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Abstract: It is of importance to design observers for multi-variable nonlinear systems with unknown parameters and partially driven by unknown inputs. Such a problem arises in systems subject to disturbances or with inaccessible inputs and in many applications such as parameter identification, fault detection and isolation or cryptography. In this paper, the problem of fixed-time observation for nonlinear dynamical systems with unknown parameter and inputs is studied. Conditions on full/partial state and parameters identification are provided by the the way of an observation algorithm based on differential geometry theory. Then, an uniform differentiator for estimating simultaneously the states and unknown parameters in fixed-time while avoiding observability singularities is designed. An example on topology identification of network systems is described to show the effectiveness of the proposed method.

1 Introduction

The design of robust observers occur in many practical applications such as systems with uncertainties or/and unknown parameters, fault and identification problems, cryptography [46, 49], and more recently the fast topology identification of network systems (an illustrative example will be given in this paper). Another important requirement is completion of all transients in finite time because strict quality requirements are imposed for design, operation and control of complex technical processes with a large number of applications (in control design for a variety of robotic and mechatronic devices, safety evaluation, aerospace applications, vehicles control systems, etc). Therefore the problems of finite-time control and finite-time observation of uncertain nonlinear systems have been intensively studied for many years (see, for example, [3, 8, 11, 16, 24]). Considering the observation problems, a convergence of observed states to the real ones in finite time is always preferable, in particular for nonlinear systems where the well-known separation principal does not apply or hybrid systems where fast commutation in switched dynamics appear ([13, 25]). Thus this type of convergence greatly simplify design and analysis. For finite-time attractivity, convergence to the limit mode is required to occur in a finite, terminal time, which is permitted to depend on the initial condition of the system. A subclass of this is fixed-time attractivity, where the terminal time admits a uniform upper bound regardless to the initial conditions [27].

The problem of both state observation and parameter identification has already been investigated extensively in the literature[5, 37]. An adaptive observer to estimate simultaneously the state and the unknown parameters is proposed in [23] and it guarantees arbitrary fast exponential convergence when the condition of persistently of excitation is satisfied. In [30], authors presented the results of their works about the parameter identification problem of linear system using multi-layered neural networks. In [43], the authors proposed a nonlinear adaptive observer with global convergence to estimate simultaneously, the state and the unknown parameters of linear time-varying systems. The results of [43] was generalized in [9] for nonlinear systems with a general nonlinear parameterization and with bounded states and unknown parameters. Results on nonlinear systems can be found in the literature using mostly observers with linearizable error dynamics (see e.g. [6, 33, 36, 47, 48]), or high-gain observers [4, 7, 10, 18, 29, 38, 44].

Most of those observers are asymptotic/exponential, which means that the estimation error converges to zero and reaches zero towards the infinite time.

However, as explained before, it is often better to have a finite-time convergence when the needed variables need to be reconstructed quickly. Motivated by this fact, this paper aims at designing an observer that can achieve both state and parameter estimation in a finite time.

Few works deal with the problem of state and parameter finite time estimation for nonlinear systems with uncertainties. This problem is a challenging one, even for accurately known systems. In many approaches, nonlinear coordinate transformations are used to transform the system into suitable observer canonical forms. Finite time convergence can be obtained under the assumptions that the system can be put into a set of triangular observable forms, where the unknown inputs act only on the last dynamics of each triangular form. This assumption is known as the observability matching condition. Then, observers based on algebraic, numerical, adaptive or sliding mode observers [2, 15, 34, 39–41] can be designed.

The present work aims at the development of a systematic method leading to the finite time observation of a very general class of uncertain nonlinear systems with unknown inputs. The contribution of this paper is twofold:

- The description of an improved version of observation algorithms given in [1, 35] to check the full/partial state observability and parameter identifiability of general nonlinear systems with unknown inputs, even if the observability matching condition is not fulfilled.
- The design of a fixed-time sliding mode observer avoiding for possible singularities of observation.

This paper is organized as follows: Problem statement and definitions are given in Section 2. Preliminary results on parameter identifiability problem are presented in Section 3. In Section 4, an observation algorithm is given to check which part of the state and parameters is observable/identifiable. A fixed-time observer is designed in Section 5 in order to estimate those states and parameters in a prescribed time. An application example for finite time identification of the topology of network systems with simulations is presented in Section 6.

2 Problem statement and definitions

Consider the following nonlinear system with unknown parameters and unknown inputs:

$$\left\{\begin{array}{l}\dot{x}=f(x)+\varphi(x)\theta+\delta(x)\vartheta\\ \dot{\theta}=0\\ y=h(x)\end{array}\right. \tag{1}$$

where $x=[x_1,\cdots,x_n]^T\in\mathbb{R}^n$ is the state of the system $y=\left[h_1,\cdots,h_p\right]^T\in\mathbb{R}^p$ is the output, $\theta=\left[\theta_1,\cdots,\theta_q\right]^T\in\mathbb{R}^q$ is the vector of the unknown but piecewise constant parameters of the system, and $\vartheta\in\mathbb{R}^m$ is the vector of unknown inputs.

The function $f(x)=[f_1(x),\cdots,f_n(x)]^T$ is assumed to be sufficiently smooth. Without loss of generality, the scalar functions $h_1(x),\cdots,h_p(x)$ are assumed to be linearly independent. The matrix $\varphi(x)\in\mathbb{R}^{n\times q}$ is known and $\delta(x)=(\delta_1(x),\ldots,\delta_m(x))\in\mathbb{R}^{n\times m}$ is the matrix of unknown inputs.

Assumption 1. *The distribution $\Delta=span\{\delta_1(x),\ldots,\delta_m(x)\}$ is assumed to be involutive [19].*

Since the unknown parameter θ is assumed to be piecewise constant, it can be considered as an additional state variable. Then, define the new variable $\xi=\begin{pmatrix}x\\ \theta\end{pmatrix}$. The system (1) is rewritten as:

$$\left\{\begin{array}{l}\dot{\xi}=f(\xi)+\delta(\xi)\vartheta\\ y=h(\xi)\\ z=P\xi\end{array}\right. \tag{2}$$

where $\xi\in\mathbb{R}^{n+q}$, $f(\xi)=f(x)+\varphi(x)\theta$. P is a constant matrix introduced to check the partial state observability and parameter identifiability *via* the algorithm described in Section 4.

The introduction of z and P allows to recast the problem of the estimation of x and/or θ in (1) into the problem to estimate z in (2) (as it will be shown in the numerical example in Section 6, where the state observability and the unknown parameter identifiability are partial).

Given that the unknown constant parameter might be regarded as an additional time-invariant state variable, the parameter identifiability in this paper is related to state observability problem. For that, we recall hereafter the definitions of algebraic observability (for the state x in (1)) and on algebraic identifiability (for the unknown parameter θ in (1)).

Definition 1. *(Algebraic observability)[12],[35]. Consider system* (1)*, the state x is said to be algebraically observable if there exists a positive integer k and a meromorphic function η such that x is algebraic over the field $\mathscr{U}\times\mathscr{Y}$. The system* (1) *is algebraically observable if the field extension $\mathscr{U}\times\mathscr{Y}\rightarrow(\mathscr{U}\times\mathscr{Y})(x)$ is purely algebraic. $x=\eta(y,\dot{y},\ddot{y},\ldots,y^{(k)})$ where $y,\dot{y},\ddot{y},\ldots,y^{(k)}$ are the $(0,1,\ldots,k)^{th}$ derivatives of the corresponding output y.*

Definition 2. *(Algebraic identifiability) For system* (1)*, the unknown parameter θ is said to be algebraically identifiable if there exist a $T>0$, a positive integer k and a meromorphic function Φ such that*

$$\Phi(\theta,y,\dot{y},\ldots,y^{(k)})=0 \tag{3}$$

and

$$rank\ \frac{\partial\Phi}{\partial\theta}=q \tag{4}$$

holds on $[0,T]$, for all $(\theta,y,\dot{y},\ldots,y^{(k)})$.

Remark 1. *According to the implicit theorem, it can be seen that if* (3) *and* (4) *are satisfied, there exists (locally) a vector $\bar{\gamma}$ such that*

$$\theta=\bar{\gamma}(y,\dot{y},\ddot{y},\ldots,y^{(k)})$$

which is similar to the definition of algebraic observability given in Definition 1.

For the compact form (2), sufficient conditions which enable to identify the unknown constant parameters are presented in the next section.

3 Preliminary results

In this section, the method proposed by Xia *et al* in [35] is recalled. Denote $\mathscr{Y}=span\{dy,d\dot{y},\ldots,dy^{(l)}\},l\in\mathbb{N}$, $\mathscr{X}=span\{dx\}$ and $\Theta=span\{d\theta\}$.

Since $\mathscr{X}\cap\mathscr{Y}$ represents the observation space of system (2), $\mathscr{X}\cap(\mathscr{Y}+\Theta)$ can be considered as the observation space with parameters. The identifiability method proposed by Xia *et al* is to eliminate firstly the state x through observability properties of the system. To this end, the so-called observability indices for (2) are defined. Set

$$\mathscr{F}_l=\mathscr{X}\cap(span\{dy,d\dot{y}\ldots,dy^{(l-1)}\})$$

for $l=1,\ldots,n$. It has been shown in [42] that $\mathscr{F}_1\subset\mathscr{F}_2\subset\cdots\subset\mathscr{F}_n$. Then, as done in [20], define $d_1=\text{rank }\mathscr{F}_1$ and $d_l=\text{rank }\mathscr{F}_l-\text{rank }\mathscr{F}_{l-1}$ for $l=1,\ldots,n$, the definition of observability indices can be given as follows.

Definition 3. *(Observability index)[45] The list of integers $(v_1,\ldots,v_p)$ are called the observability indices of* (2) *such that $v_i=card\{d_l\geq i,1\leq i\leq l\}$.*

If necessary, reorder the output components such that

$$\text{rank}\frac{\partial(y,\dot{y},\ldots,y^{(n-1)})}{\partial x}=l_1+l_2+\cdots+l_p. \tag{5}$$

Now, compute

$$dy_i^{(j-1)}=\varepsilon_{i,j}dx+\gamma_{i,j}d\theta$$

for $i=1,\ldots,p$ and $j=1,\ldots,l_i$. From (5), any $\varepsilon_{i,j}$ can be written as a linear combination of $\{\varepsilon_{1,1},\ldots,\varepsilon_{1,l_1},\ldots,\varepsilon_{p,1},\ldots,\varepsilon_{p,l_p}\}$. Then, the higher order time derivatives $dy_i^{(j)}$ can be computed and, from the implicit function theorem, dx can be substituted to obtain

$$dy_i^{(j)}=\left(\sum_{r=1}^{p}\sum_{s=1}^{l-r}\eta_{rs}dy_r^{(s-1)}\right)+\gamma_{i,j}d\theta.$$

The system is geometrically identifiable [35] if and only if there are integers l_i^*, for $i=1,\ldots,p$, such that *rank* $\Gamma_g=q$, where

$$\Gamma_g=\left[\gamma_{1,1}^T,\ldots,\gamma_{1,l_1^*}^T,\gamma_{2,1}^T,\ldots,\gamma_{p,l_p^*}^T\right]^T.$$

The parameter is then algebraically identifiable if and only if there exist p integers k_i^*, for $i=1,\ldots,p$, such that *rank* $\Gamma_a=q$, where

$$\Gamma_a=\left[\gamma_{1,l_1+1}^T,\ldots,\gamma_{1,k_1^*}^T,\gamma_{2,l_2+1}^T,\ldots,\gamma_{p,k_p^*}^T\right]^T.$$

The following theorem can be stated.

Theorem 1. *[35] The system is algebraically identifiable if and only if $\Theta\subset\mathscr{Y}$.*

Remark 2. *This method requires to calculate higher order derivatives of the outputs that may not be used later. Indeed, the determination of the higher order derivatives of the outputs of a dynamic system can very quickly become difficult for high dimensional systems.*

In the next section, a novel algorithm is proposed for parameter identification. Unlike the method proposed in [35], the higher order derivatives of the outputs will be calculated by iteration (if needed) and not in advance. In addition, a matrix P will be introduced for a partial or total identification of the states and/or the parameters for the studied system.

4 Parameter identifiability

As presented in the previous section, the parameter identifiability is related to the observability of z in (2). For this, define the observability indices for (2).

Denote

$$\mathscr{F}_i = span\{dh_1,\ldots,dL_f^{i-1}h_1,\ldots,dh_p,\ldots,dL_f^{i-1}h_p\}$$

for $1 \le i \le k$, it has been shown in [42] that $\mathscr{F}_1 \subset \mathscr{F}_2 \subset \cdots \subset \mathscr{F}_l$. Then, set $d_1 = \text{rank } \mathscr{F}_1$ and $d_l = \text{rank } \mathscr{F}_l - \text{rank } \mathscr{F}_{l-1}$ for $2 \le l \le k$, one can obtain the observability indices given in Definition 3.

Remark 3. *The system* (2) *is observable if $v_1 + \cdots + v_k = n+q$ with $n+q$ being the dimension of ξ. In this case, the state x and the unknown parameter θ of* (1) *can be estimated simultaneously.*

Considering the observability indices, the following matrix $d\Omega$ can be defined:

$$d\Omega = \frac{\partial[h_1, L_f h_1,\ldots,L_f^{v_1-1}h_1,\ldots,h_p,\ldots,L_f^{v_k-1}h_p]^T}{\partial\xi} \tag{6}$$

and the following theorem can be stated.

Theorem 2. *The unknown parameter θ of* (1) *is identifiable (or equivalently z of* (2) *is observable) if rank $d\Omega = n+q$.*

Theorem 2 is quite straightforward, and states a very restrictive sufficient condition on parameter identifiability, which requires that all states should be observable as well. In general, parameter identifiability does not depend on state observability. In other words, some parameters could be identifiable while the system state is not observable. An alternative explanation states that some of the parameters could be identified even if the the observability condition is not satisfied. This can be illustrated by the following example:

$$\begin{cases} \dot{x}_1 = x_1\theta \\ \dot{x}_2 = x_1x_2 \\ y = x_1 \end{cases} \tag{7}$$

It can be seen that θ is identifiable if $x_1 \neq 0$ (since $\dot{y} = y\theta$), while the state x_2 is not observable. This example also shows that parameter identifiability might depend on partial observable states involved in the obtained parameter identification equation. Taking this into account and in order to obtain a less strong sufficient condition of parameter identifiability, define the following sets:

$$\mathscr{F} = \Xi \cap span_\omega\{d\omega\}$$

where $\Xi = \{\mathscr{X} + \Theta\} = span\{dx, d\theta\}$, and

$$\omega = \{h_1,\ldots,L_f^{b_1}h_1,\ldots,h_p,\ldots,L_f^{b_p}h_p\}$$

where b_j for $j = 1,\ldots,p$ are positive integer which will be used for the iterations of the following algorithm introduced here to compute the minimum of time derivatives of the outputs and therefore to relax the restrictive sufficient condition stated in Theorem 2 and in [1].

Denote $\Xi = \{\mathscr{X} + \Theta\}$. One has

$$\mathscr{F} = \Xi \cap span_\omega\{d\omega\}$$

According to Frobenius Theorem [19], a nonsingular involutive distribution is completely integrable if an only if it is involutive. Thus, the codistribution Δ is spanned by exact differentials. This allows to define a new chart of coordinates with two subsystems, where one of them is observable and unaffected by the unknown inputs. Using the following algorithm, which is an extension of [1], one can then recursively: (i) evaluate which part of the system is observable/identifiable; (ii) obtain the expression of the observable states and identifiable parameters in function of the outputs and a finite number of their time derivatives.

Algorithm 1.

Input : f, δ, h *and* P *defined in (2);*
Output: $\partial\Omega(\xi)$, $\mathscr{L}$ *and* $\mathscr{K}(\xi)$*;*

Initialization :
- $b_j = 0$, $j = 1,\ldots,p$;
- $\omega_0 = \{\omega_{1,0},\ldots,\omega_{p,0}\}$ *with* $\omega_{j,0} = \{h_j\}$;
- $\mathscr{F}_0 = \{\mathscr{F}_{1,0},\ldots,\mathscr{F}_{p,0}\} = \Xi \cap span_{\omega_0}\{d\omega_0\}$ *with* $\mathscr{F}_{j,0} = \Xi \cap span_{\omega_{j,0}}\{d\omega_{j,0}\}$;
- *Define* $\Delta = span(\delta_1(x),\ldots,\delta_m(x))$;
- $i = 1$ *et* $l = 0$;

Iteration i:

[Step 1:] Compute $\mathscr{L}_i = span_{\omega_{i-1}}\{d\omega_{i-1}\}$ *and* $\Delta^\perp$ *the annihilator of the involutive distribution* Δ *defined by* $\Delta^\perp = span\{\bar{\vartheta} \in \mathscr{L}_i | \bar{\vartheta}\delta = 0, \forall\delta \in \Delta\}$;

[Step 2:] Compute $\partial\Omega_i = \frac{\partial\omega_{i-1}^T}{\partial\xi}$;

[Step 3:] For the given matrix P, compute the matrix $\mathscr{K}_i(\xi)$ *such that* $P = \mathscr{K}_i(\xi)\partial\Omega_i$. *Denote* $\mathscr{K}_i^j(\xi)$ *as the jth line of* $\mathscr{K}_i(\xi)$;

[Step 4:] Check: if all elements $\mathscr{K}_i^j(\xi) \notin \mathscr{L}_i$, *go to Step 5; otherwise, go to Step* 10;

[Step 5:] Compute $V_i = \left(L_f^{b_1}h_1,\ldots,L_f^{b_p}h_p\right)^T$ *and*

$$\Gamma_i = \begin{pmatrix} \Gamma_{1,i} \\ \vdots \\ \Gamma_{p,i} \end{pmatrix} = \begin{pmatrix} L_{\bar{g}_1}L_f^{b_1}h_1 & \cdots & L_{\bar{g}_m}L_f^{b_1}h_1 \\ \vdots & \ddots & \vdots \\ L_{\bar{g}_1}L_f^{b_p}h_p & \cdots & L_{\bar{g}_m}L_f^{b_p}h_p \end{pmatrix};$$

Check: if $\Gamma_i = 0$, *go to Step* 7*; otherwise, go to Step* 6*;*

[Step 6:] Compute $\Delta^\perp \cap \mathscr{L}_i$. *Define* $\Upsilon_i = \{\bar{\vartheta} \in \Delta^\perp \cap \mathscr{L}_i | \bar{\vartheta}V_i \notin \omega_{i_1}\}$. *If* $\Upsilon_i \neq 0$, *then one can define a new output which is independent of the unknown inputs as follows. Increment l. Compute* $\bar{y} = \bar{\vartheta}V_i$, $y_{p+l} = \bar{y}$ *mod* ω_{i_1} *and* $\Gamma_i = (\Gamma_{1,i},\ldots,\Gamma_{p+l,i})^T$ *with* $\Gamma_{p+l,i} = 0$;

[Step 7:] $\omega_i = \{\omega_{1,i},\ldots,\omega_{p+l,i}\}$ *with* $\omega_{j,i} = \{h_j,\ldots,h_j^{(b_j+1)}\}$, $j = 1,\ldots,p+l$; $\mathscr{F}_i = \{\mathscr{F}_{1,i},\ldots,\mathscr{F}_{p+l,i}\} = \Xi \cap span_{\omega_i}\{d\omega_i\}$ *with* $\mathscr{F}_{j,i} = \Xi \cap span_{\omega_{j,i}}\{d\omega_{j,i}\}$;

[Step 8:] Check $\mathscr{F}_{j,i} \not\subset \{\mathscr{F}_{i-1} \cup \{\mathscr{F}_i \backslash \mathscr{F}_{j,i}\}\}$ *and* $\Gamma_{j,i} = 0$, *then* $b_j = b_j + 1$;

[Step 9:] Check: if $\mathscr{F}_{i-1} \subset \mathscr{F}_i$, *then do* $i = i+1$ *and go to Step* 1*; otherwise, go to Step* 10;

[Step 10:] Return $\partial\Omega(\xi) = \partial\Omega_i(\xi)$, $\mathscr{L} = \mathscr{L}_i$ *and* $\mathscr{K}(\xi) = \mathscr{K}_i(\xi)$.

Based on the return of Algorithm 1, the following result can be stated.

Theorem 3. *Apply Algorithm* 1*. If there exists* $\mathscr{K}(\xi)$ *such that* $P = \mathscr{K}(\xi)\partial\Omega$ *and the components of the elements of all the rows of* $\mathscr{K}(\xi)$ *satisfy* $\mathscr{K}^j(\xi) \in \mathscr{L}$, *then z is observable.*

Proof: By definition $\mathscr{L}_i$ defines the observable space of system (2) at each iteration i. After applying Algorithm 1, if $\mathscr{K}^j(\xi) \in \mathscr{L}$, all elements in $\mathscr{K}(\xi)$ are spanned by $\mathscr{L}$, which implies that one can always find a matrix P such that $z = P\xi$ is observable. □

Remark 4. *It is known that any matrix $\partial\Omega \in \mathbb{R}^{r\times(n+q)}$ satisfying rank$\partial\Omega = r$ and $n+q > r$ is right invertible, i.e., there exists a matrix $[\partial\Omega]_R^{-1}$ such that $\partial\Omega[\partial\Omega]_R^{-1} = I_r$. Therefore, the matrix $\mathscr{K}(\xi)$ can be obtained as follows:*

$$\mathscr{K}(\xi) = P[\partial\Omega]_R^{-1}. \tag{8}$$

Proposition 1. *Apply Algorithm 1. If there exists $\mathscr{K}(\xi)$ such that $P = \mathscr{K}(\xi)\partial\Omega$ and if the components of the elements of all the rows of $\mathscr{K}(\xi)$ satisfy $\mathscr{K}^j(\xi) \in \mathscr{L}$, then there exists a function $F(y, \dot{y}, \ldots, y^{(k)})$ such that:*

$$z = F(y, \dot{y}, \ddot{y}, \ldots, y^{(k)}). \tag{9}$$

with $k \in \mathbb{N}$.

Proof: From $z = P\xi$, one obtains

$$dz = d(P\xi) = Pd\xi.$$

Considering $P = \mathscr{K}(\xi)\partial\Omega$ and replacing P in the previous equation, one gets

$$dz = \mathscr{K}(\xi)d\Omega \text{ with } d\Omega = \partial\Omega d\xi.$$

Since dz is a closed one-form, $\mathscr{K}(\xi)d\Omega$ is also a closed one-form [26]. Due to the fact that $\mathscr{K}(\xi)$ and $d\Omega$ represent the space and co-space generated by y and its higher order derivatives, hence there exists a function $F(y, \dot{y}, \ldots, y^{(k)})$ for a certain $k \in \mathbb{N}$ such that

$$z = F(y, \dot{y}, \ddot{y}, \ldots, y^{(k)}).$$

□

It is clear that the condition presented in Theorem 3 is less restrictive than the one given in Theorem 2, since Theorem 3 does not need to compute all higher order time derivatives in advance.

Note that, in many cases, the function F is a rational function given by

$$F(y, \dot{y}, \ddot{y}, \ldots, y^{(k)}) = \frac{\bar{g}(y, \dot{y}, \ddot{y}, \ldots, y^{(k)})}{g(y, \dot{y}, \ddot{y}, \ldots, y^{(k)})} \tag{10}$$

For the system (7) of the previous example, one has:

$$\theta = F(y, \dot{y}) = \frac{\dot{y}}{y}$$

Due to the existence of the singularity when $y = 0$, it is therefore not possible to compute directly θ, even if we can use some efficient differentiators, such as higher order sliding mode differentiators [21]. In the next section, it will be shown how to overcome the singularity problem by designing a fixed-time sliding mode observer.

5 Fixed-time sliding mode observer

Assume that the condition in Theorem 3 is satisfied. It is possible to find a function F which yields an algebraic equation to estimate z.

According to Proposition 1 and the relation (10), singularity problems can appear in (9) if $g(y, \dot{y}, \ddot{y}, \ldots, y^{(k)})$ crosses zero at some values of the output y and its higher order time derivatives. A technique to overcome such a problem is the persistent excitation condition.

Definition 4. *(Persistent excitation)[31]. For the equation (10), the function $u(y, \dot{y}, \ddot{y}, \ldots, y^{(k)})$ is said to be persistently exciting (PE) if and only if there exist positive constants T and ε_1 such that for all $t \geq 0$,*

$$\int_{t-T}^{t} u^2 d\tau > \varepsilon_1 I_n$$

where T represents the excitation period of $u(y, \dot{y}, \ddot{y}, \ldots, y^{(k)})$.

There are several approaches in the literature that can be used to estimate the involved parameters as long as the Persistent of Excitation (PE) condition is satisfied. The so-called Least-Square method is one of the simplest methods to apply, which however is not suitable for noisy systems [37]. Another way to estimate the unknown parameter is to use an adaptive algorithm as given in [23] or the normalized gradient algorithm [37]. These methods can only provide asymptotic convergence. Another contribution of this paper is to propose a method to estimate the parameters of system (2) in finite-time or fixed-time (non-asymptotic convergence [22]).

One contribution of this paper is to design an observer providing fixed-time stability of the origin and allowing to adjust the global settling time of the closed-loop system, while avoiding observability singularities. Precisely, this paper defines a sliding manifold ensuring that in sliding motion:

- the estimate of the state z is obtained in finite time,
- the trajectories of the system satisfy the Persistent Excitation condition given in Definition 4 (and thus overcome the observability singularity problem).

To achieve the main goal of this work, define the cost function Q as follows:

$$Q(t) = \int_{t-T}^{t} [\bar{g}(\tau) - z(\tau)g(\tau)]^2 d\tau \tag{11}$$

Hence, the aim is to design an observer to minimize the error (cost function) Q. For this, differentiating Q with respect to z, one has:

$$\frac{\partial Q(t)}{\partial z(t)} = -2\int_{t-T}^{t} g(\tau)[\bar{g}(\tau) - z(\tau)g(\tau)]d\tau$$

Note that the minimal value of Q occurs only if $\frac{\partial Q(t)}{\partial z(t)} = 0$, thus one can define the sliding manifold S as follows:

$$S(t) = \int_{t-T}^{t} g(\tau)[\bar{g}(\tau) - z(\tau)g(\tau)]d\tau = -\frac{1}{2}\frac{\partial Q(t)}{\partial z(t)} \tag{12}$$

and consequently the objective is fulfilled if a sliding motion appears in finite time on $S = 0$.

The following theorem can be stated.

Theorem 4. *The following dynamics*

$$\dot{\hat{z}}(t) = \left[\frac{\partial S(\hat{z}(t), t)}{\partial \hat{z}(t)}\right]_L^{-1} \left[-k_1\lceil S\rfloor^{\alpha} - k_2\lceil S\rfloor^{\beta} - \frac{\partial S(\hat{z}(t), t)}{\partial t}\right] \tag{13}$$

where $\lceil S\rfloor^{\alpha} = \mathrm{sign(S)|S|}^{\alpha}$ and $\lceil S\rfloor^{\beta} = \mathrm{sign(S_i)|S_i|}^{\beta}$ with $k_1 > 0$, $k_2 > 0$, $0 < \alpha < 1$ and $\beta > 1$, converges to $z(t)$ in a fixed-time , i.e., $\|z(t) - \hat{z}(t)\| = 0$, $\forall\, t \geq T_f$ where $T_f \leq \frac{1}{k_1(1-\alpha)} + \frac{1}{k_2(1-\beta)}$ is a positive constant independent of any initial condition.

Proof: The function Q takes its minimum value (zero) when $\frac{\partial Q}{\partial z}=0$, which implies that $S=0$. Since

$$\begin{aligned} S(t) &= \int_{t-T}^{t} g(\tau)[\bar{g}(\tau)-\hat{z}(\tau)g(\tau)]d\tau \\ &= \int_{t-T}^{t} g(\tau)[z(\tau)g(\tau)-\hat{z}(\tau)g(\tau)]d\tau \\ &= \int_{t-T}^{t} g^2(\tau)[z(\tau)-\hat{z}(\tau)]d\tau \end{aligned}$$

$\|z(t)-\hat{z}(t)\|=0$ if and only if $S=0$. Therefore, it remains to prove that S converges to zero in fixed-time.

Consider the dynamics of S:

$$\frac{dS(\hat{z}(t),t)}{dt} = \frac{\partial S(\hat{z}(t),t)}{\partial t} + \frac{\partial S(\hat{z}(t),t)}{\partial \hat{z}(t)}\dot{\hat{z}}(t)$$

Replacing $\dot{\hat{z}}(t)$ by the observer (13) into the above equation, one obtains

$$\dot{S} = -k_1\lceil S\rfloor^{\alpha} - k_2\lceil S\rfloor^{\beta} \tag{14}$$

Define the Lyapunov function $V=|S|$. Its time derivative is given by:

$$\begin{aligned} \dot{V} &= S\dot{S} = S\left(-k_1\lceil S\rfloor^{\alpha} - k_2\lceil S\rfloor^{\beta}\right) \\ &= -k_1|S|^{\alpha} - k_2|S|^{\beta} \\ &= -k_1 V^{\alpha} - k_2 V^{\beta} \end{aligned}$$

Since $k_1>0$, $k_2>0$, $0<\alpha<1$ and $\beta>1$, following the lines of Lemma 1 given in [27] it can be proven that S converges to zero in a fixed-time $T_f \leq \frac{1}{k_1(1-\alpha)} + \frac{1}{k_2(1-\beta)}$ independently of the initial condition $S(0)$. Therefore, it has been proven here that the observer (13) provides an estimate of z in fixed-time i.e. $\|z(t)-\hat{z}(t)\|=0$, $\forall\ t\geq T_f$. □

Remark 5. *In this algorithm, the value of the outputs and their successive time derivatives up to a certain order has to be known in finite time. This is done using fixed-time step-by-step super twisting sliding mode observers following the lines of the work [14, 17, 32]).*

To show the efficiency of the proposed algorithm, an illustrative example and simulation results are presented in the next section.

6 Application to topology identification of network systems

The Internet of Things reached its peak in the early 21st century. Indeed, several objects that exist naturally or developed by engineers can be interconnected by physical or wireless links (Bluetooth, radio-frequency, internet, ...), by interaction forces (gravitation) or by biomolecular links [28]. Thus, knowing the network topology in a fast way becomes essential, especially since it is often necessary to interact on these networks in order to improve them.

Consider a network of k nonlinear dynamical subsystems ($k\in\mathbb{N}, k\geq 2$) with unknown constant parameters connections, as shown in Fig. 1:

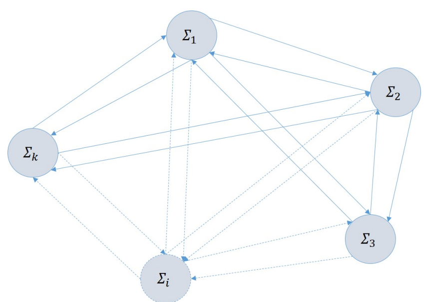

Fig. 1: *Example of nonlinear systems network topology*

It is assumed that the dynamics of each nonlinear subsystem Σ_i for $1\leq i\leq k$ involved in the considered network is given by:

$$\Sigma_i : \begin{cases} \dot{x}_i = \bar{f}_i(x_i) + \sum_{j=1, j\neq i}^{k} \varphi_{j,i}(x_j)\theta_{j,i} + \delta_i(x_i)\vartheta_i \\ = \bar{f}_i(x_i) + \varphi_i(x_i)\theta_i + +\delta_i(x_i)\vartheta_i \\ \dot{\theta}_i = 0 \\ y_i = \bar{h}_i(x_i) \end{cases} \tag{15}$$

where $x_i = \left[x_{i,1},\cdots,x_{i,n_i}\right]^T \in \mathbb{R}^{n_i}$ represents the state of Σ_i and $y_i\in\mathbb{R}^{p_i}$ is the output. $\theta_{j,i}\in\mathbb{R}$ represents the unknown but piecewise constant topology connection between Σ_j and Σ_i. The vector $\varphi_{j,i}(x_j)\in\mathbb{R}^{n_i}$ represents the information of the subsystem Σ_j injected into Σ_i via the connection $\theta_{j,i}$. Therefore, $\theta_i = \left[\theta_{1,i},\cdots,\theta_{i-1,i},0,\theta_{i+1,i},\cdots,\theta_{k,i}\right]\in\mathbb{R}^k$ and $\varphi_i(x_i) = \left[\varphi_{1,i}(x_1),\cdots,\varphi_{i-1,i}(x_{i-1}),0,\varphi_{i+1,i}(x_{i+1}),\cdots,\varphi_{k,i}(x_k)\right]\in\mathbb{R}^{n_i\times k}$.

For the sake of simplicity, denote $n=\sum_{i=1}^{k} n_i$, $p=\sum_{i=1}^{k} p_i$. Define $x=\left(x_1^T,\ldots,x_k^T\right)^T\in\mathbb{R}^n$ and $y=\left(y_1^T,\ldots,y_k^T\right)^T\in\mathbb{R}^p$.

The total number of possible interconnections is given by $q=k(k-1)$, i.e the maximum number of parameters to be estimated.

The system can be recasted in system 1 with $\varphi(x) = diag(\varphi_1(x),\ldots,\varphi_k(x))\in\mathbb{R}^{n\times k^2}$ and $\theta = \begin{pmatrix}\theta_1\\ \vdots \\ \theta_k\end{pmatrix}\in\mathbb{R}^{k^2}$.

Consider the example of a network of three interconnected nonlinear systems with unknown inputs (see Figure 2):

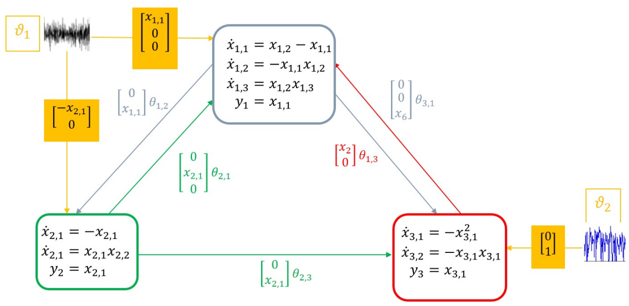

Fig. 2: *Network of three nonlinear systems*

The parametric coefficients of interconnections are represented by the unknown parameter $\theta=(\theta_{2,1},\theta_{3,1},\theta_{1,2},\theta_{1,3},\theta_{2,3})^T$. The dynamics of the whole network system is described by the following interconnected systems:

$$\begin{cases} \dot{x}_{1,1} &= x_{1,2} - x_{1,1} + x_{1,1}\nu_1 \\ \dot{x}_{1,2} &= -x_{1,1}x_{1,2} + x_{2,1}\theta_{2,1} \\ \dot{x}_{1,3} &= -x_{1,2}x_{1,3} + x_{3,1}\theta_{3,1} \\ \dot{x}_{2,1} &= x_{2,1} - x_{2,1}\nu_1 \\ \dot{x}_{2,2} &= x_{2,1}x_{2,2} - x_{1,1}\theta_{1,2} \\ \dot{x}_{3,1} &= -x_{3,1}^2 + x_{1,2}\theta_{1,3} \\ \dot{x}_{3,2} &= -x_{3,1}x_{3,2} + x_{2,1}\theta_{2,3} + \nu_2 \\ y_1 &= x_{1,1},\ y_2 = x_{2,1},\ y_3 = x_{3,1} \end{cases} \tag{16}$$

By denoting $x=(x_{1,1},x_{1,2},x_{1,3},x_{2,1},x_{2,2},x_{3,1},x_{3,2})^T$ as the state of the whole system and $y=(y_1,y_2,y_3)^T$ as its output, the system (16) can then be rewritten as follows:

$$\begin{cases} \xi = \begin{pmatrix} x \\ \theta \end{pmatrix} \\ \dot{\xi} = f(\xi) + \delta(\xi)\nu \\ y = (y_1,y_2,y_3)^T \end{cases}$$

with the new state

$$\xi = (x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{3,1}, x_{3,2}, \theta_{2,1}, \theta_{3,1}, \theta_{1,2}, \theta_{1,3}, \theta_{2,3})^T,$$

$$f(\xi) = \begin{pmatrix} \xi_2 - \xi_1 \\ \xi_2 + \xi_4\xi_8 \\ -\xi_3 + \xi_6\xi_9 \\ -\xi_4 \\ xi_4 + \xi_1\xi_{10} \\ \xi_6 + \xi_2\xi_{11} \\ -\xi_7 + \xi_4\xi_{12} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \text{ and}$$

$$\delta(\xi) = \begin{pmatrix} \xi_1 & 0 & 0 & -\xi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T.$$

Suppose it is aimed to estimate the vector $z = (x_{1,2}, \theta_{2,1}, \theta_{1,3})$. P is chosen as follows:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Applying the proposed Algorithm 1, the identifiability of z can be checked:

1. Initialization:
- $\omega_0 = \{\omega_{1,0}, \omega_{2,0}, \omega_{3,0}\}$, with $\omega_{1,0} = \{y_1\}$, $\omega_{2,0} = \{y_2\}$ and $\omega_{3,0} = \{y_3\}$;

- $\mathscr{F}_0 = \{\mathscr{F}_{1,0}, \mathscr{F}_{2,0}, \mathscr{F}_{3,0}\}$ with $\mathscr{F}_{1,0} = \Xi \cap span_{\omega_{1,0}}\{d\omega_{1,0}\}$, $\mathscr{F}_{2,0} = \Xi \cap span_{\omega_{2,0}}\{d\omega_{2,0}\}$, $\mathscr{F}_{3,0} = \Xi \cap span_{\omega_{3,0}}\{d\omega_{3,0}\}$;

- $\Delta = \begin{pmatrix} \xi_1 & 0 & 0 & -\xi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$ which annihilator is

$$\Delta^\perp = span\{d\xi_2, d\xi_3, d\xi_5, d\xi_6, d\xi_8, d\xi_9, d\xi_{10}, d\xi_{11}, d\xi_{12}, \xi_4 d\xi_1 + \xi_1 d\xi_4\}$$

- $b_1 = b_2 = b_3 = 0$ and $i = 1$.

2. Step 1 (i=1). One has:

- $\mathscr{L}_1 = span_{\omega_0}\{d\omega_0\} = span_{\omega_0}\{d\xi_1, d\xi_4, d\xi_6\}$;

- $\partial\Omega_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$;

- and $\mathscr{K}_1(\xi) = P[\partial\Omega_1]_R^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Note that $\mathscr{K}_1^j(\xi) \notin \mathscr{L}_1$. Then, compute

- $V_1 = \begin{pmatrix} \xi_2 - \xi_1 \\ \xi_4 \\ -\xi_6^2 + \xi_1\xi_{11} \end{pmatrix}$ and $\Gamma_1 = \begin{pmatrix} \xi_1 & 0 \\ -\xi_4 & 0 \\ 0 & 0 \end{pmatrix}$;
- $\Delta^\perp \cap \mathscr{L}_1 = span\{d\xi_6, \xi_4 d\xi_1 + \xi_1 d\xi_4\}$;
- $\Upsilon = span\{\bar{\vartheta} \in \Delta^\perp \cap \mathscr{L}_1 | \bar{\vartheta}V_1 \notin \omega_0\} = span\{\xi_4 d\xi_1 + \xi_1 d\xi_4\}$.
- One can check that $\Upsilon \neq O$. Then increase $l = 1$ and define $b_4 = 0$, $\bar{y} = \bar{\vartheta}V_1 = (\xi_2 - \xi_1)\xi_4 + \xi_4\xi_1$ and $y_4 = \bar{y} \bmod \omega_0 = \xi_2$ for $\xi_4 \neq 0$.

Calculate also:

- $\omega_1 = \{\omega_{1,1}, \omega_{2,1}, \omega_{3,1}, \omega_{4,1}\}$ with $\omega_{1,1} = \{y_1, \dot{y}_1\}$, $\omega_{2,1} = \{y_2, \dot{y}_2\}$, $\omega_{3,1} = \{y_3, \dot{y}_3\}$; and $\omega_{4,1} = \{y_4, \dot{y}_4\}$

- $\mathscr{F}_1 = \{\mathscr{F}_{1,1}, \mathscr{F}_{2,1}, \mathscr{F}_{3,1}, \mathscr{F}_{4,1}\}$ with $\mathscr{F}_{j,1} = \Xi \cap span_{\omega_{j,1}}\{d\omega_{j,1}\}$;

It can be checked that:
- $\Gamma_{1,1} \neq 0$ and $\Gamma_{2,1}$ then $b_1 = b_2 = 0$;
- $\mathscr{F}_{3,1} \not\subset \{\mathscr{F}_0 \cap \{\mathscr{F}_1 \backslash \mathscr{F}_{3,1}\}\}$ and $\Gamma_{3,1} = 0$, so increase b_3, hence $b_3 = 1$;
- $\mathscr{F}_{4,1} \not\subset \{\mathscr{F}_0 \cap \{\mathscr{F}_1 \backslash \mathscr{F}_{4,1}\}\}$ and $\Gamma_{4,1} = 0$, so increase b_4 which gives us $b_4 = 1$;

Note that $\mathscr{F}_0 \subset \mathscr{F}_1$. Then go to the second iteration.

3. Step 2 (i=2). One has:

- $\mathscr{L}_2 = span_{\omega_1}\{d\omega_1\} = span_{\omega_1}\{d\xi_1, d\xi_2, d\xi_4, d\xi_6, d\xi_8, d\xi_{11}\}$;

- $\partial\Omega_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_{11} & 0 & 0 & 0 & -2\xi_6 & 0 & 0 & 0 & 0 & \xi_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\xi_2 & -\xi_1 & 0 & \xi_8 & 0 & 0 & 0 & \xi_4 & 0 & 0 & 0 & 0 \end{pmatrix}$;
- and $\mathscr{K}_2(\xi) = P[\partial\Omega_2]_R^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{\xi_2}{\xi_4} & \frac{-\xi_8}{\xi_4} & 0 & 0 & \frac{1}{\xi_4} & \frac{1}{\xi_4} \\ 0 & 0 & \frac{2\xi_6}{\xi_2} & \frac{1}{\xi_2} & \frac{-\xi_{11}}{\xi_2} & 0 \end{pmatrix}$.

One can check that for $j = 1,2,3$, $\mathscr{K}_2^j(\xi) \in \mathscr{L}$, then stop the algorithm and note $\partial\Omega = \partial\Omega_2$ and $\mathscr{K}(\xi) = \mathscr{K}_2(\xi)$.

The algorithm ends in a positive way. Thus the condition of Theorem 3 is satisfied. This means that the state $x_{1,2}$ is observable and the unknown parameters $\theta_{2,1}$ and $\theta_{1,3}$ of the system (16) are also identifiable. This means that the proposed observer can be designed to estimate both the state $x_{1,2}$ and the unknown parameters $\theta_{2,1}$ and $\theta_{1,3}$.

From the matrix $P = \mathscr{K}(\xi)\partial\Omega$ and $z = P\xi$, one has

$$dz = Pd\xi = K(\xi)d\Omega \text{ with } d\Omega = \partial\Omega d\xi.$$

Since $d\Omega = (dy_1, dy_2, dy_3, d\dot{y}_3, dy_4, d\dot{y}_4)^T$, one obtains

$$\begin{cases} d\xi_2 & = dy_4 \\ d\xi_8 & = \frac{\xi_2}{y_2}dy_1 - \frac{\xi_8}{y2}dy_2 + \frac{1}{y_2}dy_4 + \frac{1}{y_2}d\dot{y}_4 \\ d\xi_{11} & = \frac{2\xi_6}{y_4}dy_3 + \frac{1}{y_4}d\dot{y}_3 - \frac{\xi_{11}}{y_4}dy_4 \end{cases}$$

which leads to

$$\begin{cases} dxi_2 = dy_4 \\ d(\xi_8 y_2) = d(y_1 y_4 + \dot{y}_4) \\ d(xi_{11} y_4) = d(y_3^2 + \dot{y}_3) \end{cases}$$

Therefore, one has

$$\begin{cases} z_1 = x_{1,2} = y_4 \\ y_2 z_2 = y_2\theta_{2,1} = y_1 y_4 + \dot{y}_4 \\ y_4 z_3 = y_4\theta_{1,3} = y_3^2 + \dot{y}_3 \end{cases}$$

and finally:

$$\begin{cases} x_{1,2} & = y_4 \\ \theta_{2,1} & = \frac{y_1 y_4 + \dot{y}_4}{y_2} \\ \theta_{1,3} & = \frac{y_3 + \dot{y}_3}{y_4} \end{cases}$$

with $y_4 = x_{1,2} = \frac{\dot{y}_1 y_2 + y_1 \dot{y}_2}{y_2}$ for $y_2 \neq 0$.

It is therefore clear that singularity problems can appear in the estimation of the parameters $\theta_{2,1}$ and $\theta_{1,3}$.

The sliding variables in (12) are given by:

$$S_1 = \int_{t-T}^{t} (y_4 - z_1)\, ds$$
$$S_2 = \int_{t-T}^{t} y_2 \left((y_1 y_4 + \dot{y}_4) - y_2 z_2\right) ds$$
$$S_3 = \int_{t-T}^{t} y_4 ((y_3 + \dot{y}_3) - y_4 z_3) ds$$

Then, the fixed-time observer of type (13) can be designed to estimate the state $x_{1,2}$ and the identifiable parameters $(\theta_{2,1}, \theta_{1,3})$. In this simulation, the initial conditions are

$$x(0) = [1,1,1,1,1,1,1]^T,$$
$$z(0) = [1,3,5]^T \text{ and}$$
$$\hat{\theta}(0) = [3,0,0,5,0]^T.$$

The simulation results are presented in the following figures.

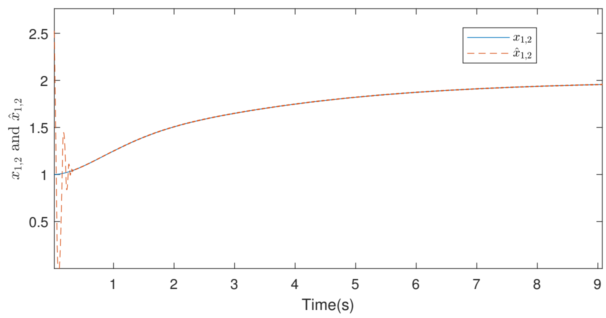

Fig. 3: *State $x_{1,2}$ and its estimate $\hat{x}_{1,2}$*

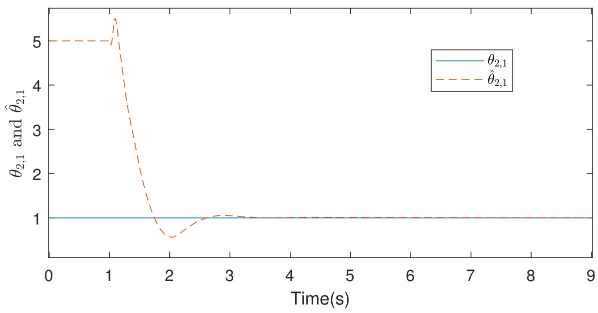

Fig. 4: *Parameter $\theta_{2,1}$ and its estimate $\hat{\theta}_{2,1}$*

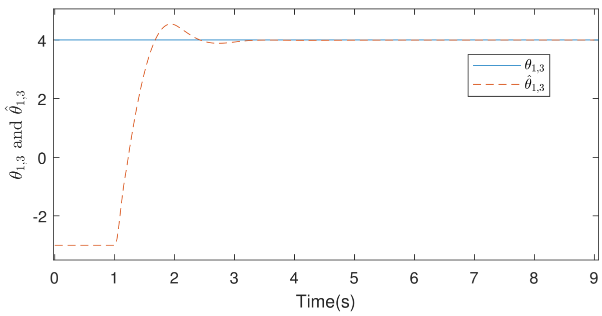

Fig. 5: *Parameter $\theta_{1,3}$ and its estimate $\hat{\theta}_{1,3}$*

The figures 3, 4 and 5 show that the estimates of the state $x_{1,2}$ and the unknown parameters $\theta_{2,1}$ and $\theta_{1,3}$ converge well in finite-time. This also shows that the singularity problem has been overcome for the estimation of the unknown parameters.

7 Conclusion

This paper investigated full and/or partial identifiability of uncertain nonlinear systems with unknown inputs. An observer with fixed-time convergence was designed to estimate simultaneously the values of the observable states and the identifiable parameters while avoiding observability singularity. A numerical example with applications to the fixed-time identification of the topology of the network of dynamical systems was presented to highlight the proposed approach. Further work aims at applying the given observation method for fast topological identification of network systems. To this end, fixed-time decentralized observers will have to be designed using graph theory. The results given here will also be extended to networks of nonlinear systems with time-delay implying other geometric tools such as Öre rings and extended Lyapunov functions (as Lyapunov-Krasovskii functionals).

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