Oceanic Dynamics under Location Uncertainty: Towards a consistent stochastic modeling

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Oceanic Dynamics under Location Uncertainty

Towards a consistent stochastic modelling

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Objective

- **Why stochastic modeling ?**
  - → **Take into account small-scale / unresolved processes**
  - → **Uncertainty Quantification, Ensemble Forecasts**
- **Why consistent model ?**
  - → **Both scales must respect appreciate physical laws**
  - → **Losing consistency may provide wrong statistics**

1. **Governing Equations**
   - **Location Uncertainty Principles**
   - **Barotropic Quasi−Geostrophic Model under LU**
2. Experimental Evaluations
   - Structure−preserving of Rossby wave
   - Ensemble forecasting verification of SQG_{MU}
   - Time−statistics of wind−driven circulation
3. Summary

- **Stochastic flow :**

$$\mathrm{d}\boldsymbol{X}_t = \underbrace{\boldsymbol{u}(\boldsymbol{X}_t, t)\mathrm{d}t}_{\text{large-scale resolved}} + \underbrace{\boldsymbol{\sigma}(\boldsymbol{X}_t, t)\mathrm{d}\boldsymbol{B}_t}_{\text{small-scale unresolved}}$$

- **Functional process :**

$$\underbrace{\boldsymbol{\sigma}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{B}_t}_{\text{correlated in space, uncorrelated in time}} = \int_D \breve{\boldsymbol{\sigma}}(\boldsymbol{x}, \boldsymbol{y}, t)\mathrm{d}\boldsymbol{B}_t(\boldsymbol{y})\mathrm{d}\boldsymbol{y}$$

correlated in space
uncorrelated in time

symmetric kernel

space-time white noise

- **Variance tensor :**

$$\boldsymbol{a} \triangleq \boldsymbol{\sigma}\boldsymbol{\sigma}^T = \mathbb{E}\left[\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t(\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t)^T\right]/\mathrm{d}t$$

homogeneous / heterogeneous

- **Transport of a random tracer :**

$(\nabla \cdot \boldsymbol{\sigma} = 0)$

Balanced Energy $\xrightarrow{(\nabla \cdot \boldsymbol{u}^\star = 0)}$ $\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\int_D \frac{1}{2}\theta^2 = 0}$

$$\mathrm{D}_t\theta \triangleq \mathrm{d}_t\theta + \underbrace{\boldsymbol{u}^\star}_{\text{corrected drift}} \cdot \nabla\theta\mathrm{d}t + \underbrace{\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t \cdot \nabla\theta}_{\text{multiplicative noise}} - \underbrace{\frac{1}{2}\nabla \cdot (\boldsymbol{a}\nabla\theta)\mathrm{d}t}_{\text{subgrid diffusion}} = 0$$

$\boxed{\boldsymbol{u} - \frac{1}{2}\nabla \cdot \boldsymbol{a}}$

- **Evolution of the potential vorticity with source processes :**

$$\mathrm{D}_t q = S_1(\boldsymbol{\nabla}\boldsymbol{u})\mathrm{d}t + S_2(\boldsymbol{\nabla}\boldsymbol{u})\mathrm{d}\boldsymbol{B}_t$$

- **Evolution of the stream function :**

$$q = \Delta\psi - \psi/L_R^2 + f$$

- **Strong incompressible constraints :**

$$\boldsymbol{u} = \boldsymbol{\nabla}^{\perp}\psi, \qquad \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t = \boldsymbol{\nabla}^{\perp}\varphi\mathrm{d}B_t, \qquad \boldsymbol{\nabla}\cdot\boldsymbol{u}^{\star} = 0$$

➡ **Conservation of total energy :** $\boxed{\dfrac{\mathrm{d}}{\mathrm{d}t}\displaystyle\int_D \frac{1}{2}\Big[\|\boldsymbol{\nabla}\psi\|^2 + (\psi/L_R^2)^2\Big] = 0}$

$$\left[\ \text{Sketch : } \int_D \psi\boldsymbol{\sigma}\circ\mathrm{d}\boldsymbol{B}_t\cdot\boldsymbol{\nabla}(q-f) = \int_D \psi S_2(\boldsymbol{\nabla}\boldsymbol{u})\circ\mathrm{d}\boldsymbol{B}_t\ \right]$$

- **Transport of a prognostic passive tracer :**

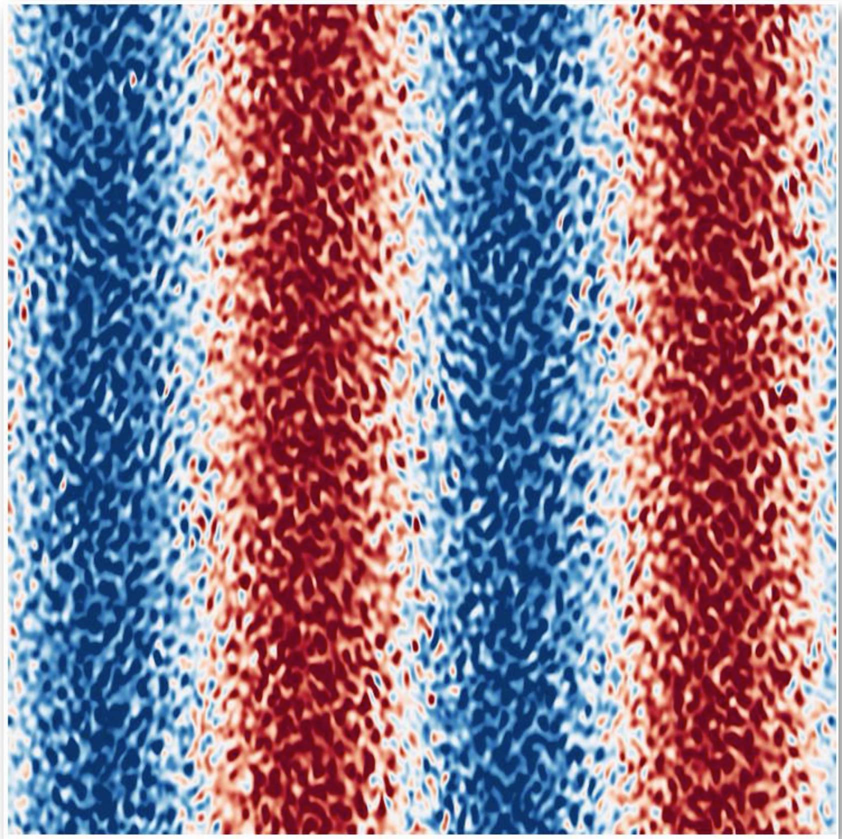
$$\mathrm{D}_t q' = 0$$

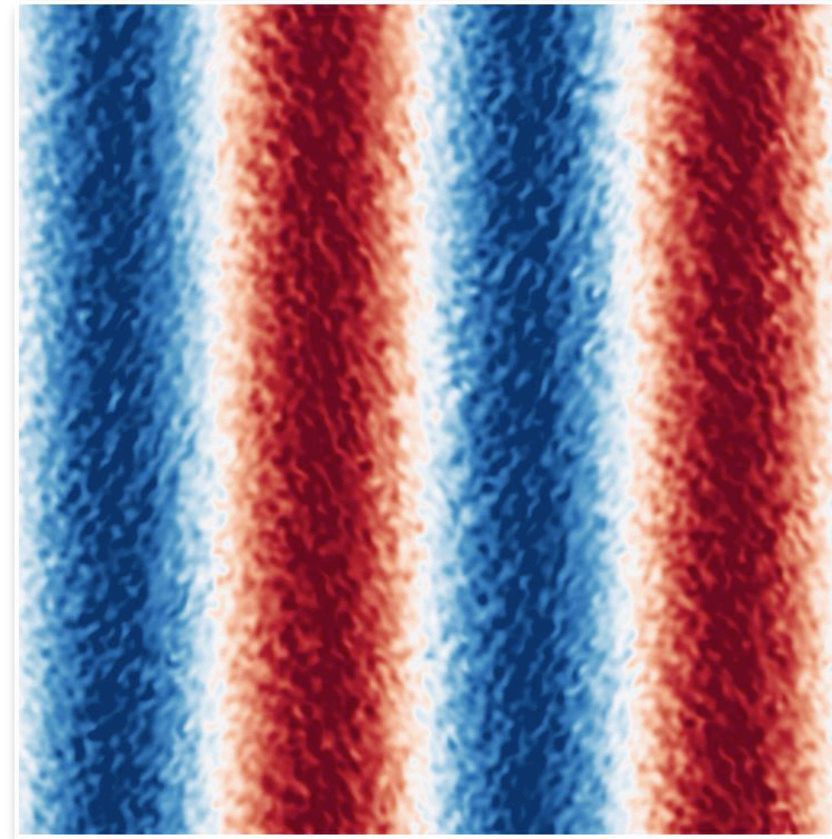
➡ **Conservation of tracer energy :** $\boxed{\dfrac{\mathrm{d}}{\mathrm{d}t}\displaystyle\int_D \frac{1}{2}(q')^2 = 0}$

Structure–preserving of Rossby wave

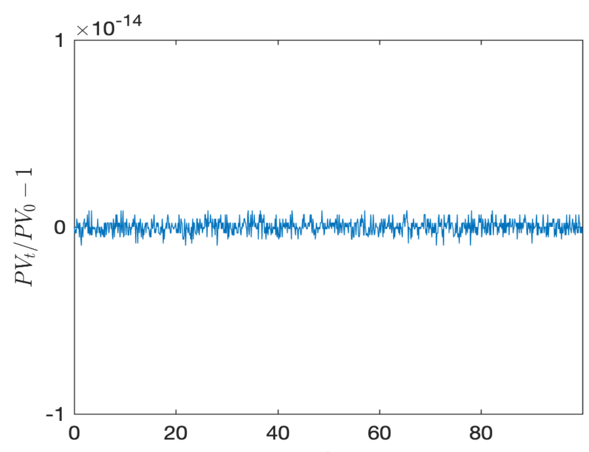
- **Flowchart :** $q \longleftrightarrow \psi \longrightarrow q'$

  (**Potential Vorticity**) (**Stream Function**) (**Passive Tracer**)

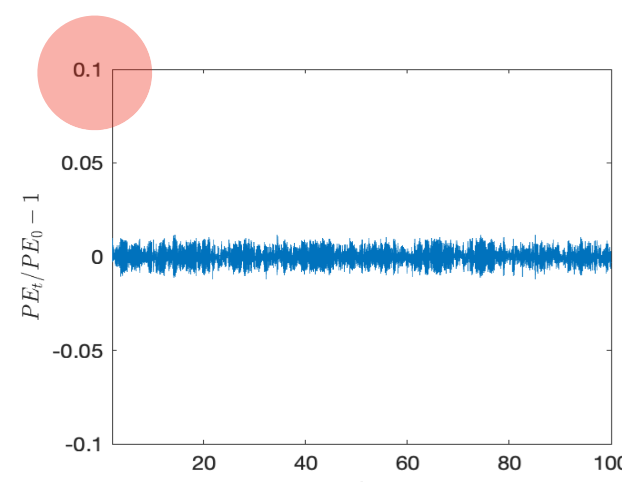




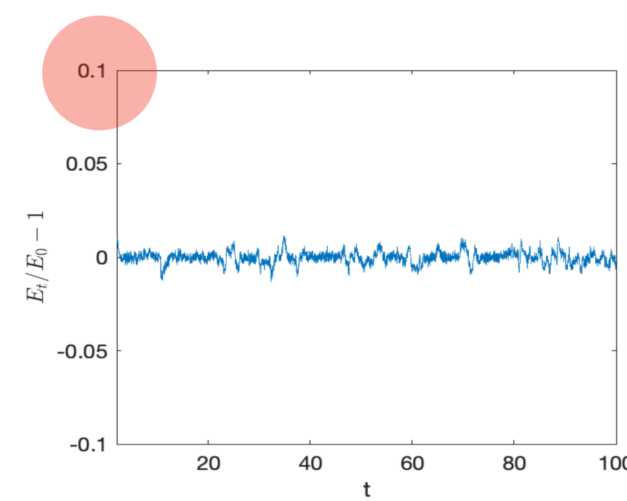
- **Illustration of conservations :**



Passive Tracer

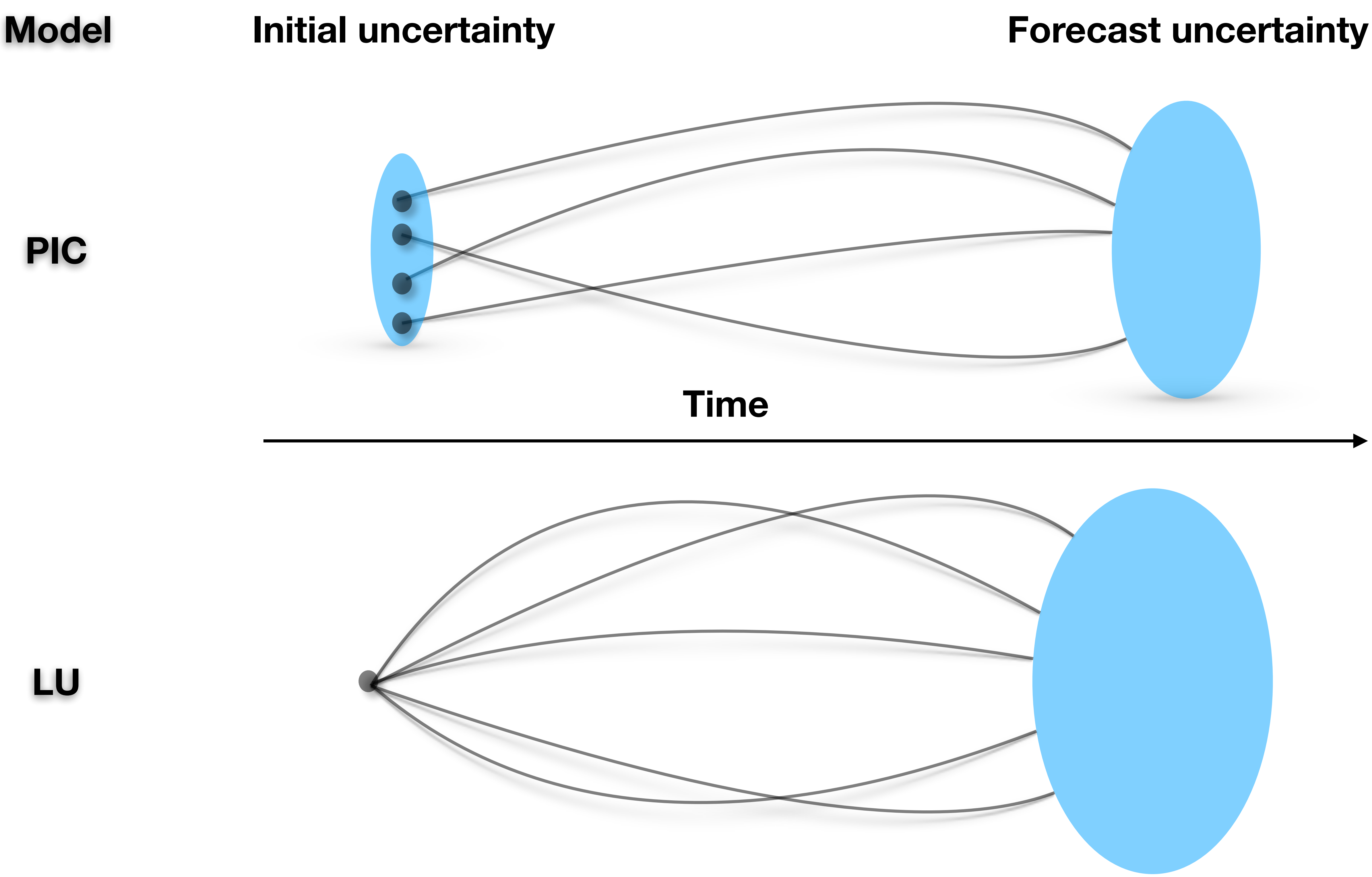


Tracer Energy



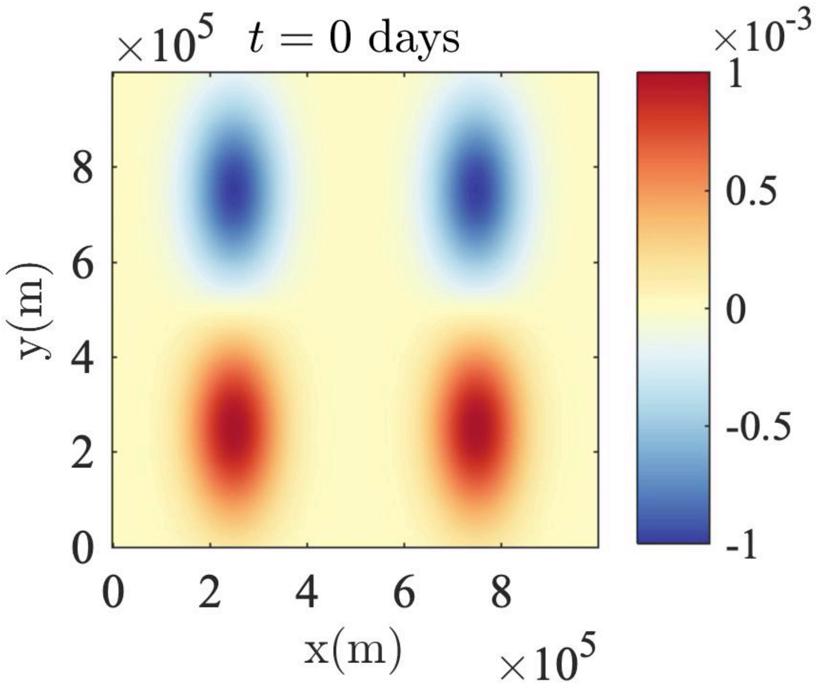
Total Energy

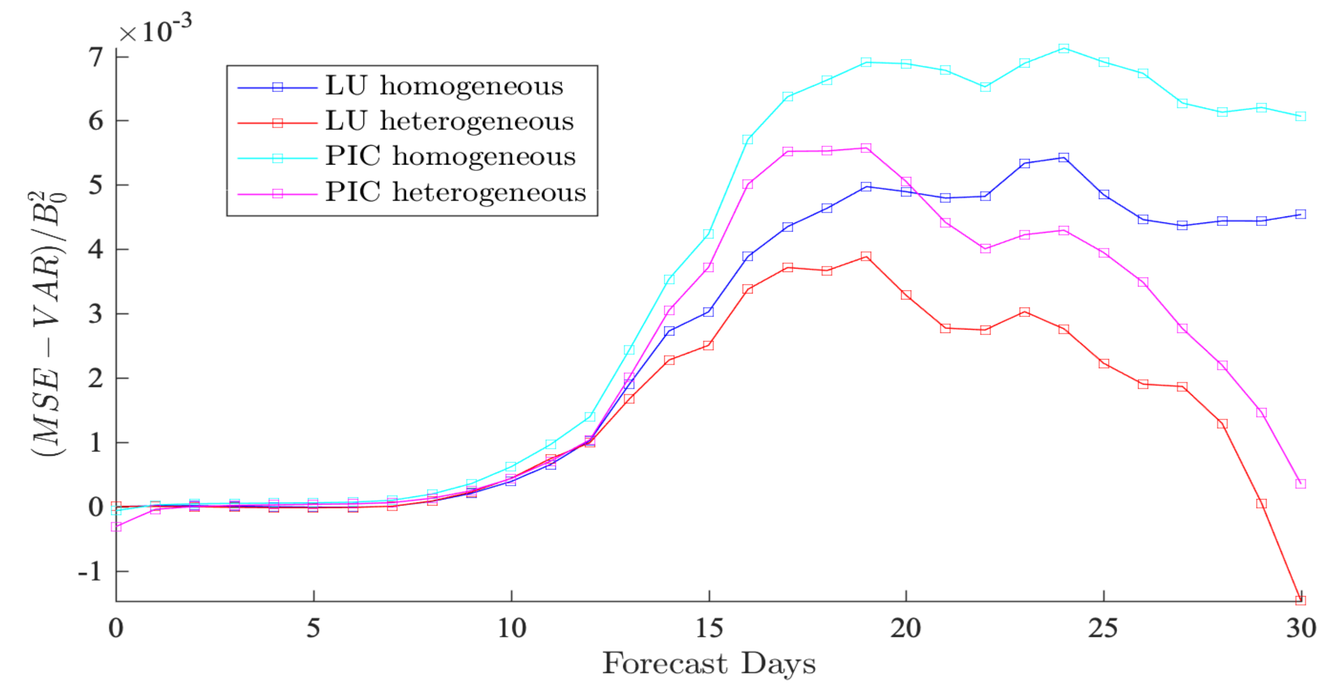
- **Illustration of ensemble prediction systems :**

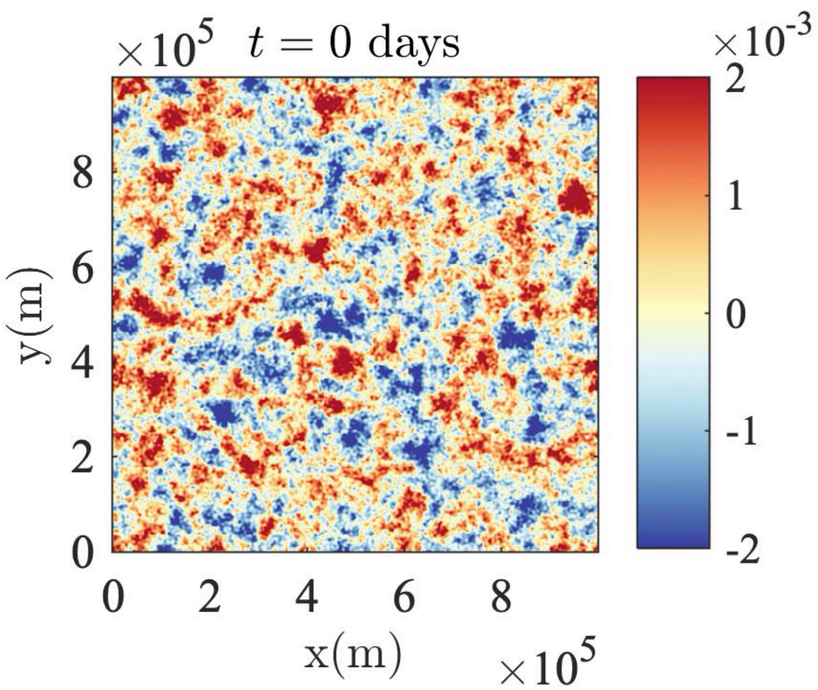


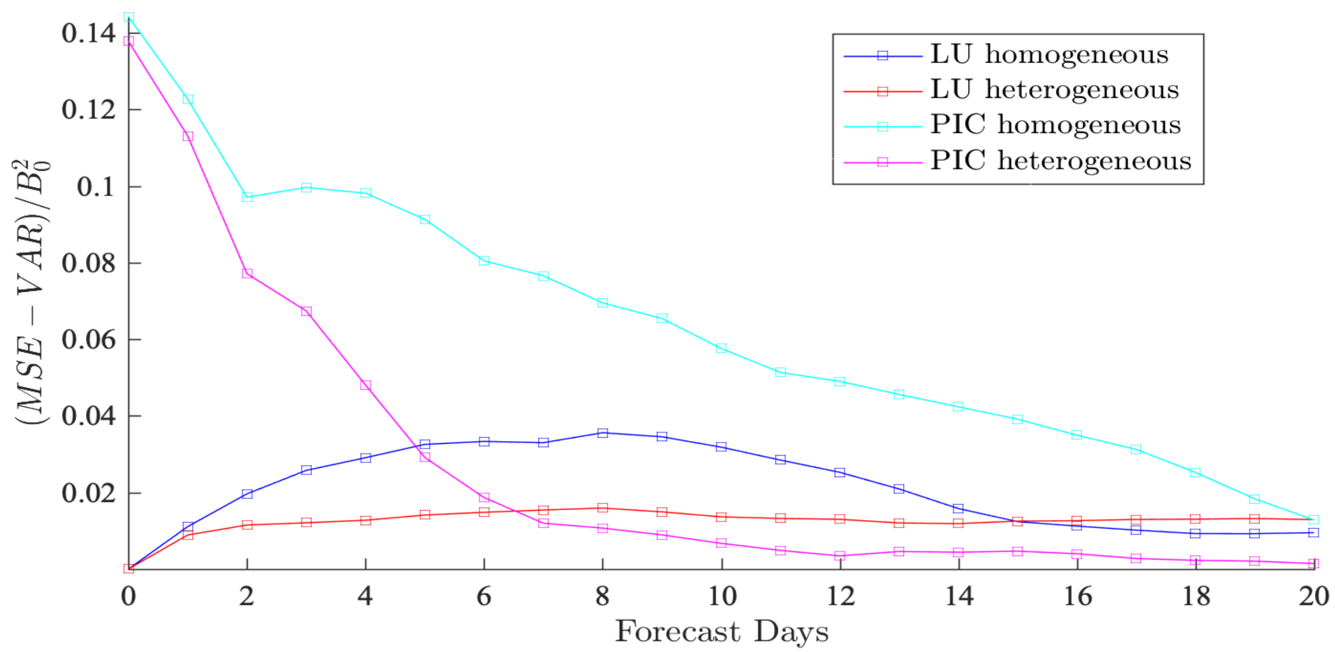
Ensemble forecasting verification of SQG_{MU} (Resseguier et al. 2019)

- **Reliability — Ensemble Spread match MSE of the Ensemble Mean :**





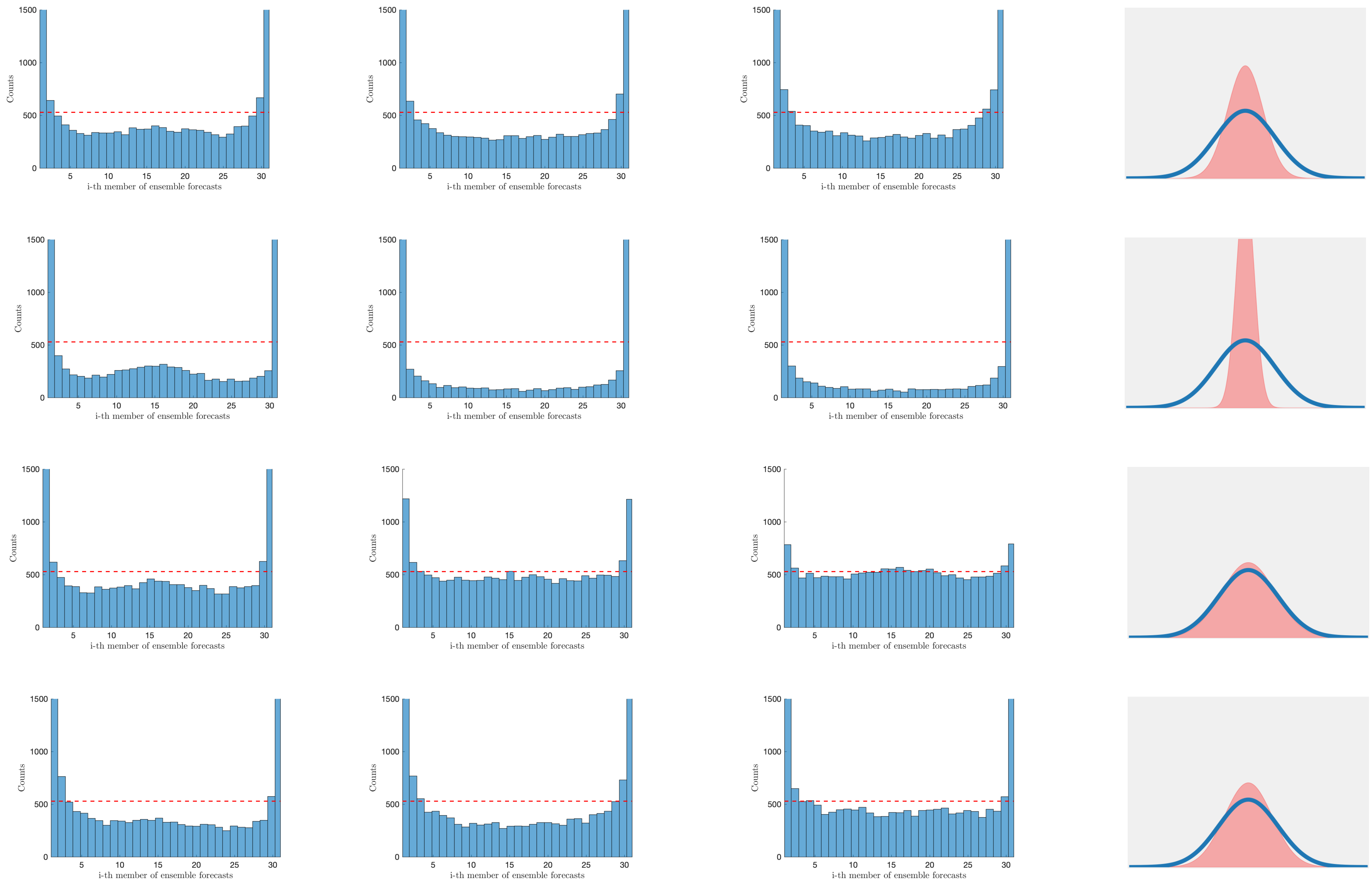




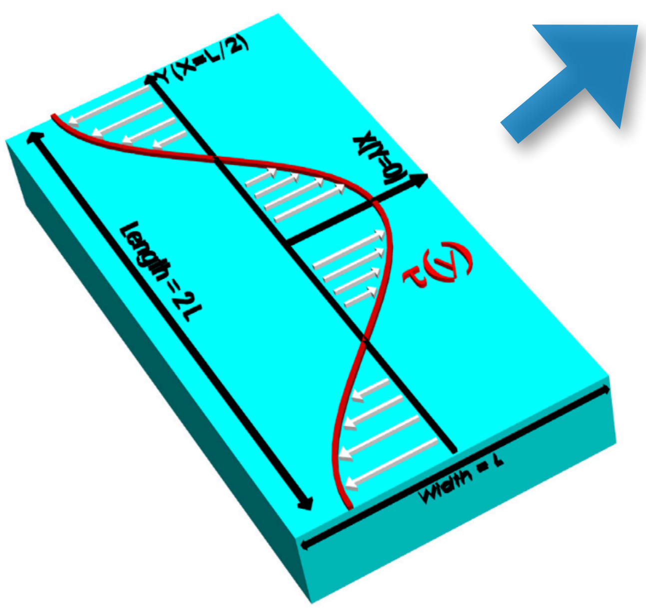
Ensemble forecasting verification of SQG_{MU} (Resseguier et al. 2019)

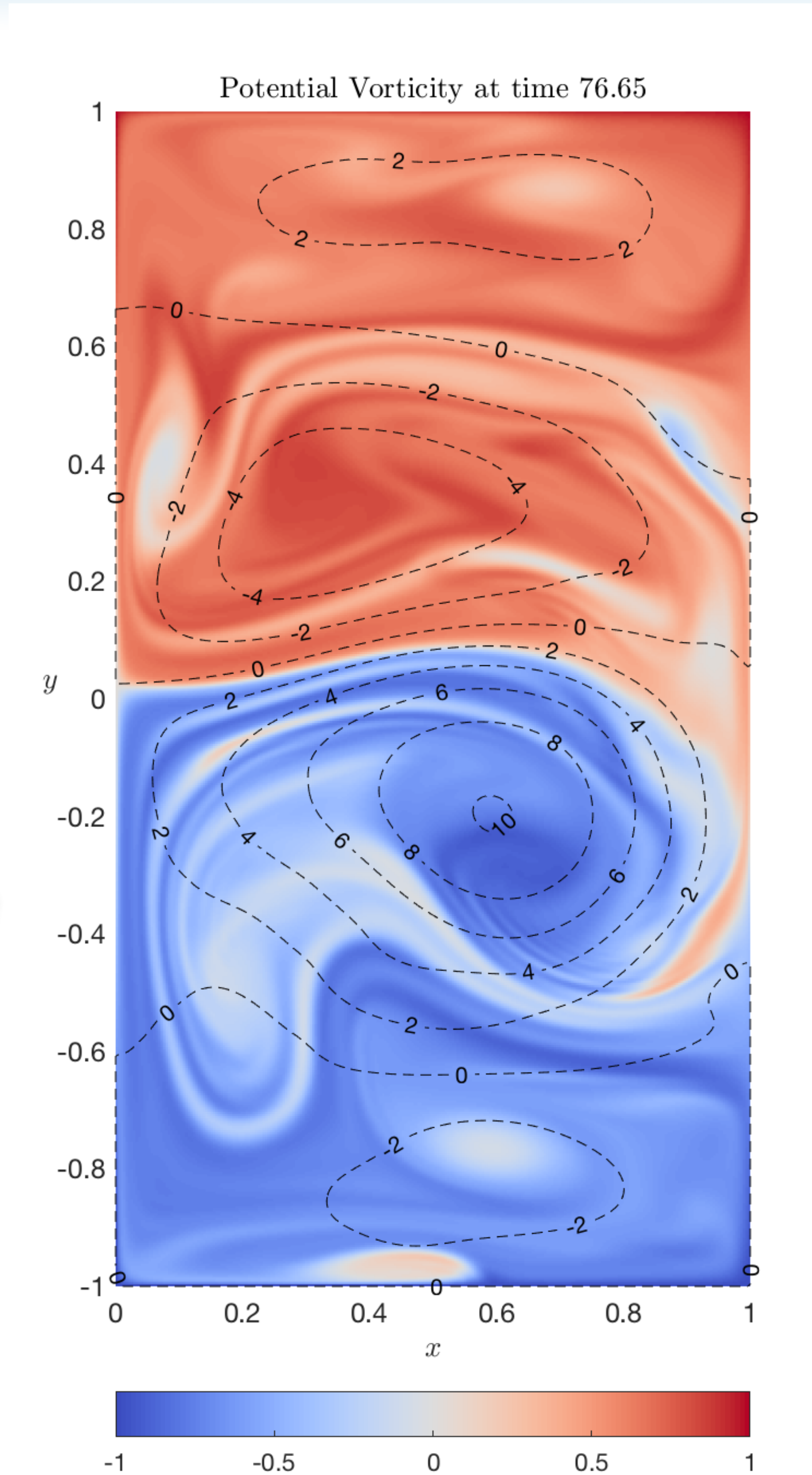
- **Reliability — Ranked Histogram of Ensemble Members :**

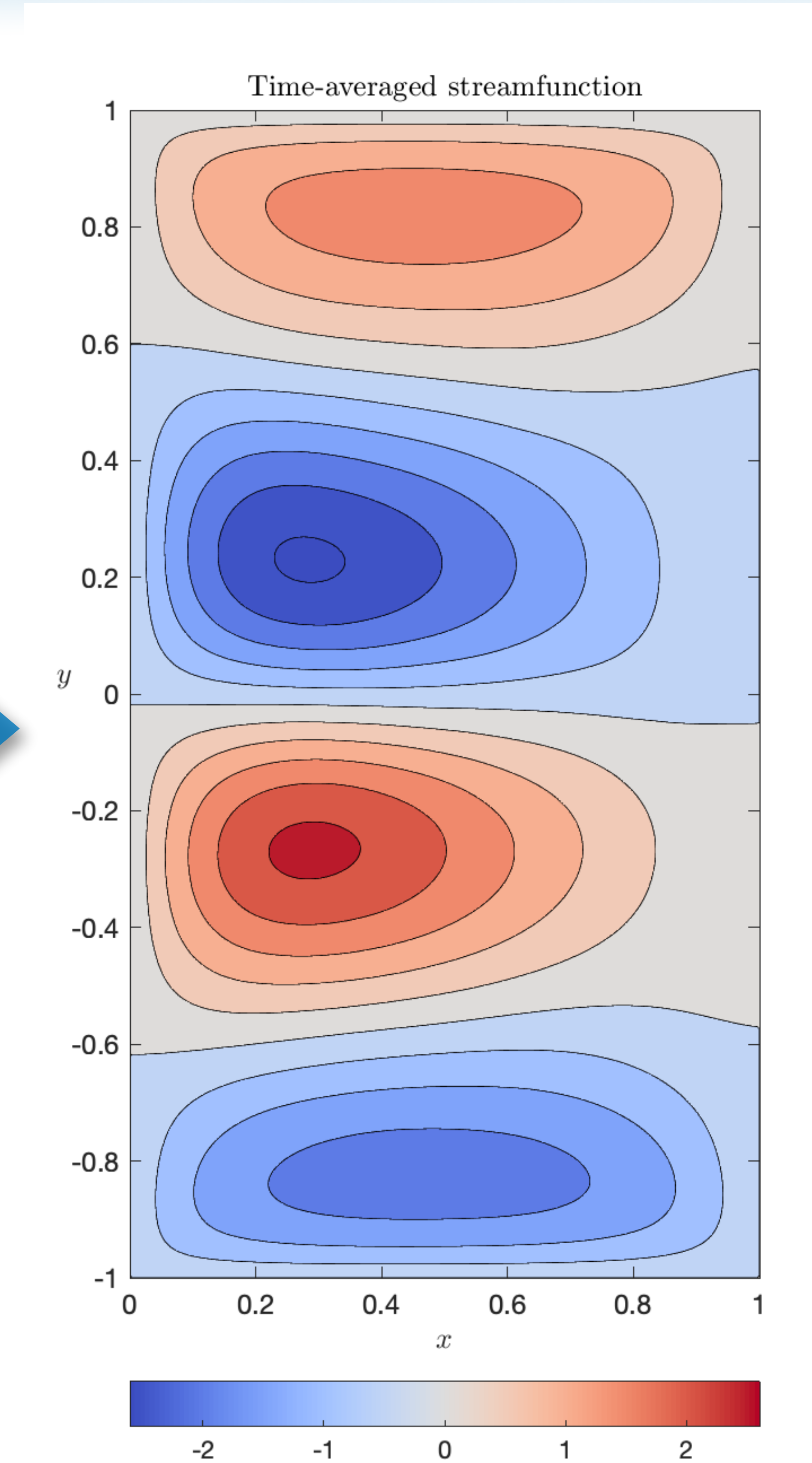
Model	Day 10	Day 20	Day 30	Reliability
LU homogeneous				
PIC homogeneous				
LU heterogeneous				
PIC heterogeneous				



Time–statistics of wind–driven circulation

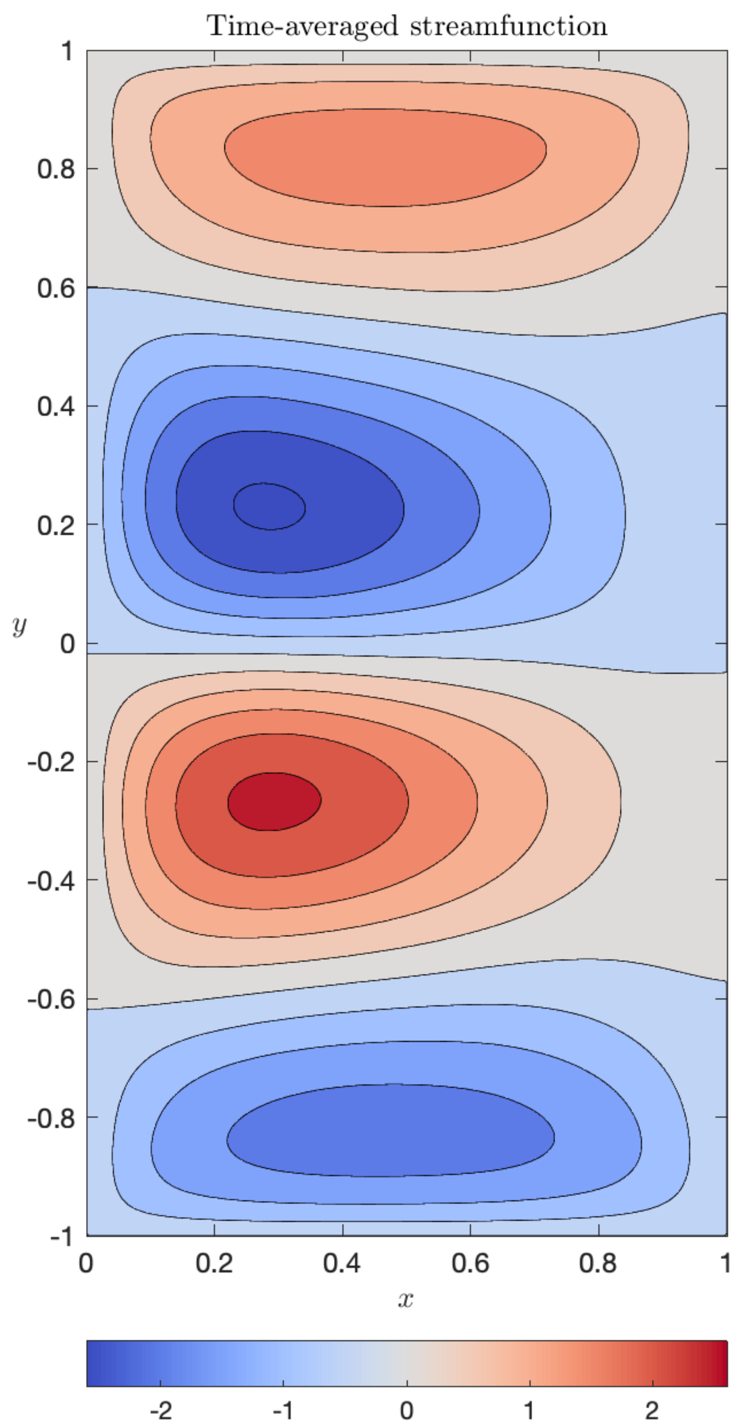




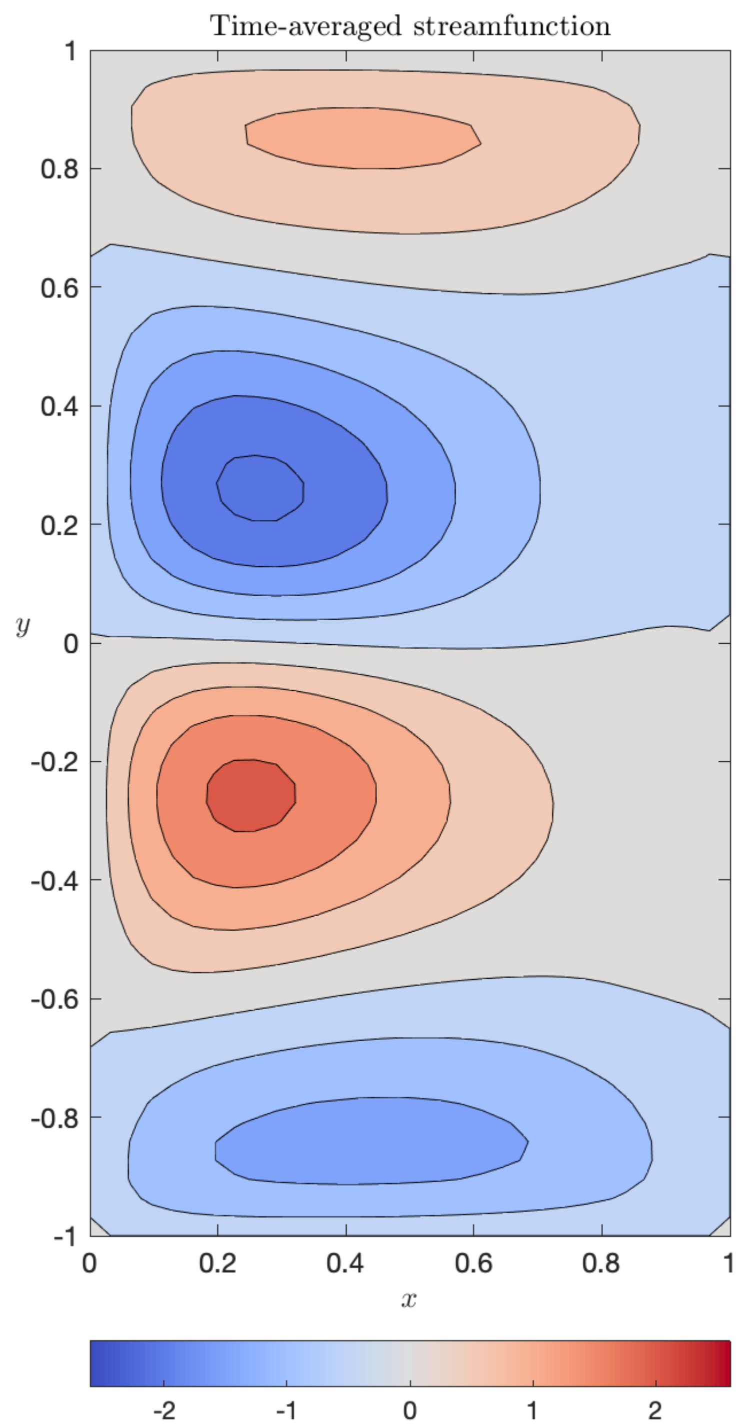


DNS 256×512 with $Re = 450$ and $Ro = 0.0016$

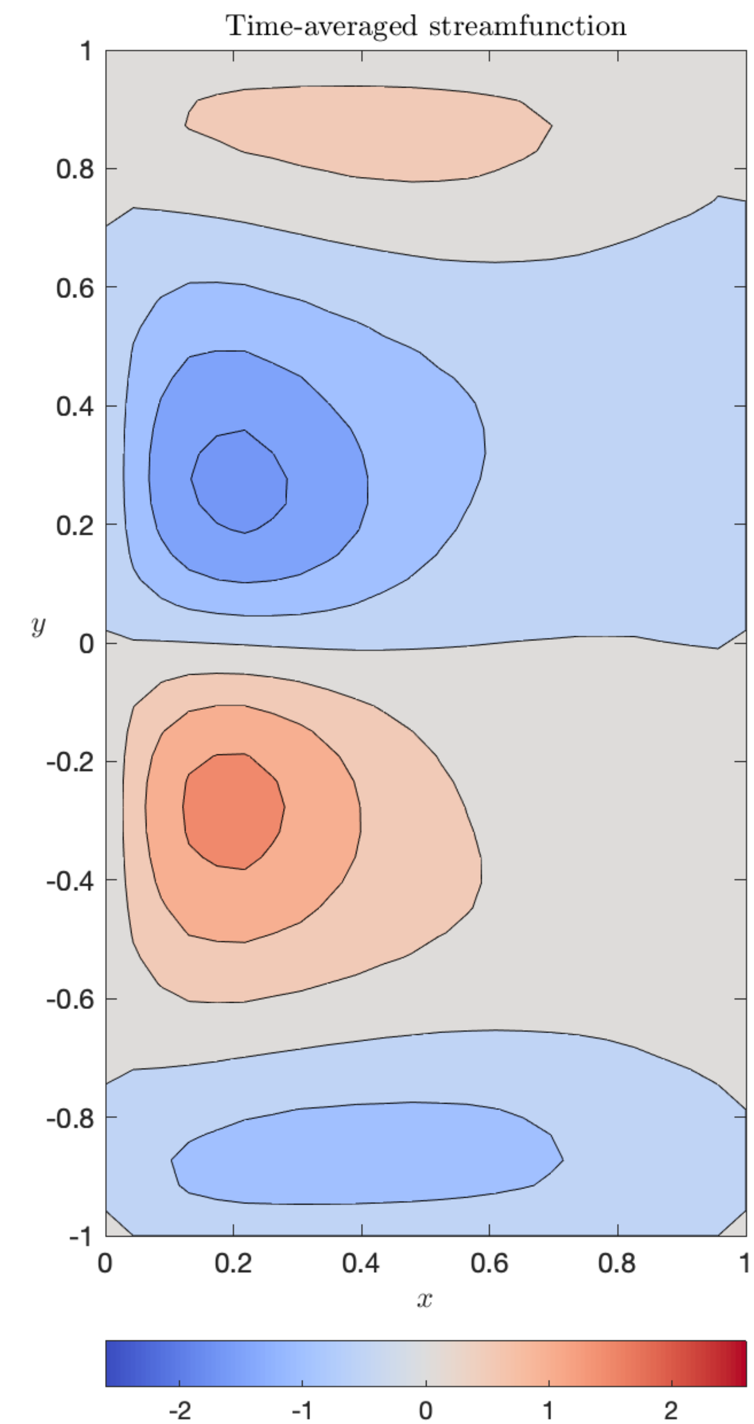
Time–statistics of wind–driven circulation



DNS 256 $\times$ 512



LU 32 $\times$ 64



LES_{LU} 32 $\times$ 64

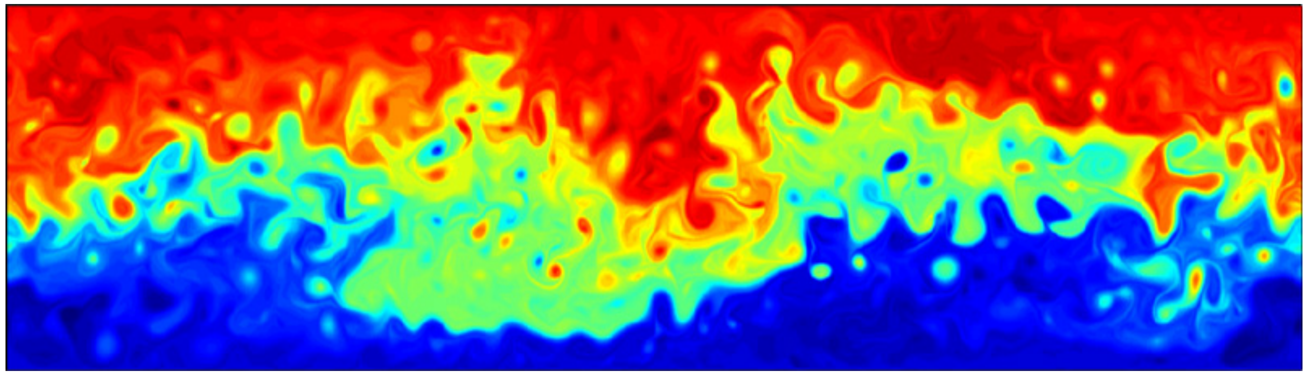
Truth

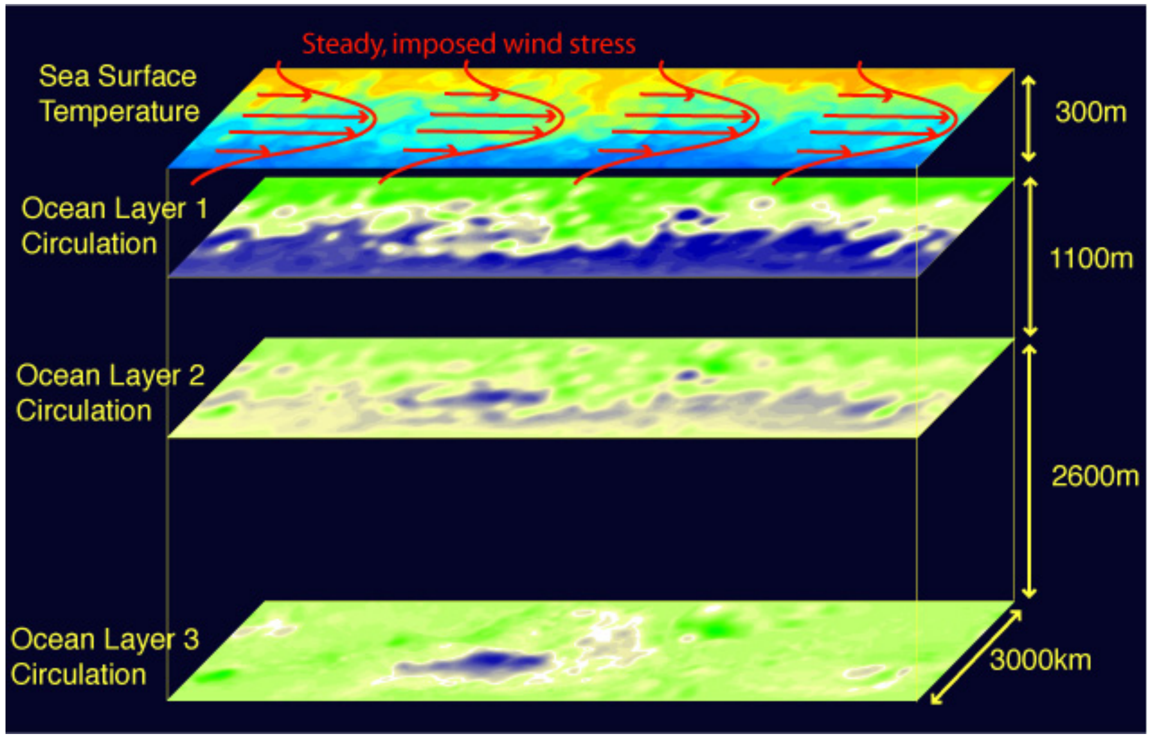
Outline

- **A consistent stochastic model :**
  - → **Physical conservation laws satisfied**
  - → **Better ensemble spread represented**
  - → **Time-averaged profil well described on coarse mesh**

- **Future works :**
  - → **Multi-layers QG model under LU**

    (Q-GCM Projects : Hogg et al. 2003)



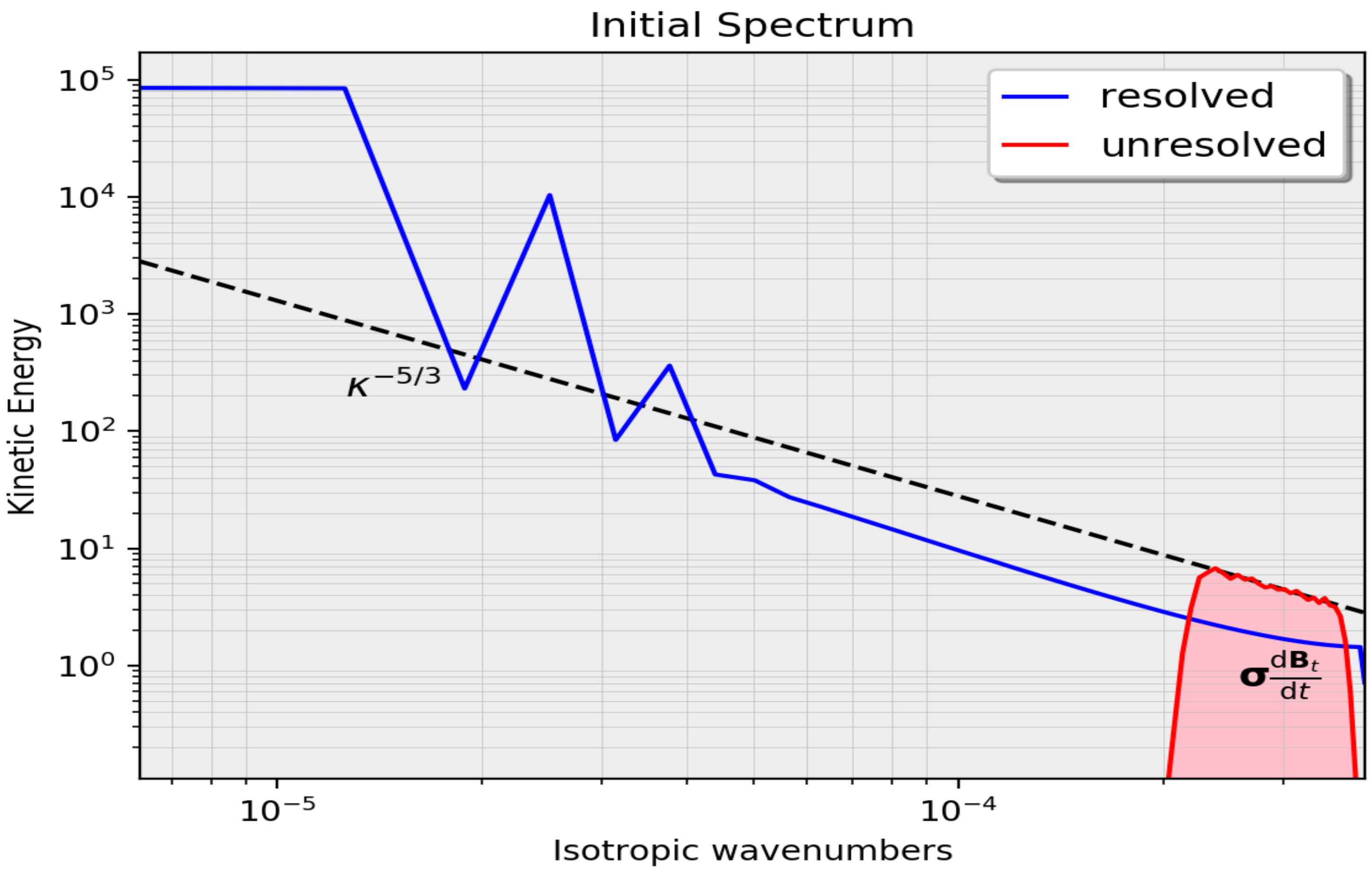


  - → **Data Assimilation with Particle Filters**

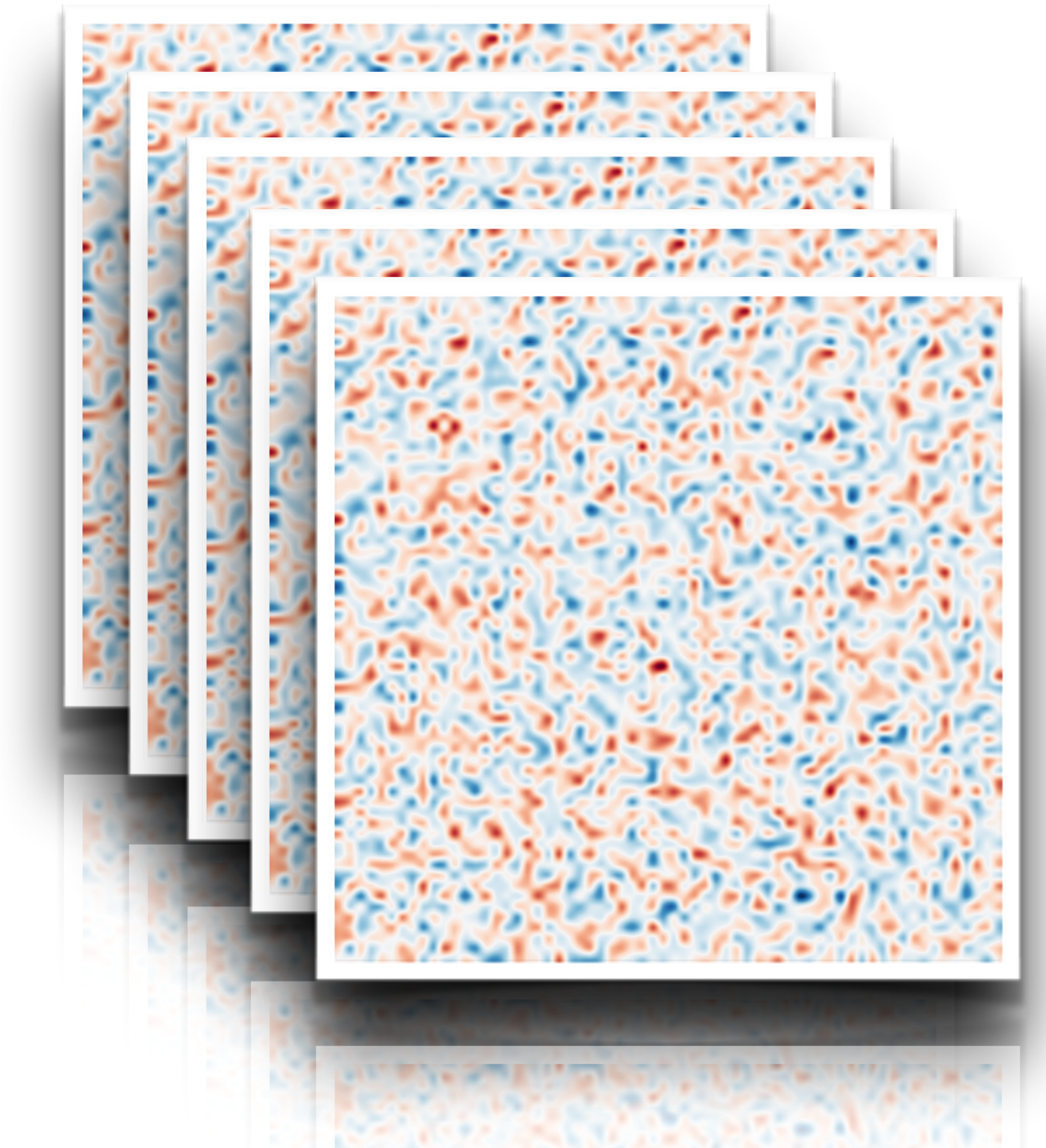
    (Cotter et al. 2018)

Thanks for Your Attention !

Homogeneous parameterization



Ensemble of random stream functions



low-pass band filter $\star$ white noise

Heterogeneous parameterization