



Oceanic Dynamics under Location Uncertainty: Towards a consistent stochastic modeling

Long Li, Werner Bauer, Etienne Mémin

► To cite this version:

Long Li, Werner Bauer, Etienne Mémin. Oceanic Dynamics under Location Uncertainty: Towards a consistent stochastic modeling. Workshop Conservation Principles, Data and Uncertainty in Atmosphere-Ocean Modelling, Apr 2019, Potsdam, Germany. hal-02708028

HAL Id: hal-02708028

<https://inria.hal.science/hal-02708028>

Submitted on 1 Jun 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Oceanic Dynamics under Location Uncertainty

Towards a consistent stochastic modelling

Long Li Werner Bauer Etienne Mémin

Fluminance Group
Inria Rennes France & Université Rennes 1 France

- **Why stochastic modeling ?**

- ➔ **Take into account small-scale / unresolved processes**

- ➔ **Uncertainty Quantification, Ensemble Forecasts**

- **Why consistent model ?**

- ➔ **Both scales must respect appreciate physical laws**

- ➔ **Losing consistency may provide wrong statistics**

1 Governing Equations

- Location Uncertainty Principles
- Barotropic Quasi—Geostrophic Model under LU

2 Experimental Evaluations

- Structure—preserving of Rossby wave
- Ensemble forecasting verification of SQG_{MU}
- Time—statistics of wind—driven circulation

3 Summary

- Stochastic flow :

$$d\mathbf{X}_t = \underbrace{\mathbf{u}(\mathbf{X}_t, t)dt}_{\text{large-scale resolved}} + \underbrace{\boldsymbol{\sigma}(\mathbf{X}_t, t)d\mathbf{B}_t}_{\text{small-scale unresolved}}$$

- Functional process :

$$\underbrace{\boldsymbol{\sigma}(\mathbf{x}, t)d\mathbf{B}_t}_{\text{correlated in space, uncorrelated in time}} = \int_D \underbrace{\check{\boldsymbol{\sigma}}(\mathbf{x}, \mathbf{y}, t)}_{\text{symmetric kernel}} \underbrace{d\mathbf{B}_t(\mathbf{y})}_{\text{space-time white noise}} d\mathbf{y}$$

- Variance tensor :

$$\underbrace{\mathbf{a} \triangleq \boldsymbol{\sigma}\boldsymbol{\sigma}^T}_{\text{homogeneous / heterogeneous}} = \mathbb{E}[\boldsymbol{\sigma}d\mathbf{B}_t(\boldsymbol{\sigma}d\mathbf{B}_t)^T] \big/ dt$$

- Transport of a random tracer :

$$(\nabla \cdot \boldsymbol{\sigma} = 0)$$

Balanced Energy

→

$\frac{d}{dt} \int_D \frac{1}{2} \theta^2 = 0$

$$D_t \theta \triangleq d_t \theta + \underbrace{\mathbf{u}^* \cdot \nabla \theta dt}_{\text{corrected drift}} + \underbrace{\boldsymbol{\sigma} d\mathbf{B}_t \cdot \nabla \theta}_{\text{multiplicative noise}} - \underbrace{\frac{1}{2} \nabla \cdot (\mathbf{a} \nabla \theta) dt}_{\text{subgrid diffusion}} = 0$$

$\mathbf{u} - \frac{1}{2} \nabla \cdot \mathbf{a}$

- **Evolution of the potential vorticity with source processes :**

$$D_t q = S_1(\nabla \mathbf{u}) dt + S_2(\nabla \mathbf{u}) d\mathbf{B}_t$$

- **Evolution of the stream function :**

$$q = \Delta \psi - \psi / L_R^2 + f$$

- **Strong incompressible constraints :**

$$\mathbf{u} = \nabla^\perp \psi, \quad \sigma d\mathbf{B}_t = \nabla^\perp \varphi d\mathbf{B}_t, \quad \nabla \cdot \mathbf{u}^* = 0$$

- ➔ **Conservation of total energy :**

$$\frac{d}{dt} \int_D \frac{1}{2} \left[\|\nabla \psi\|^2 + (\psi / L_R^2)^2 \right] = 0$$

$$\left[\text{Sketch : } \int_D \psi \sigma \circ d\mathbf{B}_t \cdot \nabla (q - f) = \int_D \psi S_2(\nabla \mathbf{u}) \circ d\mathbf{B}_t \right]$$

- **Transport of a prognostic passive tracer :**

$$D_t q' = 0$$

- ➔ **Conservation of tracer energy :**

$$\frac{d}{dt} \int_D \frac{1}{2} (q')^2 = 0$$

1 Governing Equations

- Location Uncertainty Principles
- Barotropic Quasi—Geostrophic Model under LU

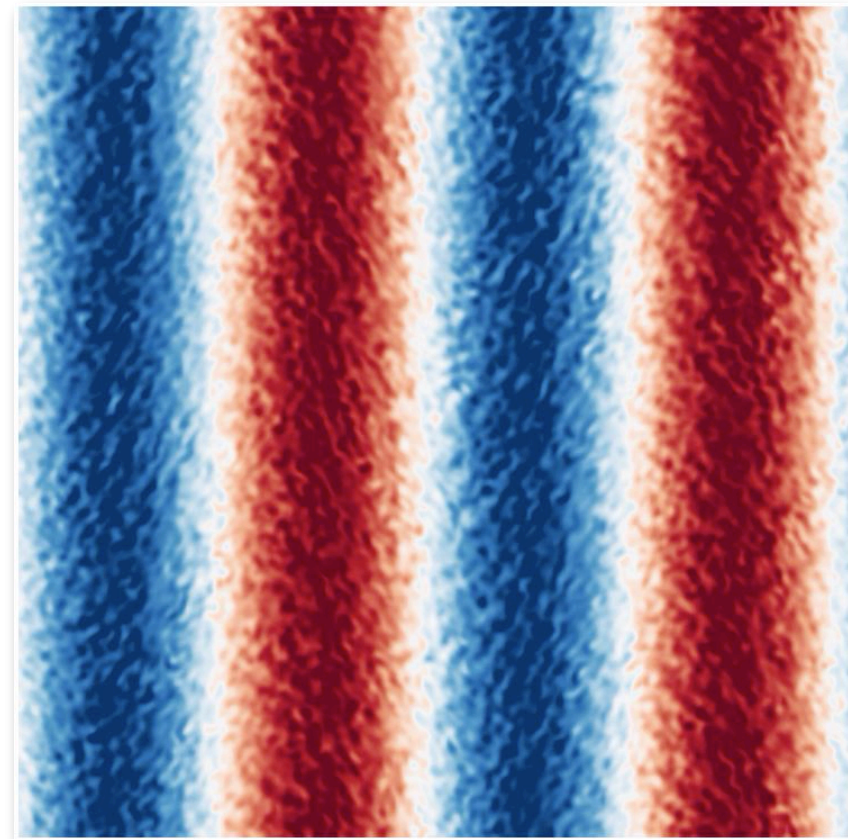
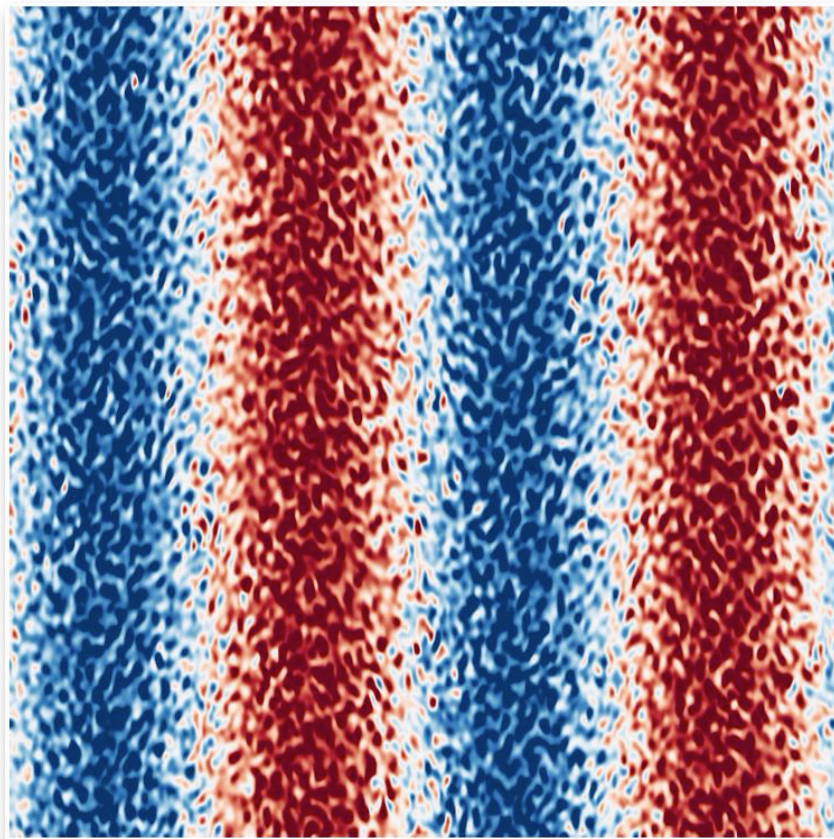
2 Experimental Evaluations

- Structure—preserving of Rossby wave
- Ensemble forecasting verification of SQG_{MU}
- Time—statistics of wind—driven circulation

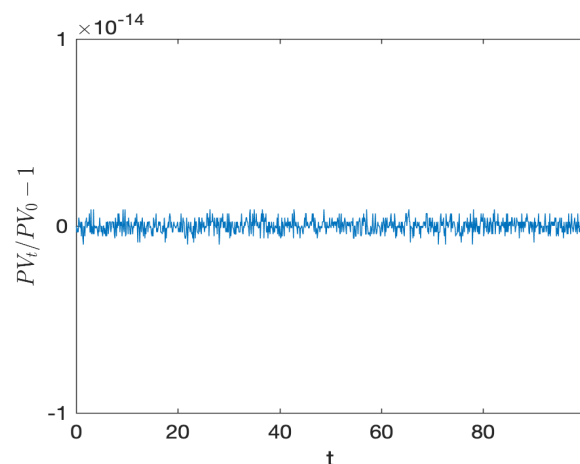
3 Summary

Structure-preserving of Rossby wave

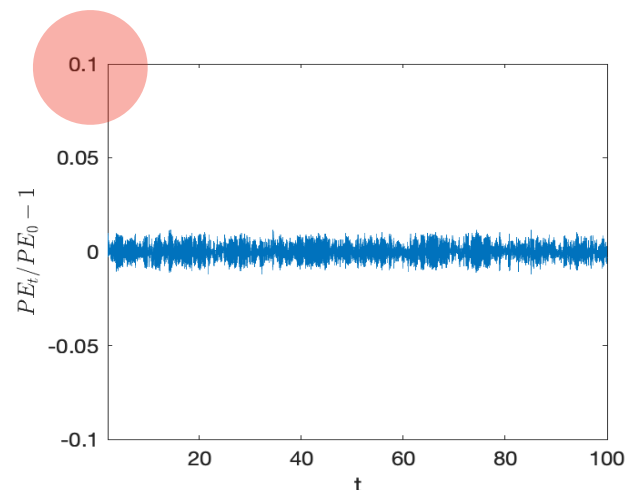
- Flowchart :



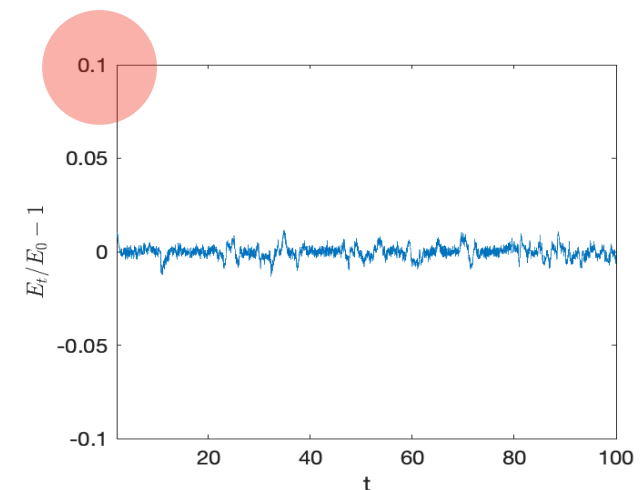
- Illustration of conservations :



Passive Tracer



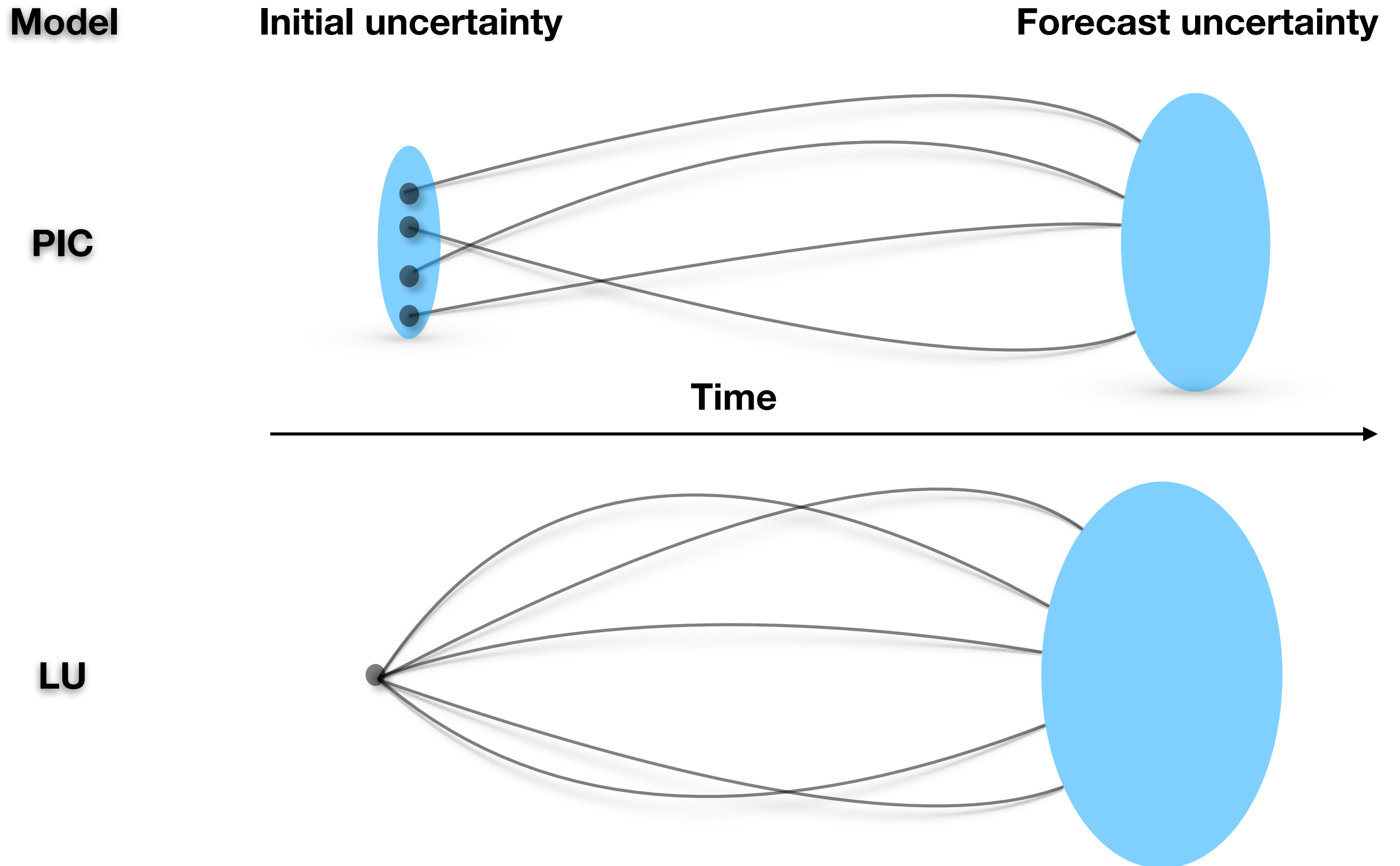
Tracer Energy



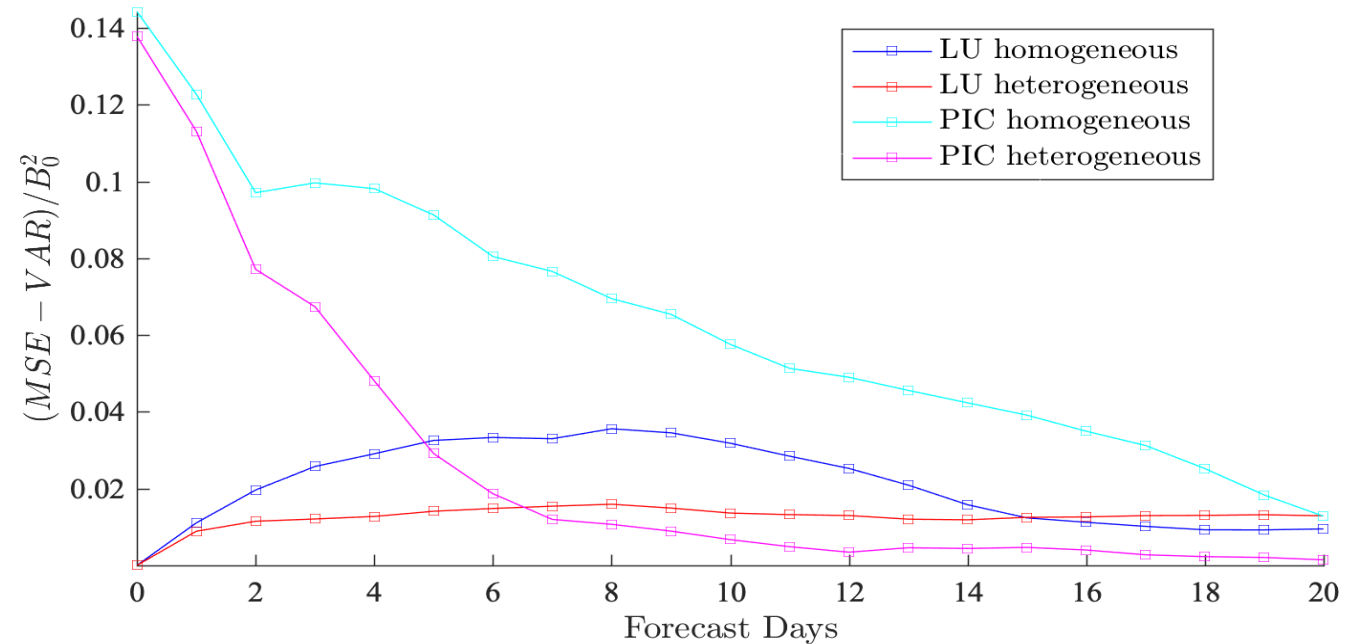
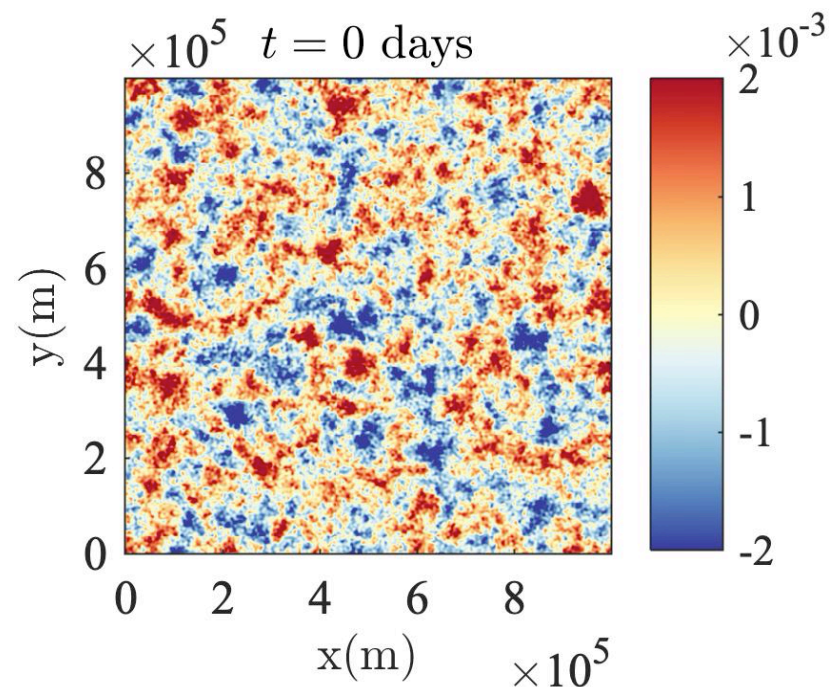
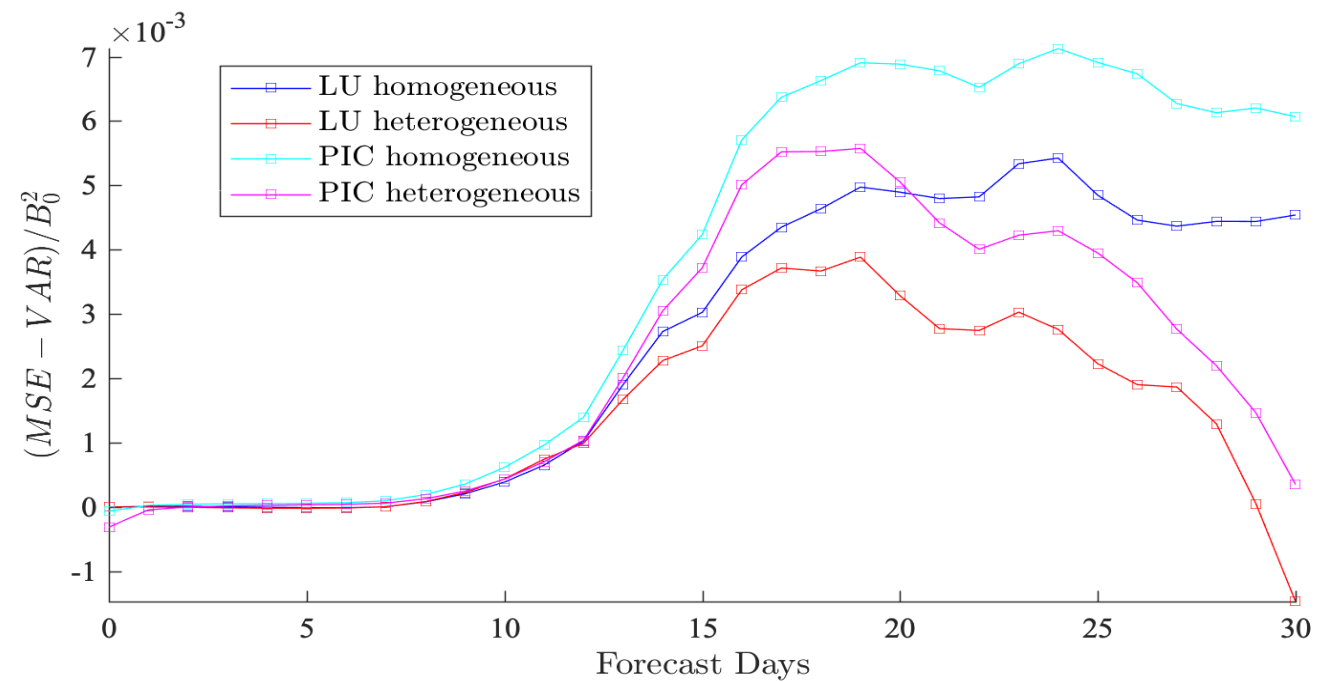
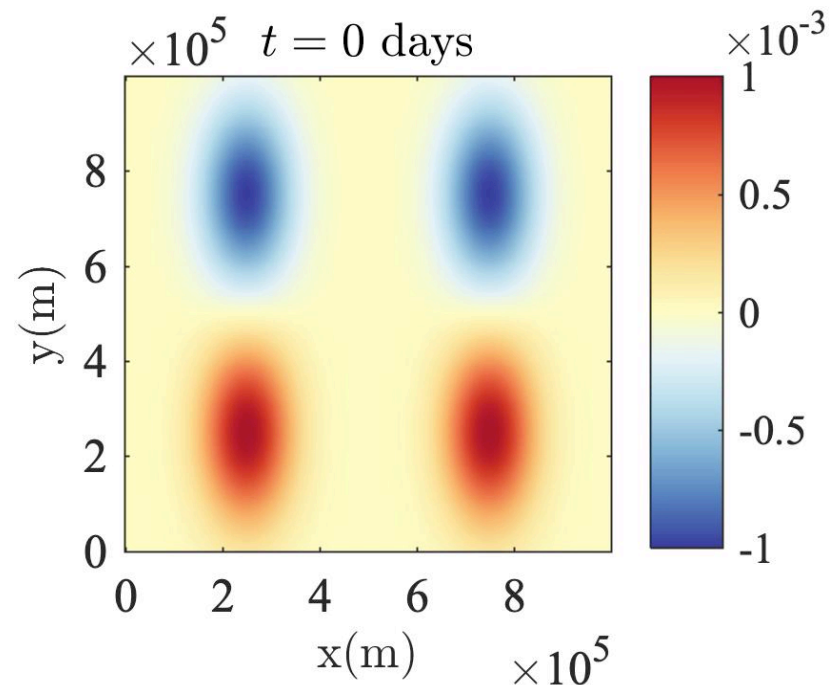
Total Energy

Ensemble forecasting verification of SQG_{MU}

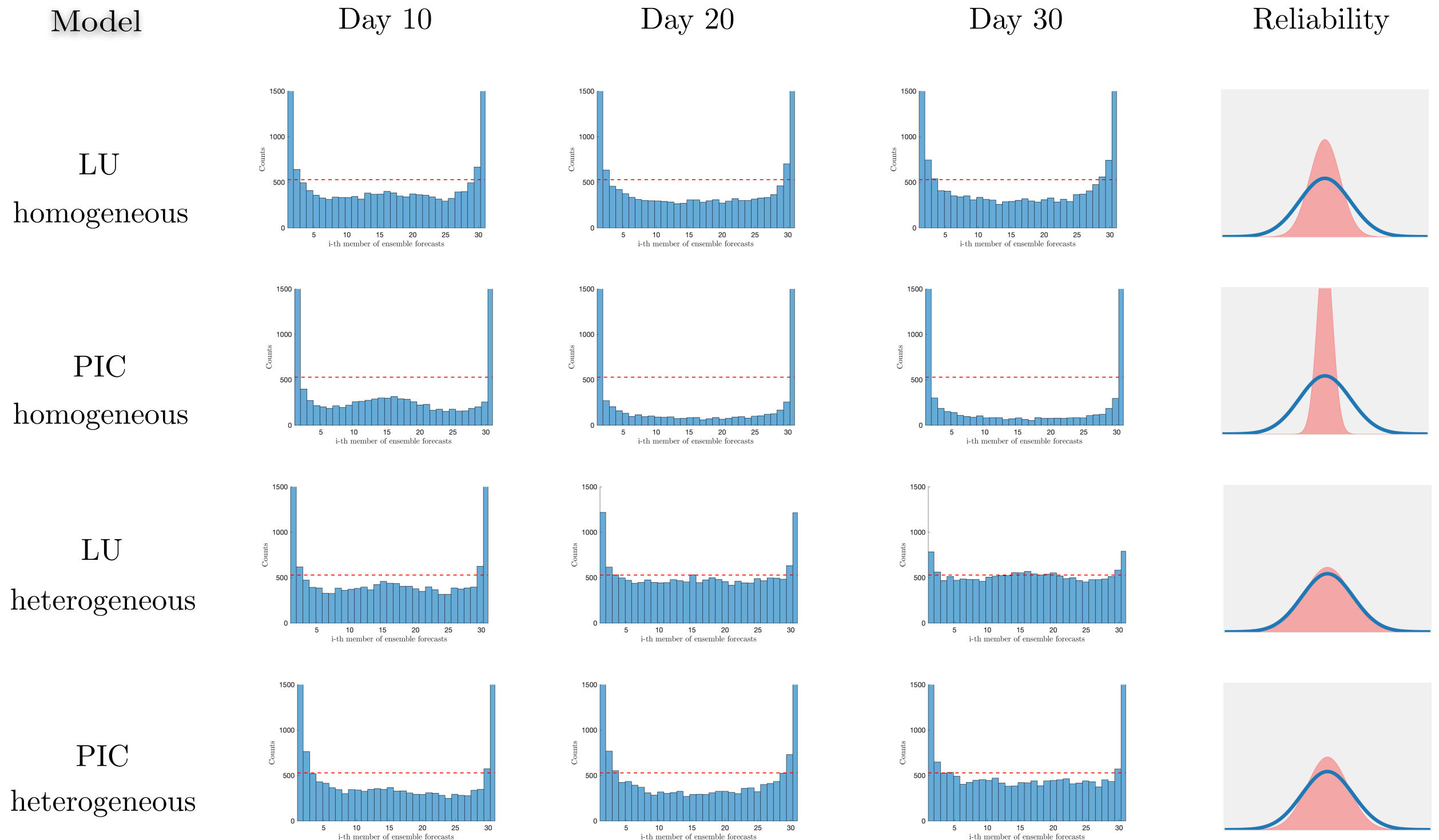
- Illustration of ensemble prediction systems :



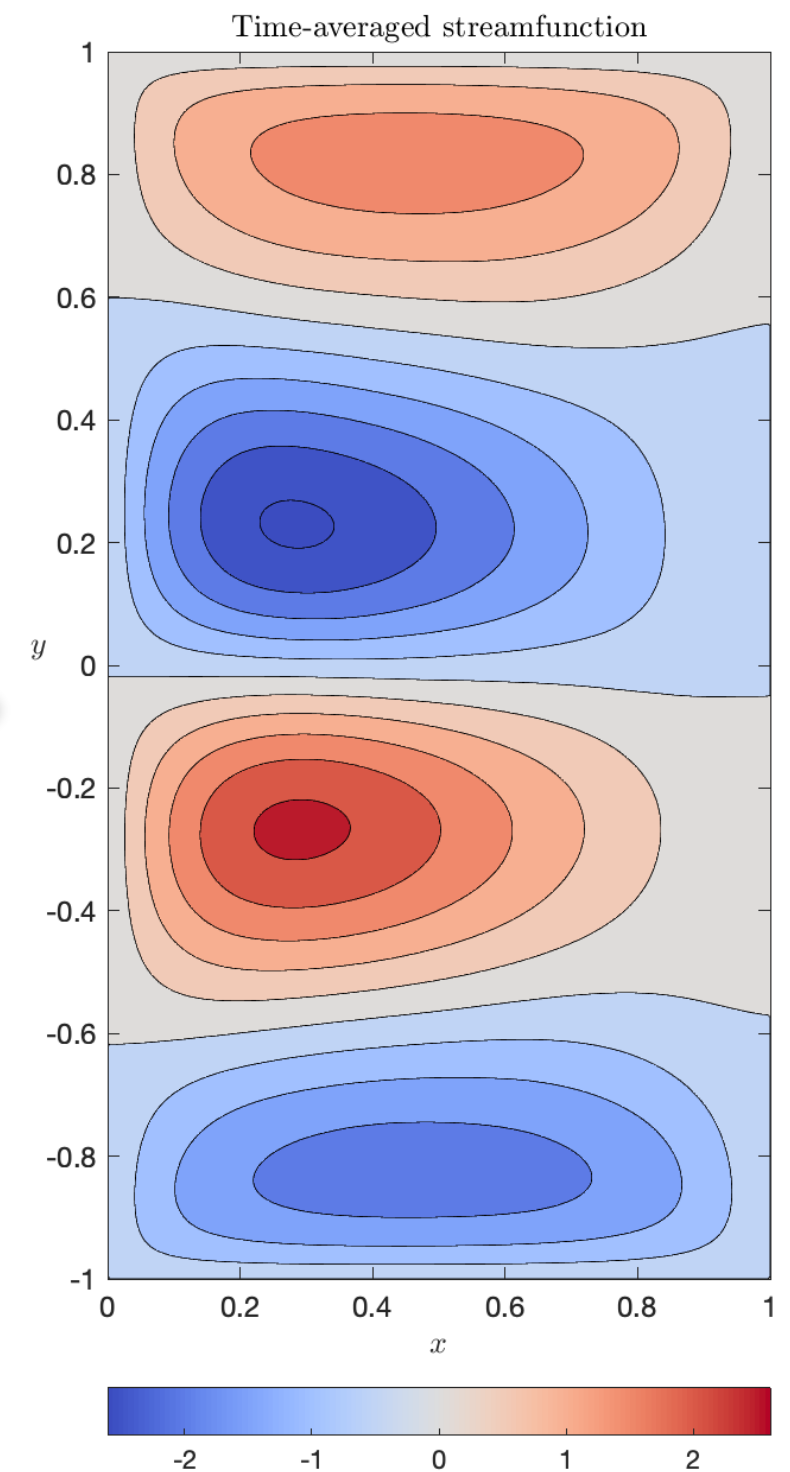
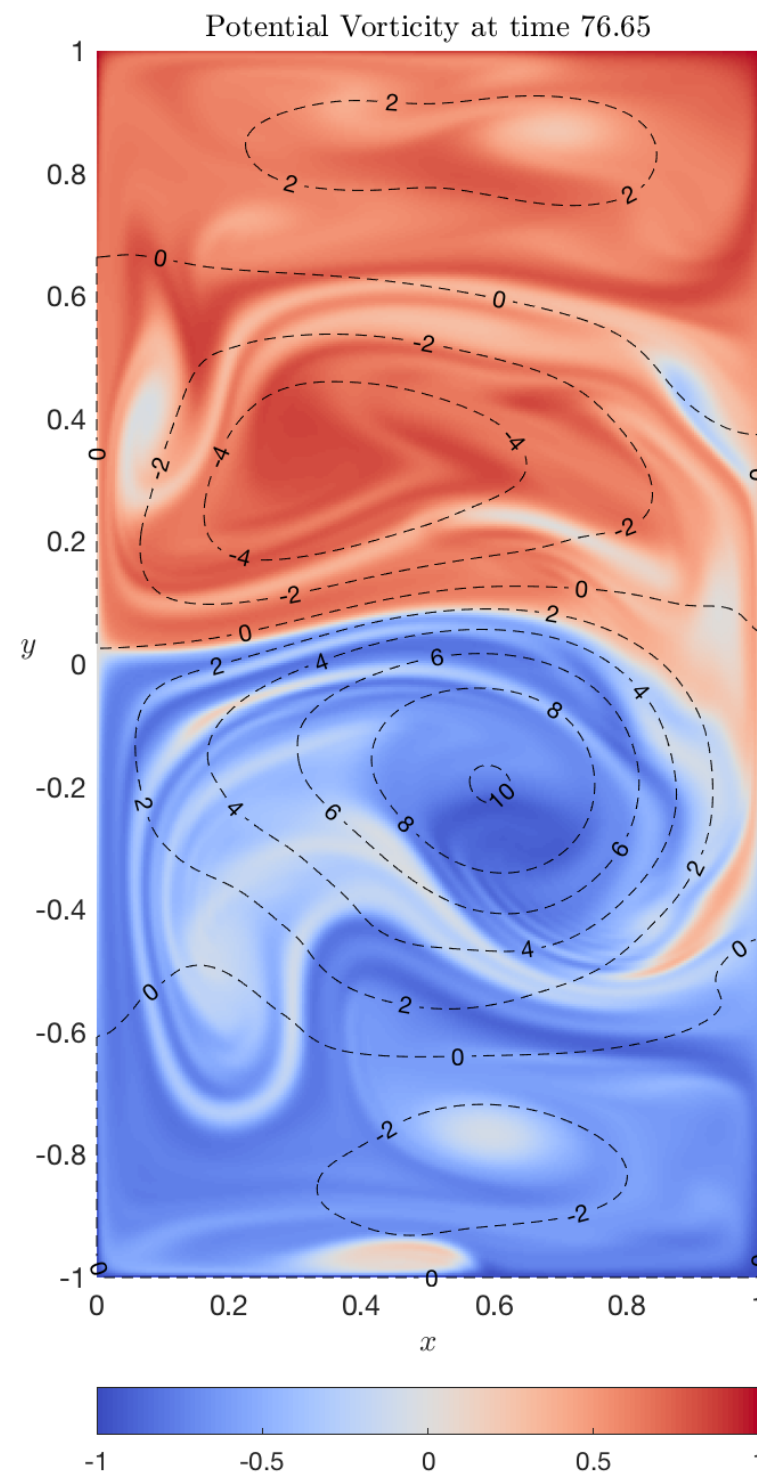
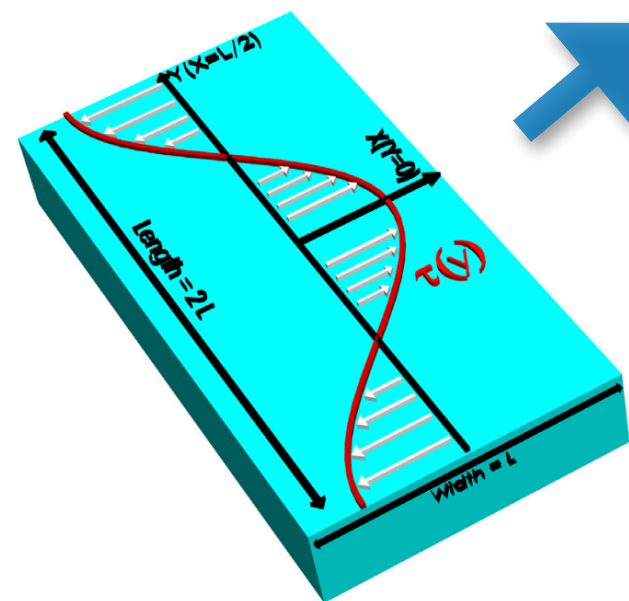
Reliability — Ensemble Spread match MSE of the Ensemble Mean :



● Reliability — Ranked Histogram of Ensemble Members :

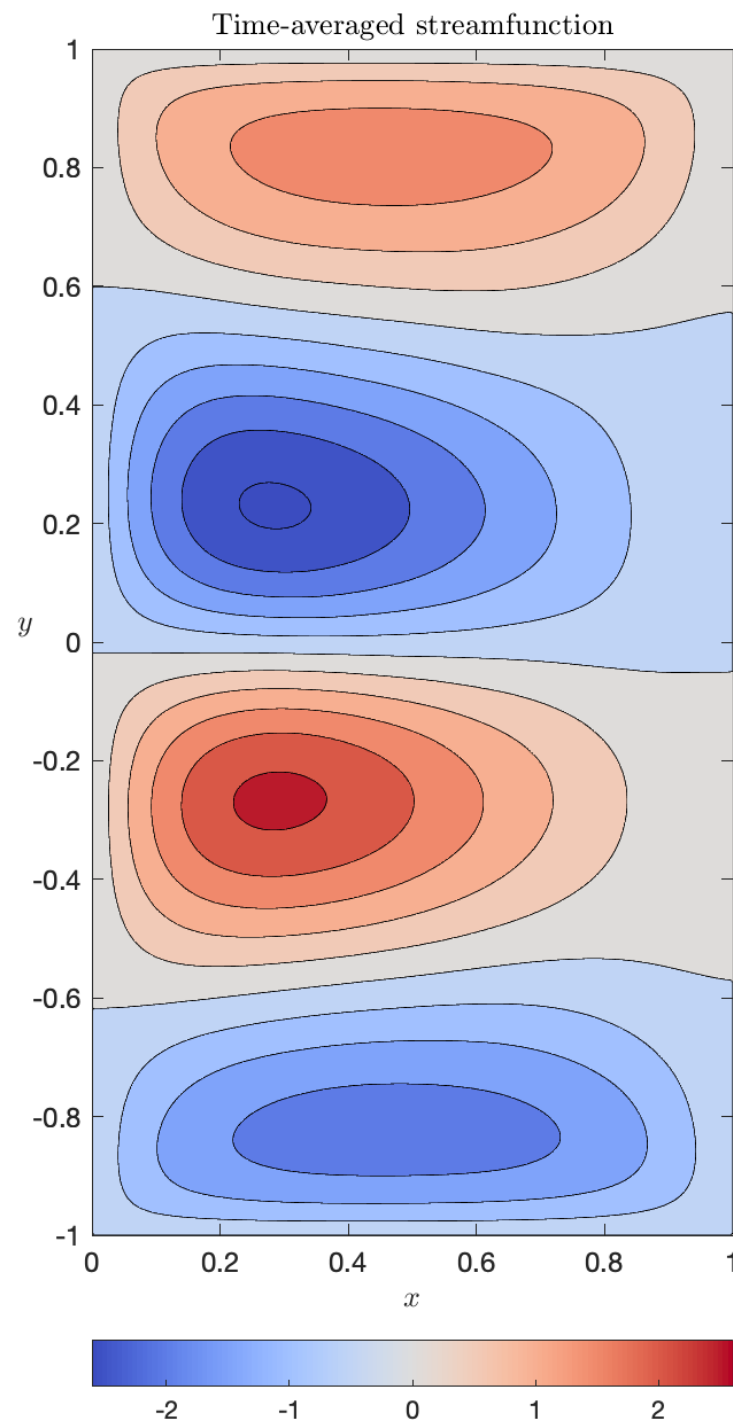


Time-statistics of wind-driven circulation

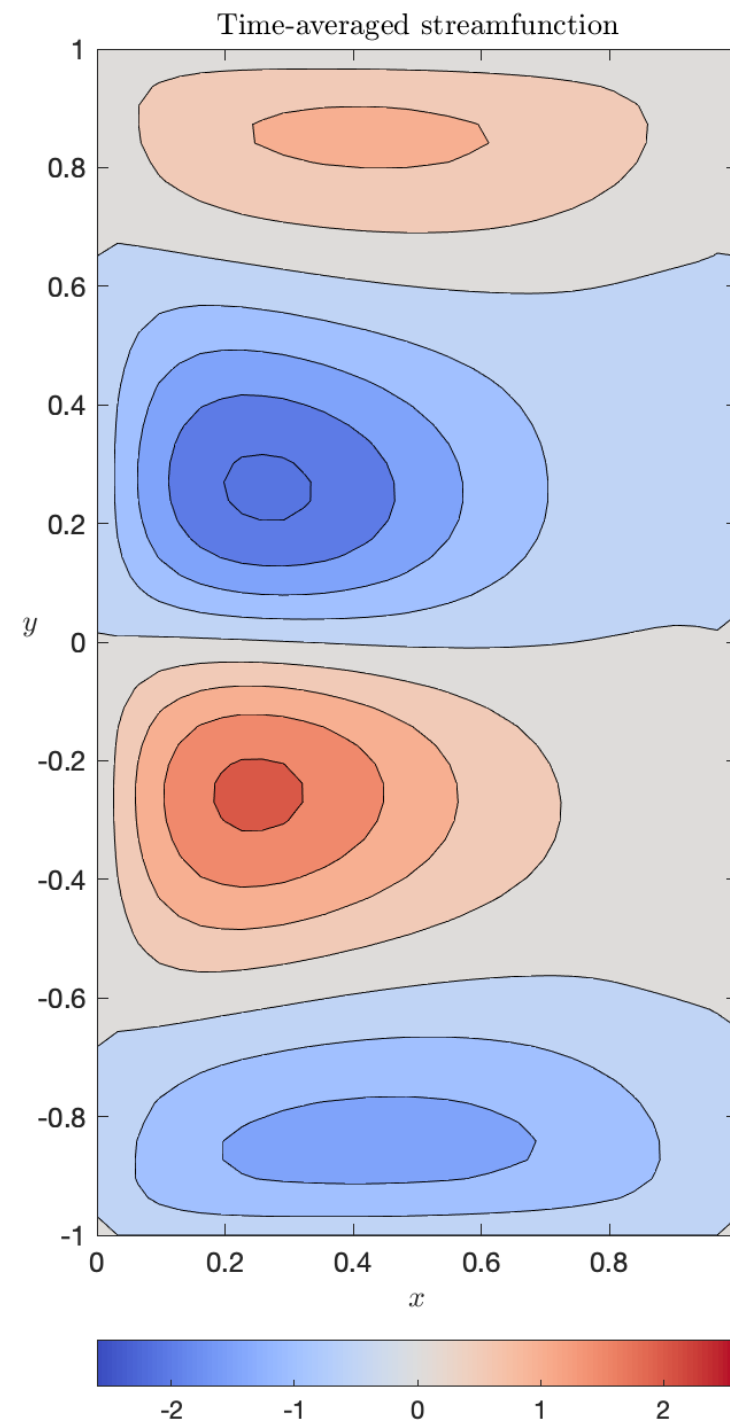


DNS 256×512 with $Re = 450$ and $Ro = 0.0016$

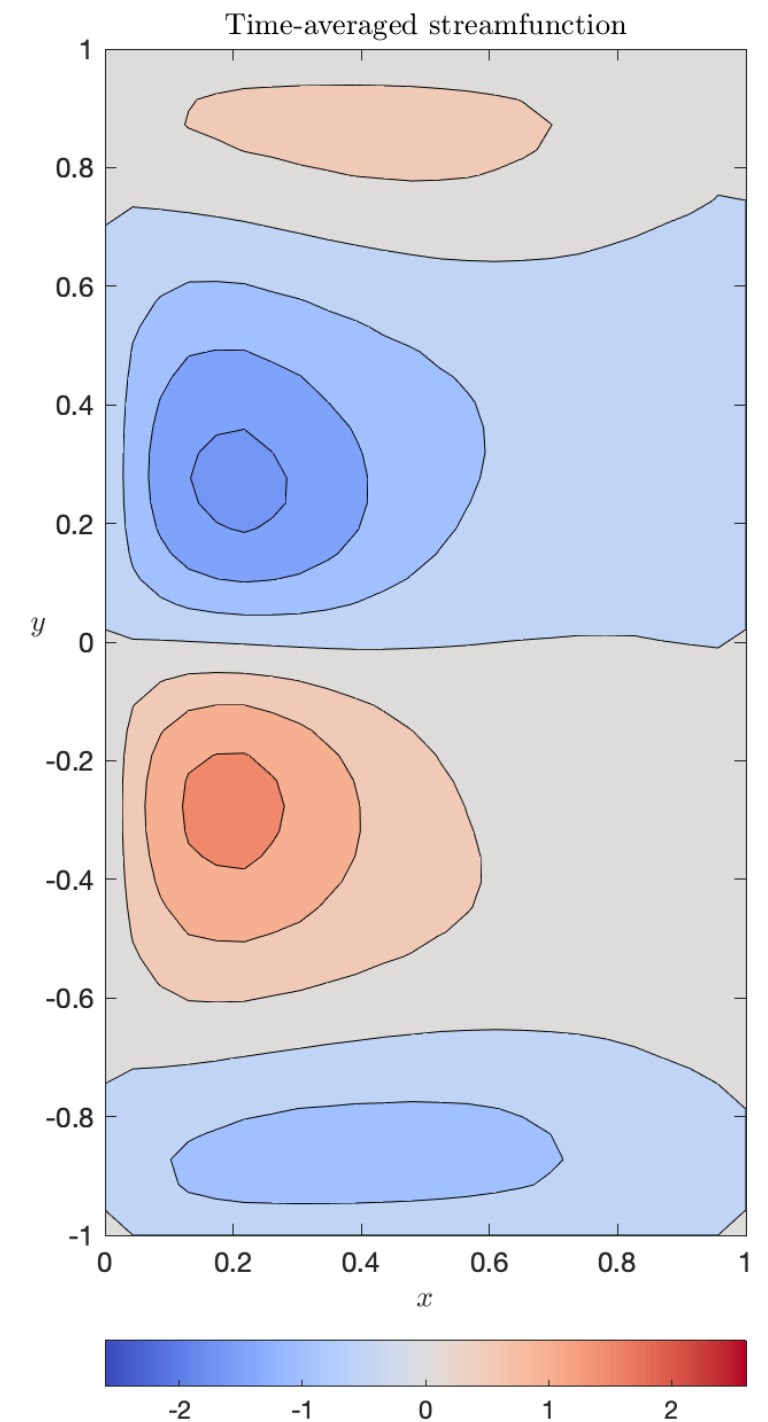
Time-statistics of wind-driven circulation



DNS 256×512



LU 32×64



LES_{LU} 32×64

Truth



1 Governing Equations

- Location Uncertainty Principles
- Barotropic Quasi—Geostrophic Model under LU

2 Experimental Evaluations

- Structure—preserving of Rossby wave
- Ensemble forecasting verification of SQG_{MU}
- Time—statistics of wind—driven circulation

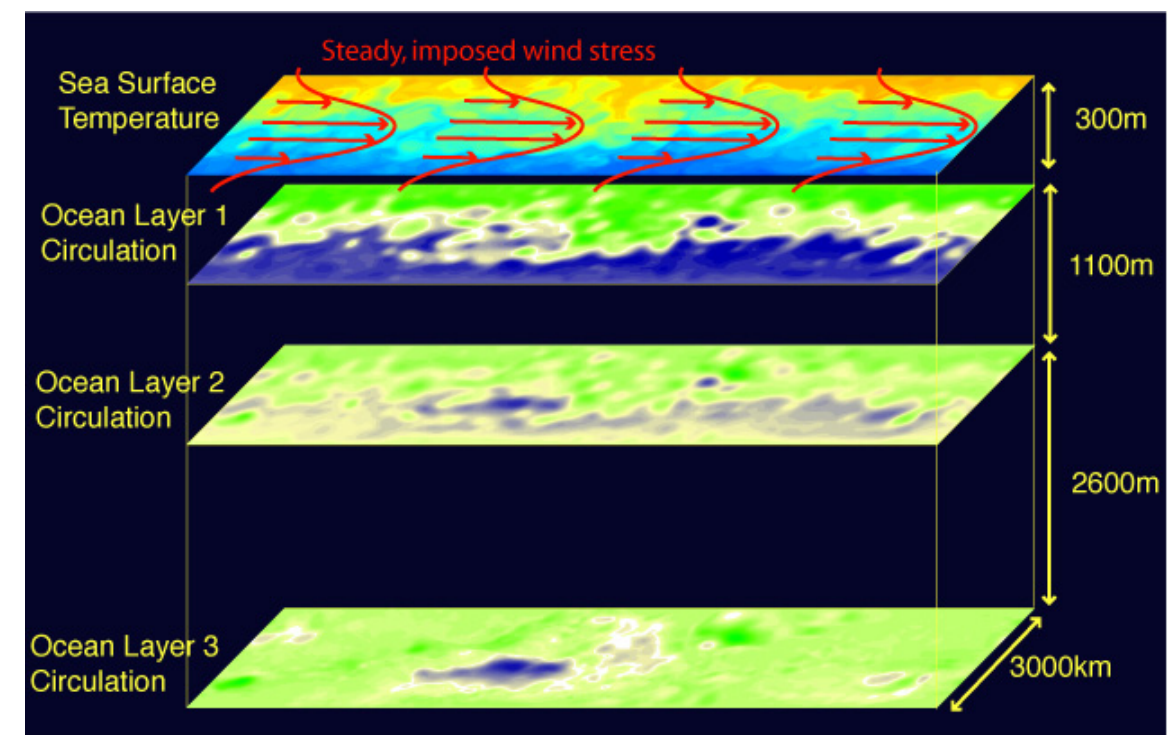
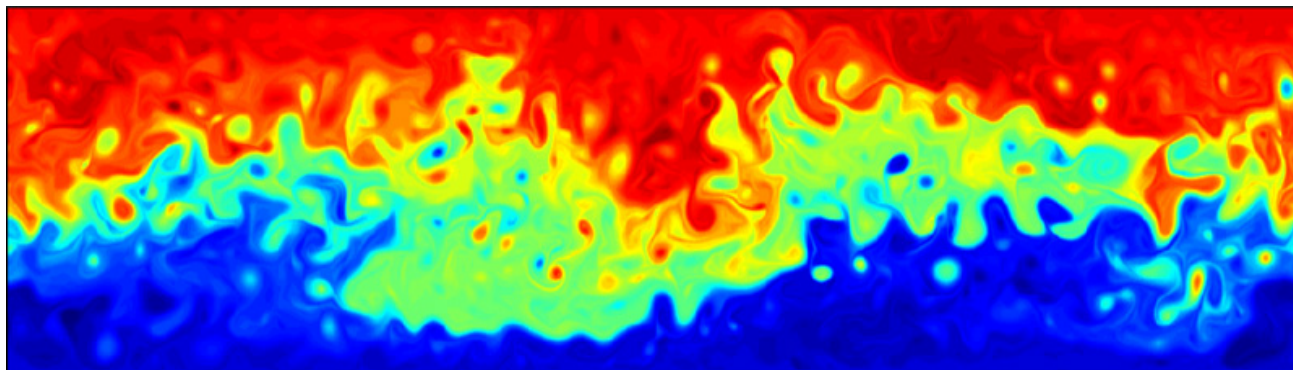
3 Summary

- **A consistent stochastic model :**

- ➔ **Physical conservation laws satisfied**
- ➔ **Better ensemble spread represented**
- ➔ **Time-averaged profil well described on coarse mesh**

- **Future works :**

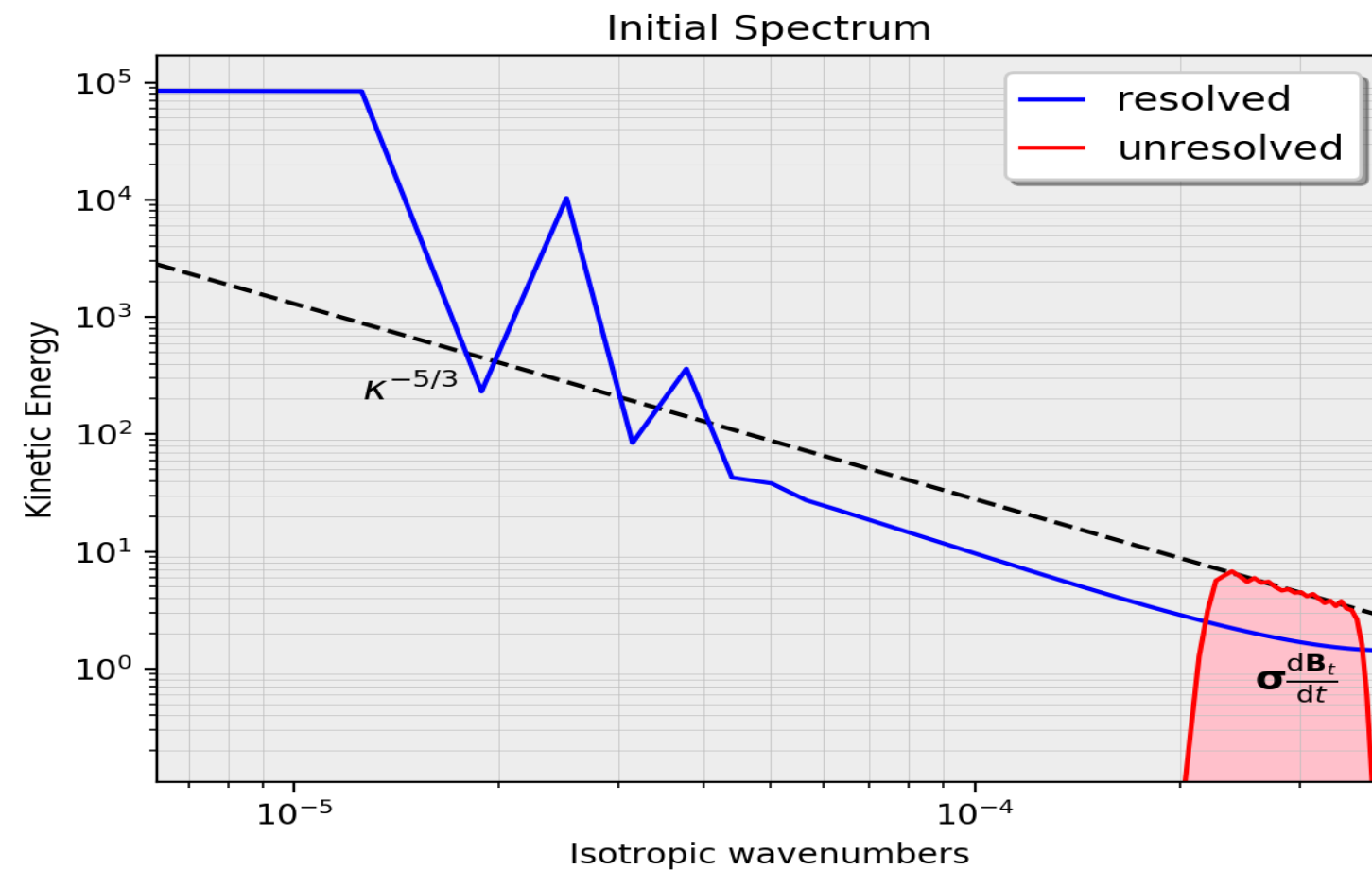
- ➔ **Multi-layers QG model under LU**
(Q-GCM Projects : Hogg et al. 2003)



- ➔ **Data Assimilation with Particle Filters**
(Cotter et al. 2018)

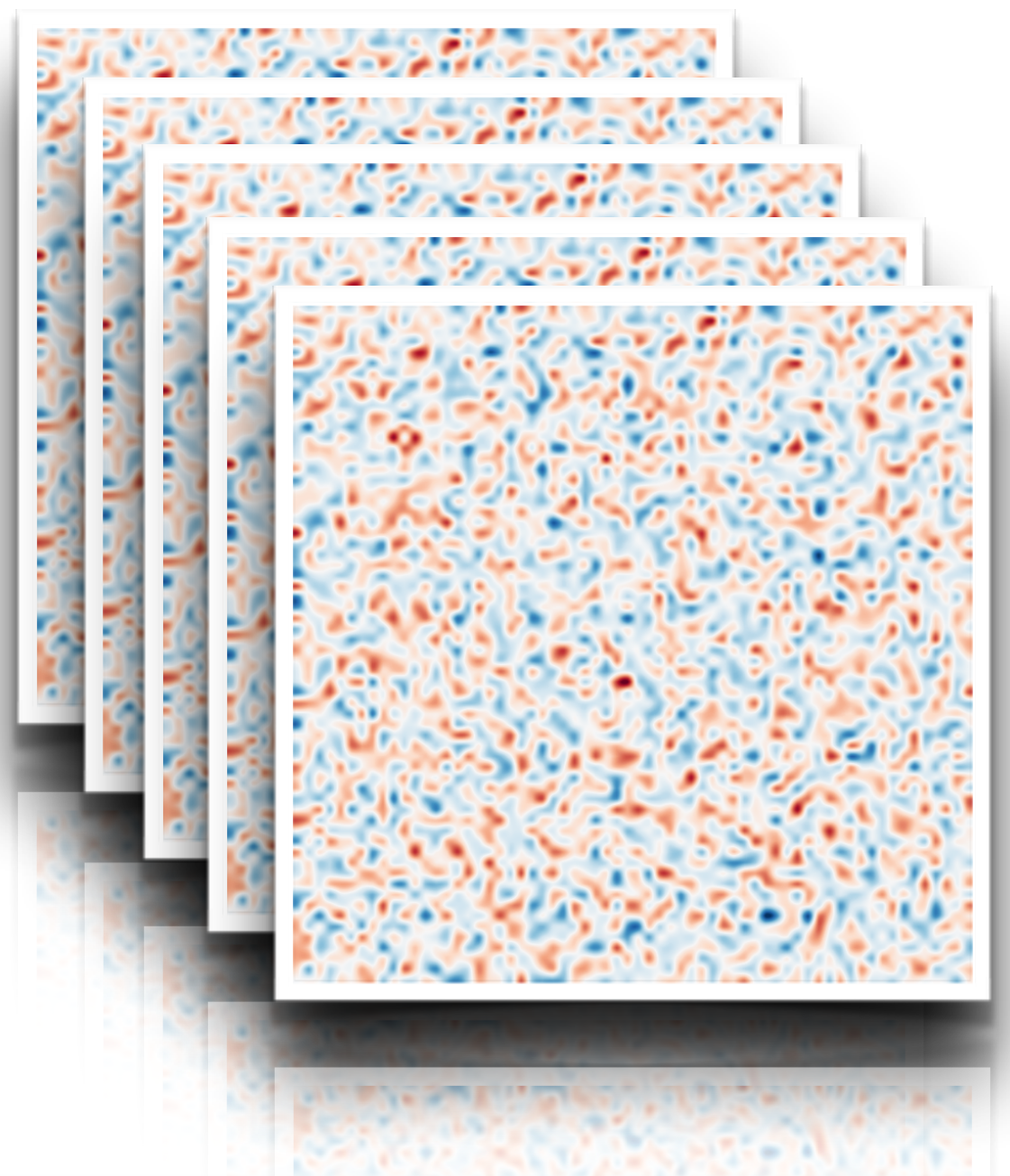
Thanks for Your Attention !

Homogeneous parameterization



low-pass band filter $\star$ white noise

Ensemble of random stream functions



Heterogeneous parameterization

