,\ Q_{55} = -(\eta - qh^2) I_{3n+p},
$$
is feasible with respect to P^{-1}, $\mathcal{U}$, $\mathcal{W}$, Σ, K_4, F_4, S_4, S_5, α, β, η and δ for some given $q > 0$, $c > 0$, $\varpi > 0$, $\mu > 0$ and $\rho > 0$, where
$$
\mathcal{A}_0 = \begin{bmatrix} 0 & I_n & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 \\ A_{31} & A_{32} & A_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \mathcal{I} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_n \end{bmatrix},
$$
and the matrices $\mathcal{B}$, $\mathcal{G}$, $\mathcal{D}$, $\mathcal{M}$, C and Γ are defined in Theorem 6. Then for
$$
[\, K_1\ hK_2\ K_3\ K_5 \,] = \mathcal{U}P,\ [\, F_1\ hF_2\ F_3\ F_5 \,] = \mathcal{W}P,
$$
$$
[\, S_1\ hS_2\ S_3\ S_6 \,] = \Sigma P,
$$
the system (2) with the control and adaptive laws
$$
\begin{aligned} u(t) = & -(K_1 + K_2) y_1(t) + K_2 y_1(t-h) \\ & -K_3 y_2(t) - K_4 \phi_2(y_2(t)) - K_5 \hat{\theta}_1(t) \\ & -\Omega(t) \hat{\theta}_2(t), \end{aligned} \tag{9}
$$
$$
\begin{aligned} \dot{\hat{\theta}}_1(t) = & (F_1 + F_2) y_1(t) - F_2 y_1(t-h) \\ & +F_3 y_2(t) + F_4 \phi_2(y_2(t)) + F_5 \hat{\theta}_1(t), \end{aligned} \tag{10}
$$
$$
\begin{aligned} \dot{\hat{\theta}}_2(t) = & \Omega^\top(t) [(S_1 + S_2) y_1(t) - S_2 y_1(t-h) \\ & +S_3 y_2(t) + S_4 \phi_2(y_2(t)) + S_6 \hat{\theta}_1(t)] \\ & -S_5 \hat{\theta}_2(t), \end{aligned} \tag{11}
$$
is practically ISS.

The last important observation is that for the algorithms (3), (4), (5) the variables $\mathscr{K}$, $\mathcal{F}$ and $\mathcal{S}$ have an additional linear constraint (*i.e.*, $K_1 = -K_5$, $F_1 = -F_5$ and $S_1 = -S_6$), which is hard to formulate in terms of an LMI since $\mathscr{K} = \mathcal{U}P$, $\mathcal{W} = \mathcal{F}P$ and $\mathcal{S} = \Sigma P$, then a possible solution is to assume that the variable $\hat{\theta}_1(t)$ in the algorithms (9), (10), (11) enters with an independent gain.

Let us consider some results of application of the proposed robust adaptive output control.

5. SIMULATION RESULTS

To illustrate the proposed approach, we consider an inverted pendulum stabilization problem. We use the dynamic model and mechanical parameters of the setup previously reported in Aranovskiy et al. (2019). The notation is shown in Fig. 1.

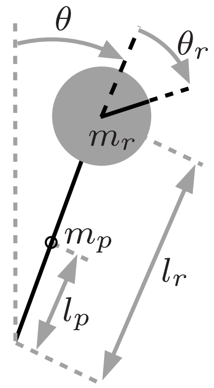
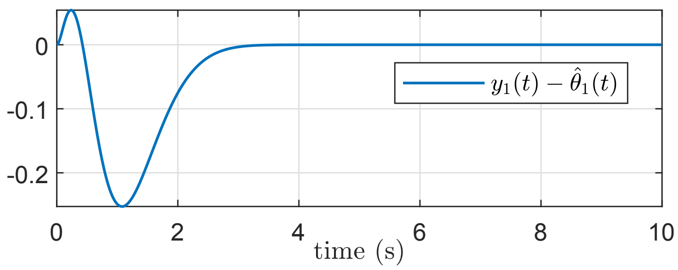


Fig. 1. The inverted pendulum notation, see Aranovskiy et al. (2019).

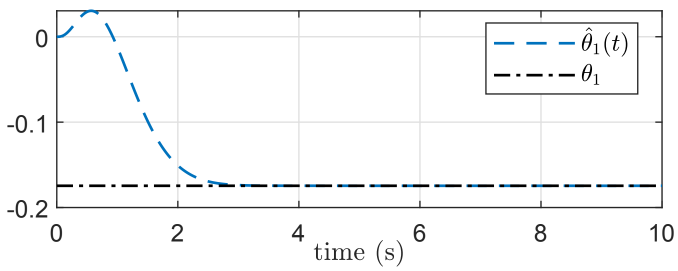
Define the state variable vector $x := \begin{bmatrix} \theta & \dot{\theta} & \dot{\theta}_r \end{bmatrix}^\top$, where θ is the angle between the pendulum and the vertical, and θ_r is the angle of the reaction wheel with respect to the pendulum; note that the reaction wheel velocity $\dot{\theta}_r$ is assumed to be available due to the equipment specifics. Following Aranovskiy et al. (2019), the system dynamics is given by
$$
\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= a_1 \sin(x_1) - b_1 u, \\ \dot{x}_3 &= -a_1 \sin(x_1) + b_2 u, \end{aligned} \tag{12}
$$
where $a_1 = 50$, $b_1 = 1.4$, $b_2 = 31$, and u is the motor current considered as the control input. The measured signals are $y_1 = x_1 + \theta_1$ and $y_2 = x_3$, where θ_1 is the unknown constant bias of the pendulum position sensor. It is straightforward to verify that the model (12) can be written in the form (2) with $\Omega(t) \equiv 0$. The goal is to drive the system to the origin and to estimate the bias θ_1. In Aranovskiy et al. (2019), the goal has been achieved constructing a nonlinear velocity observer for the state x_2 combined with the state-feedback control law; however, only local convergence has been shown. In this section, we apply the control law (3), (4). It can be verified that the LMI (7) is feasible with $h = 0.006$ and $K_1 = -68$, $K_2 = -1764$, $K_3 = -0.12$, $F_1 = 1$, $F_3 = 6 \cdot 10^{-4}$, and $K_4 = F_2 = F_4 = 0$, thus the conditions of Remark 7 are met.

For simulations, we choose the bias as $\theta_1 = -10° \approx -0.1745$. The initial conditions are chosen such that $y_1(0) = 0$, i.e., $x_1(0) = -\theta_1$, and $x_2(0) = 0$, $x_3(0) = 0$.

The simulation results of the pendulum stabilization and the θ_1 estimation with the control law (3), (4) are shown in Fig. 2 for the estimated pendulum position $y_1 - \hat{\theta}_1$ and the bias estimation $\hat{\theta}_1$. The reaction wheel velocity y_2 is depicted in Fig. 3. The simulation results illustrate the the proposed delay-based control law stabilizes the system and allows for the bias estimation.



(a) The corrected pendulum position $y_1 - \hat{\theta}_1$.



(b) The bias estimate $\hat{\theta}_1$ and the true bias value θ_1.

Fig. 2. Stabilization of the inverted pendulum.

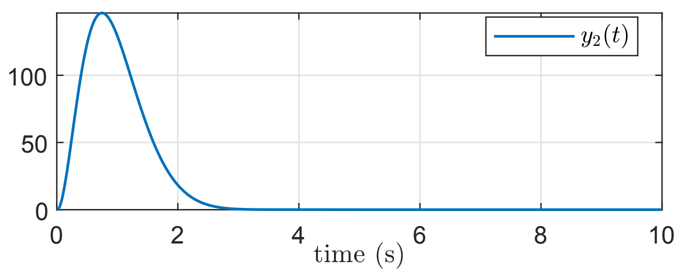


Fig. 3. Stabilization of the inverted pendulum: the reaction wheel velocity y_2.

6. CONCLUSIONS

Considering a Lipschitz nonlinear system, whose model contains external perturbation and uncertain parameters, while the measurements are available with a constant bias, the problem of robust output adaptive stabilization has been solved. Due to a severe uncertainty of the plant, the state reconstruction has been avoided by introducing artificial delays of the output in the feedback and adaptation algorithms. Applying the Lyapunov-Krasovskii approach, the conditions of practical ISS have been established, which are based on linear matrix inequalities. The efficacy of the proposed approach is demonstrated in simulations for an inertia wheel nonlinear pendulum. Extension of the proposed method to a more general class of systems is a direction of future research.

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