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***Abstract*—This paper presents a new formulation to build an interference topology for the multi-user unicast Topological Interference Management (TIM) based wireless network problem. Based on our interference topology formulation, we are able to evaluate the achievable rate's theoretical limit, in the asymptotic signal to noise ratio (SNR) regime, for the underlying wireless network and not just for its topological interference representation. This new formulation allows us to cope with the finite SNR regime and not just with the asymptotic SNR regime with the Degrees of Freedom (DoF) analysis. A new SNR independent interference threshold parameter is proposed and we evaluate the achievable symmetric rates of the wireless network in both the finite SNR regime and the asymptotic SNR regime. Finally, we present outer bound solutions on the new normalized interference threshold parameter for interference topologies with half-DoF-feasibility, considering both an orthogonal resource allocation and Interference Alignment (IA). These bounds specify if a given half-DoF-feasible interference topology can be, in terms of the achievable rate, the best topology or not. Using this result, we limit the search space in the normalized interference threshold parameter range, to find half-DoF-feasible interference topologies having the possibility to be the best topologies in terms of the achievable rate.**

***Index Terms*—TIM, wireless network, interference topology, DoF, interference threshold parameter, achievable symmetric rate.**

I. Introduction

Fast and seamless access to enormous amounts of data has become essential to the development of our society, and wireless communication has a significant role in the access of distributed information. Indeed, the Internet of Things (IoT) will require the coordination of a massive amount of communication devices in an ever more crowded spectrum, imposing very large scale networks to thrive under the limitations of interference. Dealing with interference has become all the more important nowadays in order to manage such a crowded wireless space.

Dealing with interference usually requires some sort of coordination between transmitting nodes to minimize its impact on the overall system performance. Such coordination usually comes in the form of resource management, finding orthogonal resources that can be assigned to different nodes, thus avoiding interference. Other traditional interference management techniques such as Interference Alignment (IA) [1] and interference cancellation [2] have been investigated before based on the assumption of having a perfect knowledge of critical information at the transmitters, i.e., Channel State Information at the Transmitter (CSIT). This critical information is usually not available at the transmitters, rendering the assumption of perfect knowledge for the CSIT unrealistic. In this sense, more realistic interference management schemes have been explored based on partial CSIT [3], [4].

In [5], Jafar pioneered a very interesting technique called Topological Interference Management (TIM) that has the benefit of reducing the CSIT requirement to a simple interference topology information. Such information represents only a distinction between weak and significant interference channels [5]. Consequently, an interference topology only considers the direct links as well as the significant interference links, disregarding the non-significant (i.e., weak) interference links, which greatly simplifies the analysis of such networks. TIM has shown to be promising in the study of the Degrees of Freedom (DoF) of many-node systems taking into consideration the interference through the knowledge of the topology.

An equivalence relation between the TIM problem and the index coding problem [6], [7], is shown in [5]. Optimal solutions to both TIM and index coding problem are shown to be achievable through IA. Solutions for different classes of topologies for the index coding and TIM problems have been considered in [5], [8]–[12].

The TIM problem has also been extended in various directions, where TIM schemes relying on different system configurations have been investigated in [5], [13]–[22]. For a detailed overview of the TIM for wireless networks, the interested readers are referred to [23] and the references therein.

A common point of these works is that they depend heavily on the interference topology construction model given in [5]. Therein, an interference topology is fixed whatever power levels are used by the transmitters. While fixing the topology allows for easier analysis, as developed in [5], it has an important drawback since in reality the interference topology depends on the transmission power levels of all transmitters and won't provide the same topology for different transmission powers.

In this work, we propose a new interference topology construction model for the TIM problem of a wireless network that addresses this drawback. We introduce a new finite signal to noise ratio (SNR) framework that is able to control the interference topology more accurately and with more flexibility. We restrain to the study of multi-user unicast wireless networks to simplify the normalization process proposed. The extension to groupcast will be studied in future work.

The remainder of this work is divided as follows. Section II presents the considered system model and Section III recalls the classical approach to devising the interference topology as introduced in [5]. Section IV presents the proposed interference topology construction model, and the achievable rates of wireless networks under this new model is given in Section V. In Section VI we present some fundamental limits on the new normalized interference threshold parameter for interference topologies with half DoF feasibility. Finally, a conclusion is given in Section VII.

II. System Model

Consider a wireless communication network consisting of K Transmitters (TXs), labeled, $S_1, S_2, \dots, S_K$, and K Receivers (RXs), labeled, $D_1, D_2, \dots, D_K$. Let $\mathcal{W} = \{W_1, W_2, \dots, W_K\}$ be the set of all messages to be transmitted in the network. We assume that the network supports unicast transmissions, where TX S_k, $k \in \{1, 2, \dots, K\}$, sends a unique message W_k to a unique RX D_k. Each TX S_k uses a power P_k to transmit its unique message W_k, and each RX D_k is subject to interference from every TX $S_{k'}, k' \in \{1, 2, \dots, K\}/\{k\}$. The developments in this work are also valid for a broadcast style channel where each TX S_k aims at transmitting M_k unique messages, $M_k \leq K$, to M_k unique RXs. Indeed, this setting can be considered as the point-to-point one by splitting each TX S_k into M_k independent co-located TXs.

The wireless network includes two kinds of communication links: the desired links, i.e., the links $S_k \to D_k$, $\forall k \in \{1, 2, \dots, K\}$, and the interference links, i.e, the links $S_{k'} \to D_k$, $\forall k, k' \in \{1, 2, \dots, K\}$ and $k' \neq k$.

The channel input-output relationships are defined as

$$y_i(n) = \sum_{j=1}^{K} h_{ij}(n) x_j(n) + z_i(n), \tag{1}$$

where, over the nth channel use, $x_j(n)$ is the transmitted symbol from message source S_j, $j \in \{1, 2, \dots, K\}$, $y_i(n)$ is the received symbol at message destination D_i, $i \in \{1, 2, \dots, K\}$, $z_i(n)$ is the additive noise at message destination D_i, and h_{ij} is the constant channel coefficient between message source S_j and message destination D_i. All symbols belong to the field $\mathbb{C}$. We also denote $g_{ij} = |h_{ij}|^2$, the flat fading channel gain associated to each source-destination link. The term $z_i(n)$ is the independent and identically distributed (i.i.d.) complex circularly symmetric additive white Gaussian noise term, with zero mean and variance N_0.

III. The Classical TIM Formulation

Before we introduce our formulation for the interference topology construction model for TIM, let us first introduce the classical approach for the interference topology construction in the multi-user unicast TIM problem, as proposed in [5]. Two important constraints are considered:

1) The average transmit power at each TX S_j,

$$P_j := \frac{1}{N} \left[\sum_{n=1}^{N} |x_j(n)|^2 \right], \tag{2}$$

where N is the number of channel uses, is set to ensure the following nominal interference-free SNR for all desired links $S_j \to D_j$, given as

$$\frac{g_{jj} P_j}{N_0} \geq \gamma, \quad \forall j \in \{1, 2, \dots, K\}, \tag{3}$$

where γ denotes the desired SNR target for all desired links.

2) Once the transmission powers P_j are chosen, the interference level at each D_i, is fixed and given by

$$I_i = \sum_{j=1;\ j \neq i}^{K} g_{ij} P_j. \tag{4}$$

For each destination D_i, the interference links set, $\mathcal{I}_i = \{S_j \to D_i;\ j \in \{1, 2, \dots, K\},\ j \neq i\}$, is divided into two subsets, a set of significant interference links $\mathcal{S}_i$ and a set of weak interference links $\bar{\mathcal{S}}_i$, in which $\mathcal{I}_i = \mathcal{S}_i \cup \bar{\mathcal{S}}_i$ and $\mathcal{S}_i \cap \bar{\mathcal{S}}_i = \emptyset$. This decomposition is not necessarily unique as it depends on the different possible combinations of the interference links. In [5], a set $\bar{\mathcal{S}}_i$ is chosen arbitrarily, such that its elements' sum-interference verify

$$\sum_{\{S_j \to D_i\} \in \bar{\mathcal{S}}_i} g_{ij} P_j \leq N_0, \tag{5}$$

with $\mathcal{S}_i$ taken as the complement of $\bar{\mathcal{S}}_i$.

Let us now define the interference topology of the unicast TIM problem as follows.

Definition 1 (Interference Topology)**.** *Interference topology is an undirected bipartite graph with K nodes on one side, each node representing a unique message source S_j, and K nodes on the other side, each node representing a unique message destination D_i. Each message source S_j wishes to send a unique message $W_j \in \mathcal{W}$, while each message destination D_i desires a unique message $W_i \in \mathcal{W}$. Every edge in this graph connects a node from the source side to a node on the destination side. Two kinds of edges are possible: edges representing the desired links, and edges representing the significant interference links, i.e., the links associated to all $\mathcal{S}_i$ sets. All other possible edges, i.e., the edges representing the weak interference links (i.e., the links associated to all $\bar{\mathcal{S}}_i$ sets), will be suppressed in this interference graph representation.*

Once an interference topology is determined based on constraints (3) and (5), the resource allocation process will guarantee a lower bound on the Signal to Interference Plus Noise Ratio (SINR) at each D_i, denoted as ξ_i, given as

$$\xi_i \geq \alpha \frac{\gamma}{2},\ \forall i \in \{1, 2, \dots, K\}, \tag{6}$$

where α is the loss factor imposed by non-orthogonal transmission schemes. Typically, $\alpha = 1$ when an orthogonal resource allocation is used, while $0 < \alpha < 1$ when non-orthogonal linear coding is used [5].

This model guarantees a lower bounded SINR and therefore an achievable bound on the capacity region for this network.

IV. TIM: A New Formulation for the Topology Construction

In [5], Jafar fixed an interference topology, according to Definition 1 and (5), and studied the DoF, i.e., when the transmission powers tend to infinity. While this approach provides an enticing way to evaluate the theoretical performance limit of the topological interference representation of the wireless network, it does not hold for the performance of the wireless network itself. Indeed, as transmission powers tend to infinity, the interference level of the neglected interference links at each D_i, i.e., the weak interference links associated to $\bar{\mathcal{S}}_i$, also increases, and therefore, the constraint in (5) fails.

We now propose a different formulation for the interference topology construction to cope with the aforementioned drawbacks, while retaining the elegant DoF analysis in [5]. Our objectives are threefold: first, to deal with the finite SNR regime and not only with the asymptotic one; second, to improve its accuracy when analyzing the capacity as a function of the SNR; and third, to account for a flexible interference threshold (not just N_0) that can model different SNR requirements.

A. Normalized Network Model

We start by choosing the desired SNR target γ. We assume the network performs a power control to guarantee the same SNR target for all desired links $S_j \rightarrow D_j$. Hence, the transmission powers are fixed according to

$$P_j = \frac{N_0 \gamma}{g_{jj}},\ \forall j \in \{1, 2, \ldots, K\}. \tag{7}$$

Let us formulate an equivalent system by scaling the transmission powers and the noise values such that:

1) The noise power at each destination D_i is set to 1.
2) The normalized channel gains associated to the desired links are also set to $\tilde{g}_{jj} = 1,\ \forall j \in \{1, 2, \ldots, K\}$.
3) The equivalent transmission powers are consequently set to $P'_j = \gamma,\ \forall j \in \{1, 2, \ldots, K\}$.
4) To preserve the equivalence with the initial model, the normalized interference channel gains are set to $\tilde{g}_{ij} = \frac{g_{ij}}{g_{jj}} = g_{ij},\ \forall i, j \in \{1, 2, \ldots, K\}$ and $i \neq j$.

This model corresponds to a normalization of the TIM model defined in [5].

B. SNR-Independent Interference Threshold

Based on the normalized model described in Section IV-A, the transmission power from each message source S_j is γ, and the noise level at each message destination D_i is unitary. Let us turn our interest to the interference strength at message destination D_i which is now given by

$$I_i = \sum_{j=1;\ j\neq i}^{K} \tilde{g}_{ij} \cdot \gamma, \tag{8}$$

and the SINR at D_i turns out to be

$$\xi_i = \frac{\gamma}{1 + \gamma \sum_{j=1;\ j\neq i}^{K} \tilde{g}_{ij}}. \tag{9}$$

Now let us generalize the constraint given in (5) for an arbitrary interference threshold τ. In the TIM problem, the weak interference links, associated to $\bar{\mathcal{S}}_i$, at message destination D_i, are neglected. We choose the set $\bar{\mathcal{S}}_i$, at each D_i, such that its elements sum interference verifies

$$\sum_{\{S_j \rightarrow D_i\} \in \bar{\mathcal{S}}_i} \tilde{g}_{ij} \cdot \gamma \leq \tau, \tag{10}$$

which can be rewritten as

$$\sum_{\{S_j \rightarrow D_i\} \in \bar{\mathcal{S}}_i} \tilde{g}_{ij} \leq \beta, \tag{11}$$

where $\beta = \frac{\tau}{\gamma}$ is a normalized interference threshold.

In (11), the left hand term relies only on the channel coefficients and not on γ. This means that an interference topology built for a given β is valid for any couple (γ, τ) such that $\tau/\gamma = \beta$. Finally, for each value of β, a TIM instance (i.e., an interference topology in the TIM problem) can be built and evaluated for a full range of SNRs. Note that, while a given β value corresponds to an interference topology, such a topology may be obtained through a range of β values.

Once an interference topology is determined based on constraints (7) and (11), the collective interference at D_i from the weak interference links in $\bar{\mathcal{S}}_i$ is upper bounded by $\gamma\beta$, and the linear coding resource allocation process will guarantee a lower bound on the effective SINR at each D_i, given as

$$\xi_i \geq \alpha \frac{\gamma}{1 + \gamma\beta},\ \forall i \in \{1, 2, \ldots, K\}, \tag{12}$$

where α is the non-orthogonality loss factor already mentioned.

In our model, when γ tends to infinity, the SINR ξ_i is not infinite as in Jafar's model [5], but is bounded by $\frac{\alpha}{\beta}$, $\forall i \in \{1, 2, \ldots, K\}$, which fits better with the properties of the real wireless network behind the interference topology model.

V. Achievable Rates under the New TIM Formulation

First, let us denote by $\mathcal{T}$ a possible interference topology of the wireless network defined in Section II with the message set $\mathcal{W}$, and by $\mathcal{P}$ the set of all possible interference topologies of the wireless network.

As we explained in Section IV, the theoretical limit proposed in [5] is valid for a TIM instance but not for the underlying wireless network. In this section, we are going to characterize an achievable symmetric rate per message $W_k \in \mathcal{W}$ of the wireless network defined in Section II. For a TIM problem $\mathbf{TIM}(\mathcal{T})$ with an interference topology $\mathcal{T}$, we can characterize an achievable symmetric rate per message, i.e., a rate $R_{\mathcal{T}}(\gamma)$ that can be achieved by each message $W_k \in \mathcal{W}$ through linear coding, as follows

$$R_{\mathcal{T}}(\gamma) = d_{\mathcal{T}} \cdot \log_2\left(1 + \frac{\alpha_{\mathcal{T}} \gamma}{1 + \gamma \beta_{\mathcal{T}}}\right), \tag{13}$$

where $d_{\mathcal{T}}$ is the symmetric DoF per message, i.e., the maximal DoF that can be achieved by each message $W_k \in \mathcal{W}$ through linear coding, $\alpha_{\mathcal{T}}$ is the non-orthogonal linear coding penalty factor in the TIM problem **TIM**($\mathcal{T}$), and $\beta_{\mathcal{T}}$ is a β value, defined in Section IV-B, that leads to the construction of the topology $\mathcal{T}$.

Let us remind that, according to [5], linear coding allows to increase the DoF by using non-orthogonal linear vectors, but in turns result in a degraded SINR due to the non-orthogonal linear coding penalty factor $\alpha_{\mathcal{T}}$. This penalty comes from the fact that the receiver has to project the received signal in the interference-free subspace [1]. Therefore, the following engineering tradeoff holds: either the resource allocation is restricted to an orthogonal allocation strategy providing a maximal SINR value, i.e., $\frac{\gamma}{1+\gamma\beta_{\mathcal{T}}}$, at each message destination, at the price however of a limited DoF, or the resource allocation can exploit a linear coding strategy, which usually increases the DoF, at the price of reduced individual SINRs.

We recall that a range of values of β may lead to the construction of the same interference topology $\mathcal{T}$. We are interested in finding the β value, out of all the possible β values that lead to the construction of this interference topology $\mathcal{T}$, that provides the highest achievable symmetric rate in (13) for the same $d_{\mathcal{T}}$ (fixed interference topology $\mathcal{T}$). This is going to be stated in the following definition and theorem.

Definition 2 (Lowest β Value for $\mathcal{T}$)**.** *The lowest β value that leads to the construction of an interference topology $\mathcal{T}$ is defined as*

$$\beta^{\star}_{\mathcal{T}} = \max_{i} \sum_{\{S_j \to D_i\} \in \bar{\mathcal{S}}_i} \tilde{g}_{ij}. \tag{14}$$

Theorem 1 (Achievable Symmetric Rate: Finite SNR Regime)**.** *An achievable symmetric rate, in the finite SNR regime, through linear coding schemes, of a wireless network can be defined as the highest achievable symmetric rate among the achievable symmetric rates associated to all TIM problems of all possible interference topologies of the wireless network, as follows*

$$R_{\mathcal{W}}(\gamma) = \max_{\mathcal{T} \in \mathcal{P}} R^{\star}_{\mathcal{T}}(\gamma), \tag{15}$$

*where $R^{\star}_{\mathcal{T}}(\gamma)$ is the achievable symmetric rate associated to the TIM problem **TIM**($\mathcal{T}$), evaluated as in* (13) *when $\beta_{\mathcal{T}} = \beta^{\star}_{\mathcal{T}}$, defined as*

$$R^{\star}_{\mathcal{T}}(\gamma) = d_{\mathcal{T}} \cdot \log_2\left(1 + \frac{\alpha_{\mathcal{T}}\gamma}{1+\gamma\beta^{\star}_{\mathcal{T}}}\right). \tag{16}$$

Proof. The result in (16) follows naturally from the fact that, out of all possible β values that leads to the construction of a given interference topology $\mathcal{T}$ with a fixed $d_{\mathcal{T}}$, the β value associated with the highest achievable symmetric rate $R^{\star}_{\mathcal{T}}(\gamma)$ is $\beta^{\star}_{\mathcal{T}}$ (the lowest β value), since this maximizes the log term in (13). Hence, all achievable symmetric rates given by higher β values must be lower than the one with $\beta^{\star}_{\mathcal{T}}$. □

For a sufficiently large SNR, i.e., $\gamma \to \infty$, we can state the following corollary.

Corollary 1 (Achievable Symmetric Rate: Asymptotic SNR Regime)**.** *An achievable symmetric rate, in the asymptotic SNR regime, through linear coding schemes, of a wireless network can be defined as follows*

$$R_{\mathcal{W}}(\gamma) = \max_{\mathcal{T} \in \mathcal{P}} d_{\mathcal{T}} \cdot \log_2\left(1 + \frac{\alpha_{\mathcal{T}}}{\beta^{\star}_{\mathcal{T}}}\right). \tag{17}$$

VI. Fundamental Limits on $\beta^{\star}$ for Interference Topologies with Half-DoF-Feasibility

In this section, we will present $\beta^{\star}$ bounds that are able to specify, in the TIM problem of a wireless network, which *half-DoF-feasible* interference topologies will outperform, in terms of the achievable symmetric rate, the two extreme interference topologies.

We define the two extreme interference topologies, the *fully connected* and the *fully disconnected*, as follows.

Definition 3 (Fully Connected Interference Topology $\mathcal{T}_{\text{c}}$)**.** *An interference topology $\mathcal{T}$ is said to be fully connected if and only if all interference links are significant interference links. We will denote the fully connected topology as $\mathcal{T}_c$.*

Property 1 (Lowest β for $\mathcal{T}_{\text{c}}$)**.** *The lowest β value for the fully connected interference topology $\mathcal{T}_c$ is $\beta^{\star}_{\mathcal{T}_c} = 0$.*

Definition 4 (Fully Disconnected Interference Topology $\mathcal{T}_{\text{d}}$)**.** *An interference topology $\mathcal{T}$ is said to be fully disconnected if and only if all interference links are weak interference links. We will denote the fully disconnected topology as $\mathcal{T}_d$.*

Property 2 (Lowest β for $\mathcal{T}_{\text{d}}$)**.** *The lowest β value for the fully disconnected interference topology $\mathcal{T}_d$ is*

$$\beta^{\star}_{\mathcal{T}_d} = \max_{i} \sum_{j=1;\ j\neq i}^{K} \tilde{g}_{ij} = \beta_m. \tag{18}$$

For the fully connected interference topology $\mathcal{T}_{\text{c}}$, the maximal DoF that can be achieved by each message $W_k \in \mathcal{W}$ in the TIM problem **TIM**($\mathcal{T}_{\text{c}}$) is $d_{\mathcal{T}_{\text{c}}} = 1/K$, and the resource allocation is restricted to orthogonal allocation with $\alpha_{\mathcal{T}_{\text{c}}} = 1$. Hence, for $\mathcal{T}_{\text{c}}$, a symmetric achievable rate $R^{\star}_{\mathcal{T}_{\text{c}}}(\gamma)$ per each message $W_k \in \mathcal{W}$, is defined as

$$R^{\star}_{\mathcal{T}_{\text{c}}}(\gamma) = \frac{1}{K}\log_2(1+\gamma). \tag{19}$$

The achievable rate $R^{\star}_{\mathcal{T}_{\text{c}}}(\gamma)$ for $\mathcal{T}_{\text{c}}$ defined in (19) tends to infinity when $\gamma \to \infty$. This is the unique interference topology solution for which the rate may scale with $\log(\gamma)$ but in this case the optimal allocation is $1/K$.

For the fully disconnected interference topology $\mathcal{T}_{\text{d}}$, the maximal DoF that can be achieved by each message $W_k \in \mathcal{W}$ in the TIM problem **TIM**($\mathcal{T}_{\text{d}}$) is $d_{\mathcal{T}_{\text{d}}} = 1$, and in this case, full reuse is performed with $\alpha_{\mathcal{T}_{\text{d}}} = 1$. Hence, for $\mathcal{T}_{\text{d}}$, a symmetric achievable rate $R^{\star}_{\mathcal{T}_{\text{d}}}(\gamma)$ per each message $W_k \in \mathcal{W}$, is defined as

$$R^{\star}_{\mathcal{T}_{\text{d}}}(\gamma) = \log_2\left(1 + \frac{\gamma}{1+\gamma\beta_{\text{m}}}\right). \tag{20}$$

Let us now define the half-DoF-feasible interference topologies as follows.

Definition 5 (Half-DoF-Feasible Interference Topologies)**.** *An interference topology $\mathcal{T}$ that can support in the TIM problem a symmetric DoF equal to* 0.5 *per message* $W_k \in \mathcal{W}$*, is called a half-DoF-feasible interference topology.*

In [5], Jafar showed that 0.5 is the maximal symmetric DoF per message $W_k \in \mathcal{W}$ except if the interference topology is fully disconnected. To achieve 0.5 DoF per message, an IA approach may be required. The set of all half-DoF-feasible interference topologies associated to the given wireless network is denoted as $\mathcal{T}_{\text{ia}}$. It is also known, from [5], that for any interference topology belonging to $\mathcal{T}_{\text{ia}}$, a 0.25 DoF per message $W_k \in \mathcal{W}$ at least, is achievable with orthogonal sharing. More precisely, the symmetric DoF per message $W_k \in \mathcal{W}$ that is achievable with an orthogonal sharing, for any $\mathcal{T} \in \mathcal{T}_{\text{ia}}$, belongs to $\{1/2, 1/3, 1/4\}$. Therefore, the topologies in $\mathcal{T}_{\text{ia}}$ can be categorized into three sets noted $\mathcal{T}_{1/2}, \mathcal{T}_{1/3}, \mathcal{T}_{1/4}$.

Definition 6 (Orthogonal DoF Solutions Based Half-DoF-Feasible Interference Topologies)**.** *Any interference topology $\mathcal{T} \in \mathcal{T}_{ia}$ is categorized also in $\mathcal{T}_{1/L}$, i.e., $\mathcal{T} \in \mathcal{T}_{1/L}$, where $L \in \{2,3,4\}$, if and only if the maximal DoF that can be achieved by each message $W_k \in \mathcal{W}$ through orthogonal sharing is $1/L$. The sets $\mathcal{T}_{1/L}$ verify $\mathcal{T}_{ia} = \cup_{L\in\{2,3,4\}}\mathcal{T}_{1/L}$ and $\mathcal{T}_{1/L} \cap \mathcal{T}_{1/L'} = \emptyset$, where $L' \in \{2,3,4\}$ and $L \neq L'$.*

A half-DoF-feasible interference topology $\mathcal{T} \in \mathcal{T}_{\text{ia}}$ will outperform, in terms of the achievable symmetric rate, both extreme topologies, $\mathcal{T}_{\text{c}}$ and $\mathcal{T}_{\text{d}}$, at some γ, under a condition on $\beta^\star_{\mathcal{T}}$. Conversely, for a given γ, there is a specific $\beta^\star$ range for which any interference topology $\mathcal{T} \in \mathcal{T}_{\text{ia}}$ with $\beta^\star_{\mathcal{T}}$ value within this $\beta^\star$ range will outperform in terms of the achievable symmetric rate the extreme interference topologies, and any interference topology $\mathcal{T} \in \mathcal{T}_{\text{ia}}$ with $\beta^\star_{\mathcal{T}}$ value not within this $\beta^\star$ range will be outperformed in terms of the achievable symmetric rate by the extreme interference topologies.

Specifying this $\beta^\star$ range, for a given γ, will be very helpful in avoiding an exhaustive search over the full $\beta^\star$ range, i.e., the range $0 \leq \beta^\star \leq \beta_m$, to specify the half-DoF-feasible interference topologies in $\mathcal{T}_{\text{ia}}$ that are outperforming in terms of the achievable symmetric rate the extreme interference topologies. In other words, this $\beta^\star$ range limits the search (for example, in an algorithm) to find half-DoF-feasible interference topology $\mathcal{T}$ that can be the best topology at a given γ.

In the following, we will present the $\beta^\star$ range, at a given γ, for which an interference topology $\mathcal{T} \in \mathcal{T}_{\text{ia}}$ will outperform, at the given γ, the extreme interference topologies in terms of the achievable symmetric rate if and only if $\beta^\star_{\mathcal{T}}$ is within this $\beta^\star$ range. We will present this $\beta^\star$ range for the half-DoF-feasible interference topologies when orthogonal sharing is used, i.e., for the three topology categories $\mathcal{T}_{1/L}$, $L \in \{2,3,4\}$. We will denote for this $\beta^\star$ range by $\beta^\star_L$ range. Then, we will present the $\beta^\star$ range for the half-DoF-feasible interference topologies when IA is used, i.e., for any $\mathcal{T} \in \mathcal{T}_{\text{ia}}$. We will denote for this $\beta^\star$ range by $\beta^\star_{\text{ia}}$ range.

Theorem 2 ($\beta^\star_L$ range: Orthogonal DoF Solutions Based Half-DoF-Feasible Interference Topologies)**.** *For a given γ, the $\beta^\star_L$ range, $L \in \{2,3,4\}$, for which a half-DoF-feasible interference topology $\mathcal{T} \in \mathcal{T}_{1/L}$, when orthogonal sharing is used, outperforms the extreme interference topologies in terms of the achievable rate if and only if $\beta^\star_{\mathcal{T}}$ is within this $\beta^\star_L$ range, is characterized by the following $\beta^\star_L$ outer bound.*

$$\beta^\star_L < \frac{\frac{\gamma}{2^{L\cdot\max\left[\frac{1}{K}\cdot\log_2(1+\gamma),\log_2\left(1+\frac{\gamma}{1+\gamma\beta_m}\right)\right]}-1}-1}{\gamma}. \tag{21}$$

Proof. The result in (21) follows naturally from (16), for the case of a half-DoF-feasible interference topology $\mathcal{T}$ when orthogonal sharing is used, i.e., $\mathcal{T} \in \mathcal{T}_{1/L}$, where $d_{\mathcal{T}} = 1/L$ and $\alpha_{\mathcal{T}} = 1$, and from (19) and (20) which represent the achievable symmetric rates of the extreme interference topologies. □

Theorem 3 ($\beta^\star_{\text{ia}}$ range: IA DoF Solution Based Half-DoF-Feasible Interference Topologies)**.** *For a given γ, the $\beta^\star_{ia}$ range, for which a half-DoF-feasible interference topology $\mathcal{T} \in \mathcal{T}_{ia}$, when IA is used, outperforms the extreme interference topologies in terms of the achievable rate if and only if $\beta^\star_{\mathcal{T}}$ is within this $\beta^\star_{ia}$ range, is characterized by the following $\beta^\star_{ia}$ outer bound.*

$$\beta^\star_{ia} < \frac{\frac{\frac{\gamma}{2^{2\cdot\max\left[\frac{1}{K}\cdot\log_2(1+\gamma),\log_2\left(1+\frac{\gamma}{1+\gamma\beta_m}\right)\right]}-1}}{\sin^2\frac{\pi}{Na}}-1}{\gamma}, \tag{22}$$

where N_a is the number of alignment sets when non-conflicting alignment sets are merged into one alignment set [5].

Proof. The result in (22) follows naturally from (16), for the case of a half-DoF-feasible interference topology $\mathcal{T}$ when IA is used, i.e., $\mathcal{T} \in \mathcal{T}_{\text{ia}}$, where $d_{\mathcal{T}} = 0.5$ and $\alpha_{\mathcal{T}} = \sin^2\frac{\pi}{N_{\text{a}}}$, where N_{a} is the number of alignment sets when non-conflicting alignment sets are merged into one alignment set [5], and from (19) and (20) which represent the achievable symmetric rates of the extreme interference topologies. □

To illustrate the results in Theorems 2 and 3, we graphed the outer bounds of $\beta^\star$, for $K = 10$ and with $N_{\text{a}} = 3$, as a function of γ, for the orthogonal and the IA cases, in Figure 1 and Figure 2, respectively. In Figure 1, where $L = 3$, we know that, for a given γ, there exists many half-DoF-feasible interference topologies throughout the range $0 \leq \beta^\star \leq \beta_m$. However, from Theorem 2, we know that a half-DoF-feasible interference topology $\mathcal{T} \in \mathcal{T}_{1/L}$ will outperform the extreme interference topologies if and only if $\beta^\star_{\mathcal{T}}$ is within the $\beta^\star_3$ range that is characterized by the $\beta^\star_3$ outer bound. This greatly reduces the search range for the half-DoF-feasible interference topologies from the full $\beta^\star$ range, i.e, $0 \leq \beta^\star \leq \beta_m$, to the $\beta^\star_3$ range. Since this search involves looking at a huge number of combinations of possible interference topologies for each β_m value, the reduction is of great importance in optimizing the algorithmic complexity. The same conclusion holds for the IA case in Figure 2.

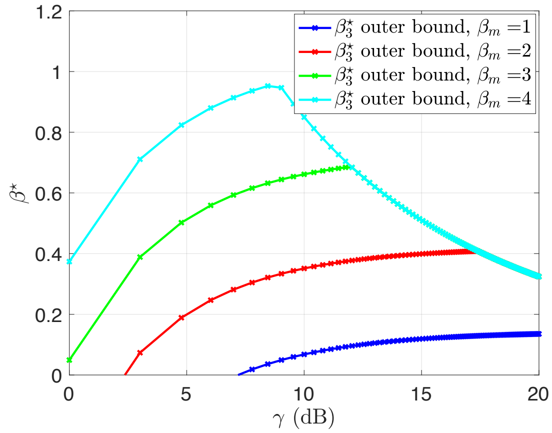

Fig. 1. $\beta_3^\star$ outer bounds for different β_m values.

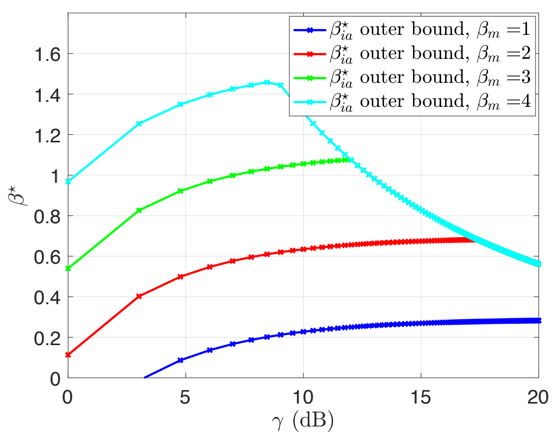

Fig. 2. $\beta_{\text{ia}}^\star$ outer bounds for different β_m values and with $N_{\text{a}} = 3$.

VII. Conclusion

In this paper, we introduce a new approach to convert a wireless network into a TIM model using a normalized interference threshold β that ensures the validity of the interference topology for all SNRs. Based on this approach, the TIM analysis exhibits the fundamental trade-off between DoF and SINR maximization. This contribution is complementary to classical work related to TIM since the focus is clearly on the formulation of a TIM problem. This approach provides a better evaluation of the achievable symmetric rate of the initial wireless network.

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