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STRUCTURAL TOPOLOGY OPTIMIZATION
IN LINEAR AND NONLINEAR ELASTICITY
USING GENETIC ALGORITHMS

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ABSTRACT

In this paper, structural topology optimization is addressed through Genetic Algorithms. A set of designs is evolved following the Darwinian *survival-of-fittest* principle. The standard crossover and mutation operators are tailored for the needs of 2D topology optimization. The genetic algorithm based on these operators is experimented on plane stress problems of cantilever plates: the goal is to optimize the weight of the structure under displacement constraints.

The main advantage of this approach is that it can both find out alternative optimal solutions, as experimentally demonstrated on a problem with multiple solutions, and handle different kinds of mechanical model: some results in elasticity with large displacements are presented. In that case, the nonlinear geometrical effects of the model lead to non viable solutions, unless some constraints are imposed on the stress field.

1 INTRODUCTION

Since the seminal work of Holland (1975) and the comprehensive study of Goldberg (1989), Genetic Algorithms (GAs) have gradually been recognized as powerful stochastic optimization algorithms. Their strength proceeds from their wide range of applications: GAs can handle non derivable, non continuous and even non analytically defined functions[1].

The main interest of stochastic methods in Engineering Sciences is to break the limits of standard deterministic methods in many optimization problems: when the search space involve both discrete and continuous domains (e.g. for the optimal design of truss structures); when the objective function or the constraints lack regularity; or when the objective function admits a huge number of local optima. In counterpart, stochastic methods are computationally expensive: GAs for instance, are slower than classical optimization methods by about one or two orders of magnitude (when comparison is possible, i.e. when classical methods apply).

This paper focuses on applying GAs to some well-studied problems in mechanical engineering, namely the structural topology optimization of cantilever plates. Section 2 briefly reviews some related works in topology optimization; their advantages and their limitations regarding the specificities of constrained optimization in mechanical engineering are discussed. Section 3 gives the broad lines of GAs; it then describes genetic operators tailored for topology optimization. Experimentations in the linear elasticity case are presented in section 4: The first problem deals with minimizing the weight of a cantilever plate under displacement constraints. Further, the standard cantilever plate is modified to highlight the ability of the GA approach to handle problems having multiple optimal or quasi-optimal solutions. A compliance-based optimization is finally achieved; it permits to compare GA-based and homogenization-based approaches. Section 5 presents the first results obtained in linear elasticity with large displacements for the cantilever plate problem. The nonlinear geometrical effects clearly show the need to take into account the stress field in the fitness function to obtain realistic solutions.

2 RELATED WORKS

The main trends in structural optimization can be sketched as follows.

A first approach consists in continuously varying the domain at hand. This approach is effective as far as a good solution can actually be obtained by a continuous transformation of the initial domain. In contrast, it does not allow to find an optimal shape that includes holes (except if the number of holes is known beforehand, of course).

An approach to *topology design*, introduced by Bendsoe and Kikushi (1988), is that of homogeniza-

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[1] e.g. functions only available through computation.

tion; it consists in dealing with a continuous density of material. In the end of this deterministic optimization, the current density is forced toward value 1 or 0, that respectively stands for material *present* or *absent*. However, this approach requires the design of the homogenized operator, as thoroughly described in Allaire and Kohn (1993), and is insofar limited to the linear elasticity case. Moreover, it cannot address loadings that apply on the actual boundary of the shape to be determined (e.g. pressure load), and hardly handles optimization for multiple loadings.

Another approach to topology design is that of stochastic optimization, such as involved in simulated annealing (Kirkpatrick et al (1983)) and genetic algorithms. Both methods have been applied to structural optimization: in the framework of discrete truss structures, for cross-section sizing by Goldberg and Samtani (1986), Hajela (1992), Lin and Hajela (1993), Schoenauer and Wu (1993) among others, as well as for topological optimization by Hajela et al (1993), Grierson and Pak (1993), Wu (1995); for the optimization of composite materials by Leriche and Haftka (1993); and more recently for structural components optimization by Anagnostou et al. (1992), Ghaddar et al. (1994) using simulated annealing, and by Jensen (1992), Chapman et al. (1994) with genetic algorithms.

This paper continues and extends the GA-based approaches of Jensen (1992) and Chapman et al. (1994) in several respects. First, specific genetic operators (crossover, mutation) are introduced. Second, a problem exhibiting multiple solutions is studied, demonstrating the ability of the GA approach to successfully find out different quasi-optimal solutions. Last but not least, this method is not limited to linear elasticity, as demonstrated by considering the large displacements model.

3 GA: RECALLS AND ADAPTATIONS

This section gives the broad lines of basic GAs; then it describes genetic operators devised for the needs of topology optimization.

3.1 Principle

Given a search space E and a *fitness* function F defined from E onto $\mathbb{R}^+$, GAs evolve a set of *p individuals* (points of E), termed *population*. This evolution crudely mimics the Darwinian evolution: according to the Darwinian *survival-of-the-fittest* principle, the fittest individuals, i.e. the near-optimal points of fitness function F will appear in population P_i for some i.

The basic step in GAs, called *generation*, is the transformation used for population P_i to give birth to population P_{i+1}. This transformation involves three steps :

- Selection builds population P_i' by copying elements of P_i ; the number of copies of an individual increases with its fitness, the total number of elements in P_i' being same as in P_i.

- Crossover applies on population P_i' to build population $P"_i$. From two individuals x and y in P_i', crossover builds two offsprings x' and y' with probability p_c (p_c usually varies from .2 to 1.). When considering a bitstring representation ($E = \{0,1\}^N$), a crossover c can be represented[2] as a bitstring itself, $c = (c_1, \dots c_N)$:

$$\begin{matrix} x_1 & .. & x_N \\ y_1 & .. & y_N \end{matrix} \rightarrow \begin{matrix} x_1\prime & .. & x_N\prime \\ y_1\prime & .. & y_N\prime \end{matrix}$$

$$\text{with } x_i\prime = \begin{cases} x_i & \text{if } c_i = 1 \\ y_i & \text{if } c_i = 0 \end{cases}$$

$$\text{and } y_i\prime = \begin{cases} y_i & \text{if } c_i = 1 \\ x_i & \text{if } c_i = 0 \end{cases}$$

- Mutation applies on population $P"_i$ to build population P_{i+1}. Mutation transforms an individual x in $P"_i$ into an offspring x' ; when considering a bitstring representation, a mutation can similarly be represented by a bitstring $m = (m_1, \dots m_N)$:

$$x_1 \;..\; x_N \;\rightarrow\; x_1\prime \;..\; x_N\prime$$

$$\text{with } x_i\prime = \begin{cases} 1 - x_i & \text{if } m_i = 1 \\ x_i & \text{if } m_i = 0 \end{cases}$$

  The probability for m_i to take value 1 is noted p_m (p_m usually varies from 10^{-2} to 10^{-4}).

3.2 Adaptation for structural topology optimization

Representation. We restrict ourselves to considering 2D domains (see sections 4 and 5). An individual is here a shape embedded into the problem domain. This domain is discretized according to a grid of quadrangular elements. An individual can then be represented by an array of bits whose dimension is the number of such elements: an element is void (respectively material) whether the corresponding bit takes value 0 (resp. 1).

Note that such arrays do not all stand for viable shapes: e.g. a sound shape must be connected to the load point(s) involved in the mechanical problem. Non-viable shapes are handled through the fitness calculation (they will be given a null fitness; see 3.3).

Further, the usual one-dimensional bitstring representation is not faithful with respect to topology: On the one hand, adjacent bits in the bitstring representation do not necessarily correspond to neighbor

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[2] Most authors only consider *one-point crossovers*, corresponding to masks (1..10..0), or *two-point crossovers*, corresponding to masks (1..10..01..1). The general case represented here is called *uniform* crossover, introduced by Syswerda (1989).

elements of the domain; And on the other hand, vertical adjacent positions are quite far from one another in the bitstring representation. This remark leads to design new genetic operators, in order to take into account the two-dimensional topology.

Crossover. The classical 1-point, 2-points and uniform crossovers were the first choice for crossover operators. As expected, both 1-point and 2-points crossovers gave poor results. And they were constantly outperformed by the uniform crossover: This can be explained by the fact that uniform crossover involves a lesser bias than 1-point and 2-point crossovers on the two-dimensional representation of 2D shapes.

Two different crossover operators have been purposely devised for a better transmission of topological properties between parents and offspring:
- the *diagonal* crossover (Figure 1) involves (a) the random selection of line D (position and slope) and (b) the exchange of the lower parts of the structures at hand;
- the *block* crossover (Figure 2), first introduced by Jensen (1992), involves here (a) the random selection of points X_1, X_2, Y_1, Y_2, defining 9 blocks in both structures, (b) the random choice of the number of blocks to exchange (2 or 3), (c) the random selection of the blocks among the 9 blocks and (c) the actual exchange of the blocks.

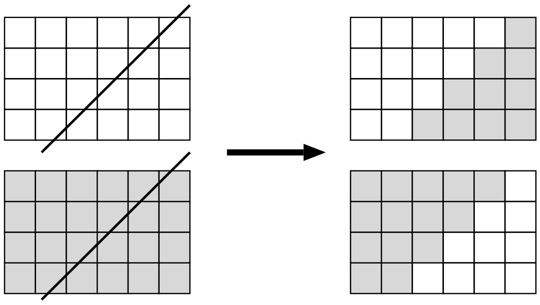


Figure 1: The diagonal crossover

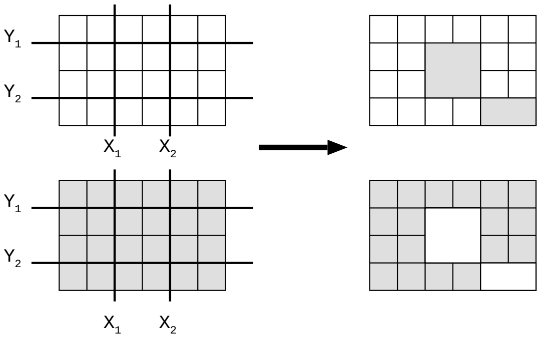


Figure 2: The block crossover

A thorough study of the influence of the crossover operator on the standard cantilever plate problem by Kane (1995) demonstrates a slight improvement of both the diagonal operator and the block operator over the uniform operator, while both significantly outperform any standard 1D crossover (e.g. 1 point crossover used by Chapman et al. (1994)). All experiments presented in this paper use the block crossover.

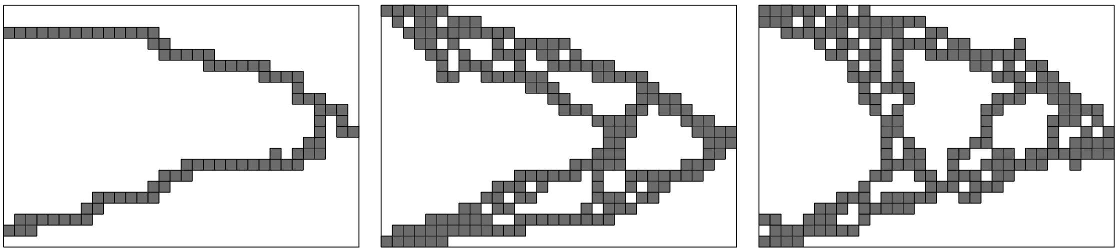
Mutation. Mutation classically proceeds by flipping randomly selected bits. Two other kinds of mutation are proposed, where the bits altered are chosen after either the topology of the current individual, or the whole current population.
In the first mode, termed *boundary mutation*, the bits flipped are preferably selected among the elements near from the boundary of the current structure (the computation of this boundary is described in section 3.3). This mutation performs slight modifications of the individual at hand. In particular, it hardly modifies the topology of the structure, and could be related to the classical "domain variation" method of shape optimization.

In the second mode, termed *epistatic mutation*, the goal is to counteract the genetic drift: It is well known that genetic population classically tends to become homogeneous over generations, i.e. the set of bits with constant value over the population increases. This loss of diversity is prejudicial to optimization, since it arbitrarily restricts the exploration. Epistatic mutation thus selects and flips bits whose value is almost constant over the population, in order to reintroduce the disappearing values.

The epistatic mutation performs statistically better than both the classical uniform mutation and the boundary mutation, as experimented thoroughly by Kane (1995), and all experiments presented in this paper use the epistatic mutation. However, when the population has converged, the boundary mutation should allow to refine the solution(s) found so far. Such an iterated schema (epistatic mutation in the beginning, boundary mutation afterwards) remains to be tested.

3.3 Fitness computation

A given individual is evaluated in a 2-step process. Some *seed* material is imposed at the point(s) where dynamical loading is applied. The connected component containing that seed is computed. Grid elements are connected if and only if they share an edge. Note that no seed material is prescribed on the part of the boundary where the plate is fixed, as it is in the approach of Chapman et al. (1994). The optimization process does choose where to hang the structure on the fixed vertical boundary. This allows for a greater flexibility in solving the optimization problem, as witnessed in the range of alternative solutions proposed for the modified cantilever plate problem (see section 4.3). On the other hand, more structures are likely to be disconnected from that fixed boundary, leading to an ill-posed problem: such structures



D_{Lim}	2.463	0.1293	0.133
D_{Max}	2.439	0.129286	0.133
$Area$	0.2075	0.4675	0.49

Figure 3: Optimal cantilever plates for different values of the displacement constraint D_{Lim}. D_{Max} is the actual maximum displacement of the structure.

are arbitrarily assigned zero fitness, and are therefore eliminated by the next Darwinian selection.

Once the component connecting the seed and the fixed boundary has been computed (if any), the FEM analysis is performed on that component, on the same regular quadrangular mesh used to represent the individuals; the FEM tool is detailed in Jouve (1993). The actual material boundary is used, in contrast with both Jensen (1992) and Chapman et al. (1994) in which the FEM analysis was done on the whole design domain, assigning a very low Young modulus to void elements. Though the results of the analysis does not differ significantly from one method to the other, computing the actual boundary allows to take into account loading applied on this boundary (e.g. pressure loading or heat exchanges).

Last, the fitness of the structure is computed (the analytic expression of the fitness is discussed in next sections), involving, in the case of elasticity:
• the weight of the structure;
• the maximum value of the displacement, or the stress field or the compliance of the structure, computed from the above FEM analysis.
Note that this approach allows as well to consider multiple loadings (with as many FEM analyses as loading cases), though no such example is given here.

4 LINEAR ELASTICITY

All experiments in this section consider the standard plane stress linear elastic model. The effects of gravity are neglected.

The general framework is that of the cantilever plate: a rectangular plate is fixed on the left vertical part of its boundary (both displacements are set to 0), and the loading is made of a single force applied on the middle of its right vertical boundary. Figure 4 shows the 2×1 cantilever plate.

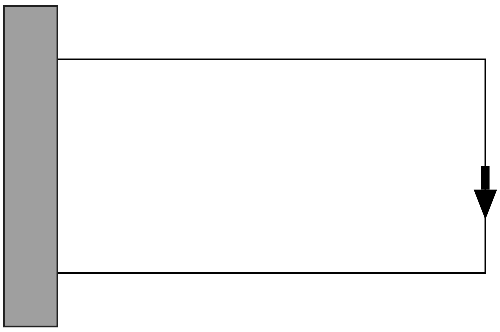


Figure 4: The standard 2×1 cantilever plate.

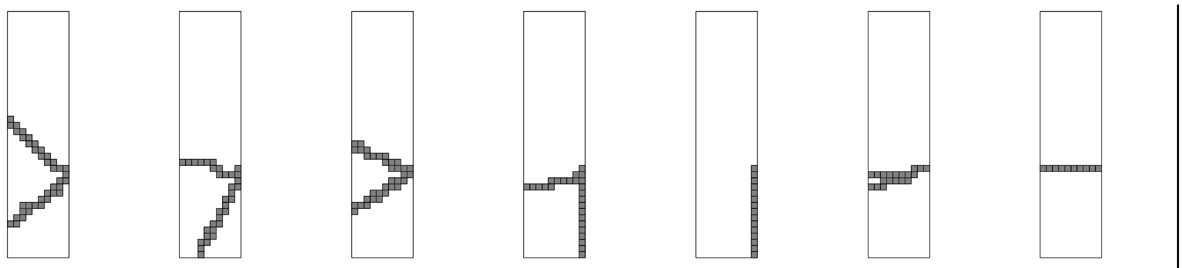
4.1 Penalized fitness

As said before, unstable structures (the connected component containing the load point does not contain any fixed edge) are assigned zero fitness. Following Jensen (1992), the fitness associated to stable structures is first defined by:

$$Fitness_{penalized} = \frac{1}{Area + \alpha(D_{Max} - D_{Lim})^{+}}$$

where $Area$ is the area of the actual structure (i.e. of the connected component used in the FEM analysis), D_{max} is the displacement of the load point (computed from the FEM analysis) and D_{Lim} is a prescribed constraint on that displacement. Positive parameter α is the *penalization factor*, and x^{+} denotes the positive part of x, max(x,0).

As pointed out by Chapman et al. (1994), the quality of the solution greatly depends on α: When α is large, the area is not that important and heavy structures are retained. When α is small, the structures optimal according to $Fitness_{penalized}$ do not satisfy the displacement constraints.



D_{Lim}	0.32	0.4		0.42	7.66		46.6
D_{Max}	0.294	0.3927	0.3919	0.3415	7.66	6.743	46.64
$Area$	0.38	0.33	0.34	0.26	0.15	0.19	0.1

Figure 5: Optimal structures on problems with multiple solutions. $\mathcal{F}$ is applied at height 15.

Nevertheless, the above fitness function, for large values of α, allows to precisely respect the constraint on the maximum displacement of the structure, which is not the case, neither with the compliance criterion (see Allaire and Kohn (1993), or section 4.5) nor with the stiffness criterion used by Chapman et al. (1994).

4.2 Iterated penalized fitness

The solution proposed to overcome the critical tuning of parameter α is a 2-step optimization: A medium value of α is first considered and a population of light structures is thereby determined. Then, during the evolution, the value of α is increased by a factor of 10 to ensure the satisfaction of the constraints. Such ideas about iterated GAs has been demonstrated powerful in truss structure optimization, by Schoenauer and Xanthakis (1993), as well as in other domains of application (see de Garis (1990), Schoenauer (1994)). All experiments presented in this paper are based on the penalized fitness, and involve this 2-step optimization.

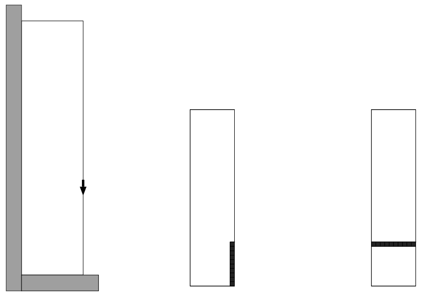
4.3 First results

Figure 3 show the first results on the 2×1 cantilever plate, discretized according to a 32×22 regular mesh. The population size for all runs is 125, and the number of generations arbitrarily fixed to 1000. One run thus require about 100000 FEM analyses, taking approximately 6 hours of a powerful HP workstation for the 32×22 discretization. The genetic operators are described in section 3.2: The block crossover is applied at a rate of 0.6, and the epistatic mutation at a rate of 0.1 (the probability to flip a given pixel varies from 0 to 0.01, depending of the diversity of the values for this particular pixel in the whole population, following a parabolic rule). All these parameters were adjusted after exhaustive tests, and details can be found in Kane (1995).

Each one of the structures on Figure 3 corresponds to a given value of the constraint on the maximum displacement D_{Lim}. And each one is the most significant result for that value of D_{Lim} obtained out of 5 runs with different random initial populations. Due to the stochastic aspect of the GA, the results differ from one run to the other. Nevertheless, on the 2×1 cantilever plate, such differences are not very important. However, note that the second structure of Figure 3, though optimized using a stronger constraint on the displacement, is lighter and more rigid than the third one. This suggested a modification of the geometry and boundary conditions to obtain a problem exhibiting known multiple solutions.

4.4 Multiple solutions

The problem considered in that section is the 1×4 cantilever plate, discretized according to a 10×40 mesh and for which both left and bottom boundaries are fixed (Figure 6-a).



(a) The problem (b) (c) Two optimal solutions

Figure 6: The 1×4 modified cantilever plate.

Depending on the height of the point where the loading is applied and on the constraint on the displacement, the same problem can have multiple solutions. A simple example of such situation is given in Figure 6 (b) and (c): the loading is applied at height 10, and, provided that the displacement constraint is large enough, both structures (a) and (b) are optimal solutions. Moreover, if the height of the loading point is fixed, and the displacement constraint D_{Lim} is gradually relaxed, different quasi-optimal solutions exist for some ranges of D_{Lim}. And the GA method is then able to find out such multiple solutions, as demonstrated in Figure 5. Here again only the most significant results are shown, for different values of D_{Lim}, while the loading is applied at height 15. The GA parameters are those of section 4.3, except for the population size (100) and the number of generations (500).

4.5 Compliance fitness

In this section, a compliance-based fitness is used in order to compare our results with those of the homogenization method:

$$Fitness_{compliance} = \frac{1}{Area + \alpha\mathcal{C}}$$

where $\mathcal{C} = \int \mathcal{F}u$ is the energy of the external load, or *compliance* of the structure under the load $\mathcal{F}$.

The comparisons are made on the 1×2 cantilever plate using a 10×20 discretization. Figure 7 shows typical solutions of the GA approach for α taking values 1, 0.1 and 0.01. The population size is 75, runs are limited to 500 generations, other parameters are described in section 4.3.

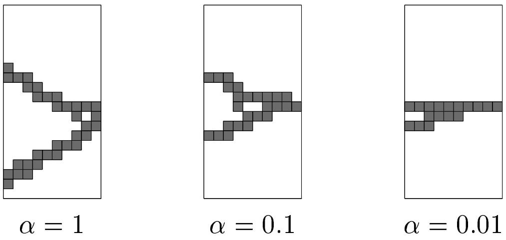

$\alpha = 1$ $\alpha = 0.1$ $\alpha = 0.01$

Figure 7: The GA solutions, for different values of α, using the compliance based fitness, on a coarse mesh.

The results of Figure 7 strongly differ from those of the homogenization method (the perfect ">" shape): Figure 8-a shows the results of the latter method, as described in Allaire and Kohn (1993), for $\alpha = 1$ on the same coarse mesh. The strength of the homogenization method lies in its convergence property, when the element size of the mesh vanishes, and whatever the parameter α, toward the optimal homogenized solution, i.e. a solution with continuous density of material between 0 (void) and 1 (pure material). However, its Achilles'heel sometimes appears when it comes to design an actual part, in which the density has to be boolean (0 or 1). The usual method of using a relaxed cost function favoring a 0/1 density at the end of homogenization might still give results far from a 0/1 material, as attested by Figure 8-a (the density of grey is proportional to the density of material of the solution). Of course, things get better when refining the mesh, and Figures 8 (b) and (c) witness that a more feasible solution can be reached easily for the 2×1 cantilever plate. But the results of Figure 7 demonstrate the ability of the GA method to provide good solutions for a given mesh, though totally different from those of the homogenization method.

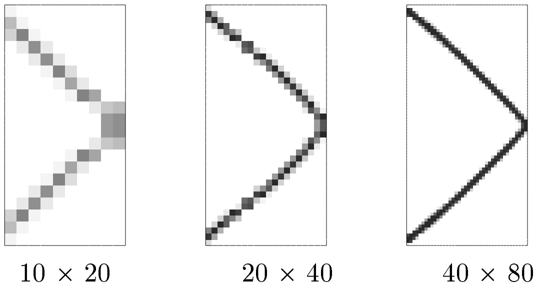

10×20 20×40 40×80

Figure 8: The homogenization solutions, for $\alpha = 1$, using different meshes.

Anyway, the aim of GA-based topological optimization is not to compete with the homogenization method, as GAs weakness is well known to be its computational cost. Next section presents results that cannot be obtained by homogenization, in a more complex mechanical background.

5 NONLINEAR GEOMETRICAL EFFECTS IN TOPOLOGY OPTIMIZATION

This section considers standard plane stress problems in the context of large displacements. The material still obeys a linear law (the extension to any other law is straightforward), but the nonlinear geometric effects due to the large displacement hypothesis are taken into account. A thorough description of the theoretical model together with the numerical algorithm can be found in Ciarlet (1988). Details on the FEM model and implementation used here can be found in Jouve (1993).

The first experiments use the penalized fitness function described in section 3.3. The idea was to use different loadings $\mathcal{F}$, with a fixed ratio $\mathcal{F}/D_{Lim}$ (D_{Lim} is the constraint on the displacement). In the purely linear case, all such problems are of course equivalent.

A typical (best) result of the GA based optimization in the large displacement context is shown in Figure 9-b, together with the result on the same problem in the pure linear model (Figure 9-a). And different values of $\mathcal{F}$ and D_{Lim} do produce similar results. But a closer look at the maximal stress of the solutions σ_{Max} gives some hint on what is hapenning: First, the large displacement model can have different solutions for the same loading, some with higher stress than others. Second, the stress field itself, on a rough domain like the structures obtained for the standard cantilever plate problem, and with such coarse discretization, presents some singularities. The value of σ_{Max} in Figure 9-b witnesses that phenomenon.

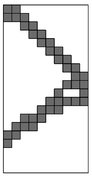
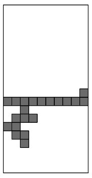
		
D_{Max}	0.022607	0.0199
σ_{Max}	0.076	0.77
$Area$	0.41	0.20
	(a): small displacements.	(b): large displacements.

Figure 9: Optimal designs with displacement constraints. $\mathcal{F} = 0.009$ and $D_{Lim} = 0.02285$.

Moreover, Table 1 gives some idea of the nonlinear geometrical effects in the case of two simple structures, the perfect ">" shape and the straight beam; both the displacement and the maximum stress are given for varying loads. The displacement is what was expected, but the stress does present weird values.

	">" shape		straight beam	
Load	D_{Max}	σ_{Max}	D_{Max}	σ_{Max}
910^{-6}	0.0002360	0.0005968	0.04161	0.03035
910^{-5}	0.00235	0.0059	0.34499	0.28344
910^{-4}	0.02286	0.05312	0.80763	1.272
910^{-3}	1.06731	10.99	1.08208	1.11683
910^{-2}	1.249	2.0763	1.6871	4.3787
910^{-1}	2.9574	break	3.177	break

Table 1: Nonlinear effects on two reference shapes. D_{Max} and σ_{Max} are respectively the maximum displacement and the maximum stress.

This suggests to incorporate a constraint on the maximal stress in the fitness function. Of course, some nice structures might be missed, like the ">" shape for $\mathcal{F} = 910^{-3}$. But, hopefully, another structure very similar to it can arise, with the same mechanical properties, and with the FEM model giving a reasonable solution among the possible solutions. Following these ideas, the fitness becomes

$$\frac{1}{Area + \alpha(D_{Max} - D_{Lim})^{+} + \beta(\sigma_{Max} - \sigma_{Lim})^{+}}$$

where σ_{Lim} is the constraint on the stress, σ_{Max} is the maximum of the stress on the structure, and β is some positive penalty parameter which has to be adjusted. Figure 10 shows the optimal designs obtained with that modified fitness: Reasonable solutions are found, as long as the stress is imposed a strong constraint.

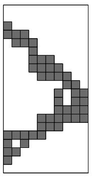
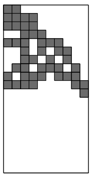
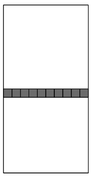
			
$\mathcal{F}$	0.009	0.018	0.09
D_{Lim}	0.22856	0.457	2.2856
σ_{Lim}	0.53	1.0622	5.3
D_{Max}	0.2143	0.4504	1.687
σ_{Max}	0.550	0.9835	4.379
$Area$	0.21	0.47	0.1

Figure 10: Optimal designs for nonlinear elasticity with displacement and stress constraints.

6 CONCLUSION AND PERSPECTIVES

The feasibility of GA-based optimal design has already been witnessed by Jensen (1992) and Chapman et al. (1994). This paper intends to emphasize on its flexibility:

When multiple quasi-optimal solutions exist, all can be obtained by this method. Actually, these were found by successive runs of the GA starting from different random populations. But it is possible, using niching techniques like the sharing scheme of Goldberg (1989), to locate in a single run more than one near-optimal solutions.

In the context of large displacements, the experiments presented in this paper acknowledge for the usual "the extension to other mechanical models is straightforward", though the fitness function had to be adjusted in order to take into account the maximal stress of the structure: there is no free lunch for nonlinearity.

Many problems remain to be addressed. The tuning of the penalization parameter, even in the context of iterated fitness functions presented in this paper, remains to be done for every new problem. The idea of evolving this parameter together with the current population will be investigated.

The comparison with the results of the homogenization method shows the need to use finer meshes. The separation of the design domain into sub-domain proposed by Chapman et al. (1994) is a promising direction. Some iterated GA on the same population of designs, the mesh being gradually refined, will also be tested.

Further research also include straightforward (!) modifications of the fitness function, to handle some multiple loading problems, and to extend this work to any nonlinear material.

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