Shape Representations and Evolution Schemes

Marc Schoenauer
CMAP – URA CNRS 756, Ecole Polytechnique
91128 Palaiseau Cedex, France
Marc.Schoenauer@polytechnique.fr

Abstract

The choice of a representation i.e. the definition of the search space, is of vital importance in all Evolutionary Optimization processes. In the context of Topological Optimum Design in Structural Mechanics, this paper investigates possible representations for evolutionary shape design. The goal is the identification of a shape in $\mathbb{R}^n$ ($n = 2$ or $n = 3$) having optimal mechanical properties. Evolutionary Computation has been demonstrated a valuable tool for TOD problems. However, all past results are based on the straightforward bitstring representation whose complexity increases with that of the underlying Mechanical model. To overcome this difficulty, different representations for shapes are introduced, and compared on the benchmark problem of TOD using various evolutionary schemes. The results are discussed with respect to the different degrees of epistasis of the representations.

1 Introduction

Representation is long acknowledged a central issue for Evolutionary Computation. From early discussions on the now out-of-date [Michalewicz 1992] problem of encoding real parameters in GAs (e.g. binary vs Gray coding [Caruna and Schaffer 1988]) to recent works on comparing representations for the TSP problem [Radcliffe and Surry 1994], EC researchers have tried to characterize the desirable properties of the mapping from the genotype space, where evolution takes place, into the phenotype space, where environmental pressure acts [Fogel 1995a, Fogel 1995b]. However, most of these works either present general recommendations and heuristics (e.g. the degree of degeneracy in the representation should be as small as possible to avoid loosing information) or focus on fixed-length representations (e.g. binary encoding for the TSP problem).

This paper studies variable-length representations for two- or three-dimensional shapes, in the field of Topological Optimum Design (TOD): the goal is to find a structure (a shape in a given *design domain*) having prescribed mechanical properties and minimal weight. The straightforward representation for shapes that has been exclusively used in past evolutionary attempts on Optimum Design problems [Jensen 1992, Chapman, Saitou and Jakiela 1994, Chapman and Jakiela 1995, Kane, Jouve and Schoenauer 1995] is a bitstring representation based on a mesh of the design domain. However, this representation hardly scales up, neither when considering 3-D shapes, nor when the accuracy of the mechanical behavior of the structure (depending on the size of the mesh) becomes a central issue.

Therefore two other representations are designed; both allow to dissociate the representation of the shape and the accuracy of the fitness computation. These representations are variable-length representation with a high degree of degeneracy; nevertheless preliminary results show they significantly outperform the (non-degenerate) bit-array representation.

Forthcoming section 2 presents the Mechanical background of the Optimum Design problem and briefly recalls the results obtained by standard deterministic approaches.

Section 3 describes how Evolutionary Computation addresses some limitations of these standard methods, by handling 2-D shapes as bitstrings (rather standing for bit-arrays). Specific evolution operators have been designed to overcome the geometrical bias of standard operators. Nevertheless this representation is intrinsically limited in the sense that the accuracy of the fitness is linked to the size of the chromosome.

Section 4 therefore introduces two new representations for 2-D and 3-D shapes, overcoming the above limitation. These are experimentally studied on a benchmark problem of TOD, and compared using the three main evolutionary schemes, namely GAs, EP and ES. Our results suggest the performance depends on two main features: the degree of epistasis of a representation, i.e. the degree the expression of one gene depends on the other genes, and the symmetry of the representation with respect to that of the optimal solutions.

Last, some avenues for further research are presented.

2 Topological Optimum Design

2.1 Background

Optimum Design in Structural Mechanics consists in finding the best design for a structure in an initial domain of $\mathbb{R}^2$ or $\mathbb{R}^3$.

More precisely, identifying a shape amounts to find a partition of the *design domain* (a subset of $\mathbb{R}^n$, $n = 2$ or $n = 3$), into two subsets; one subset is the structure while the other subset represents void[1].

The optimality criterion is determined by the mechanical properties of the structure, depending on the constitutive law (behavior) of the material; only hyper-elastic materials [Ciarlet 1978] will be considered in the following.

In most cases of Optimum Design, the goal is to minimize the weight of the structure (i.e. the amount of material), while meeting some engineering requirements for given loading cases (e.g. forces, pressure, prescribed displacements, ...). This problem is of utter importance for part manufacturers, as a small decrease in the weight of some widely used part results in large cuts in the manufacturing cost.

2.2 Deterministic state of the art

Two contexts are distinguished in Structural Optimum Design:

- When the solution is sought as the continuous deformation of a given initial shape, iterated small modifications of the shape are a deterministic way toward the solution. The methods of *domain variation*, or *sensitivity analysis* [Cea 1981], are based on gradient-like optimization techniques in that context.

- But continuous deformations do not affect the *topology* of the structure (i.e. its number of holes).
  When the topology of the solution is unknown, the problem amounts to *Topological Optimum Design* (TOD); the only deterministic method in that context, to the best of the author's knowledge, is the *homogenization method* [Bendsoe and Kikushi 1988, Allaire and Kohn 1993]. Homogenization proceeds by first relaxing the problem and considering probabilistic shapes in the design domain: the density of material ranges in $[0, 1]$ instead of being either 0 (for void regions) or 1 (for the structure itself). Theoretical results ensure that the optimal solution lies in this superset of probabilistic shapes, and can be approximated by deterministic gradient-based methods.

[1] A formally equivalent problem is that of inclusion identification [Constantinescu 1994], where the goal consists of identifying the repartition of two materials with different mechanical properties from the global behavior of the material.

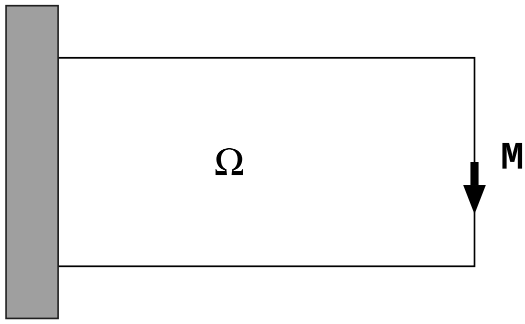

Figure 1: *A TOD benchmark problem: The* 2×1 *cantilever plate. The design domain is fixed on its left boundary and a force is applied at point M.*

The optimal probabilistic shape is then mapped into a real shape. Spectacular results in two and three dimensions have been obtained by the homogenization method [Allaire & al. 1996]. The greatest limitation of this approach is that dealing with probabilistic shapes is possible so far in the frame of linear elasticity only. Moreover, even in this restricted frame, homogenization method can handle neither multiple loading cases, nor loading applied on the unknown boundary (e.g. a uniform pressure on one side of the structure).

2.3 A TOD benchmark

The classical benchmark problem of topological optimization in two dimensions is the cantilever plate, described in Figure 1: a plate is fixed on one of its boundary, and one (or more) punctual forces are applied on the other end of the design domain. The goal is to minimize the weight of the structure while staying below some upper bounds on the maximal displacement of the points where the forces are applied.

A shape is therefore evaluated from two criterions: its weight (or equivalently its area in the two-dimensional case), and its mechanical behavior under the prescribed loading. The numerical simulation of the mechanical behavior of the structure is achieved using the Finite Element Method (FEM) [Zienkiewicz 1977, Ciarlet 1988], which implies a discretization of the shape into small elements, termed *mesh*.

Formally, the TOD problem is a constrained problem: one aims at minimizing the weight of the shape, while satisfying the prescribed mechanical requirements.

3 "Standard" Evolutionary TOD

This section briefly describes to-date results obtained on TOD problems in the frame of Evolutionary Computation.

3.1 Bitarray Representation

As stated above, the computation of the fitness of a shape involves *meshing* that shape. In that context, the most natural representation for shapes is the bit-array representation, directly based on a fixed regular mesh of the design domain into rectangular elements. Each element of the mesh is given a boolean value (0 or 1) indicating which subset it belongs to (material of void). This representation is straightforward from the point of view of both FEM (this is the simplest possible mesh of the domain) and GAs (the representation can be viewed as a bitstring). All previous stochastic works addressing the topological optimum design problem have adopted that representation, be they based on simulated annealing [Ghaddar, Y. Maday, and A. T. Patera 1996] or GAs [Jensen 1992, Chapman, Saitou and Jakiela 1994, Chapman and Jakiela 1995].

3.2 Evolution Operators

However, standard genetic operators appear ill-suited for the above representation of shapes, in the sense they suffer from geometrical bias. Bit-arrays are not bit-string: one-dimensional operators can only exchange horizontal parts of the design domain; if the good schemata involve vertical bands of the structure, these tend to be disrupted by any one-dimensional crossover, and the *building block* hypothesis therefore does not apply.

Two-dimensional crossover operators, exchanging two-dimensional regions of the shapes (e.g. rectangular blocks or regions separated by random lines), have therefore been purposely designed. It has been demonstrated on the cantilever plate problem that these specific two-dimensional crossover operators clearly outperform the standard one- and two-points bitstring crossover operators [Kane and Schoenauer 1995].

3.3 Constrained Optimization

Many methods have been designed to constrained evolutionary optimization (see [Michalewicz 1995] for a survey of such methods). In particular, the method described in [[Schoenauer and Xanthakis 1993] has been successfully applied to another problem of structural mechanics, the optimization of truss structures. But as the focus of this work is representation, the standard method of penalization was chosen, being a non specific robust way of handling constraints.

The first draft for the fitness function thus has the following expression:

$$\mathcal{F} = Area + \alpha(D_{Max} - D_{Lim})^{+} \qquad (1)$$

where D_{Max} is the maximal displacement of the structure when the prescribed force is applied (computed using the FEM), D_{Lim} the imposed limit value for the displacement and α is a positive user-supplied penalty parameter (a^{+} denotes the positive part of a). Though some mechanical difficulties in fact lead to slightly more complex expression of the fitness (e.g. structures not connecting the fixed boundary and the loading are not valid solutions, see [Kane and Schoenauer 1995, Schoenauer 1995]), equation (1) will be considered in the following for the sake of simplicity.

3.4 Successes

Using specific two-dimensional evolution operators on bitarray representation, EC has successfully tackled TOD problems that could not be addressed by other techniques [Kane, Jouve and Schoenauer 1995, Kane 1996] — at the expense of large computational time.
The most significant results are the following:

- Overall, EC can accommodate **any mechanical model** for which there exists a numerical simulation algorithm. This is confirmed as the results obtained for the large displacement model appear to be the first results in Optimum Design of structures in non-linear elasticity.

- The optimization can take into account more than one loading case (as in the design of a bicycle), as well as loading applied on the unknown boundary of the structure (as in the case of the underwater dome).

- In some situations, many optimal, or near-optimal solutions exist. Evolutionary algorithms, using for instance the sharing scheme [Goldberg and Richardson 1987], are able to provide the engineer with a range of such solutions, allowing him to take into account inarticulate criterions.

3.5 Drawbacks

The limitations of these results come from the following fact.

The accuracy of Evolutionary TOD is dictated by the size of the mesh underlying the FEM analysis and the fitness computation: the above mentioned results were obtained on rather coarse meshes (e.g. 10×20) whereas real-world problems and accurate analyses of the mechanical behavior of the shapes require much finer meshes (e.g. 100×200).

Increasing the size of the mesh would not only increase the cost of the fitness computation (which is roughly quadratic in the size of the mesh) — but also the size of the chromosomes. And increasing the size of the individuals would require to increase in turn the size of the population and the number of generations to reach the same level of convergence (Cerf [Cerf 1994, Cerf 1996] proved that the minimal size of the population for a GA to converge increases linearly in term of the size of the bitstring).

Finally the bit-array representation poorly scales up when refining the mesh — not to speak of handling 3-D shapes...

Therefore other representations have been designed in order to overcome this limitation, and dissociate the complexity of the representation and the accuracy of the evaluation.

4 Variable-length representations for shapes

4.1 The Voronoï representation

A possible way of representing shapes comes from computational geometry, more precisely from the Voronoï diagram theory. The ideas of Voronoï diagrams are already well-known in the FEM community, as a powerful tool to generate good meshes [George 1991]. However, the representation of shapes by Voronoï diagrams and their evolutionary optimization seems to be original.

4.1.1 Voronoï diagrams

Consider a finite number of points $V_0, \ldots, V_N$ (the *Voronoï sites*) of a given subset of $\mathbb{R}^n$ (the design domain). To each site V_i is associated the set of all points of the design domain for which the closest Voronoï site is V_i, termed *Voronoï cell.* The *Voronoï diagram* is the partition of the design domain defined by the Voronoï cells. Each cell is a polyhedral subset of the design domain, and any partition of a domain of $\mathbb{R}^n$ into polyhedral subsets is the Voronoï diagram of at least one set of Voronoï sites (see [Preparata and Shamos 1985, Boissonnat and M. Yvinec 1995] for a detailed introduction to Voronoï diagrams, and a general presentation of algorithmic geometry).

Consider now a (variable length) list of Voronoï sites, each site being labeled 0 or 1. The corresponding Voronoï diagram represents a shape (a partition of the design domain into two subsets), if each Voronoï cell is labeled as the associated site (here the Voronoï diagram is supposed regular, i.e. to each cell corresponds exactly one site). Example of Voronoï representations can be seen in Figure 2. The Voronoï sites are the dots in the center of the cells. Note that Voronoï representation of shapes does not depend in any way on the mesh that will be used to compute the behavior of the shapes. Furthermore, Voronoï diagrams being defined in any dimension, the extension of this representation to $\mathbb{R}^3$ and $\mathbb{R}^n$ is straightforward.

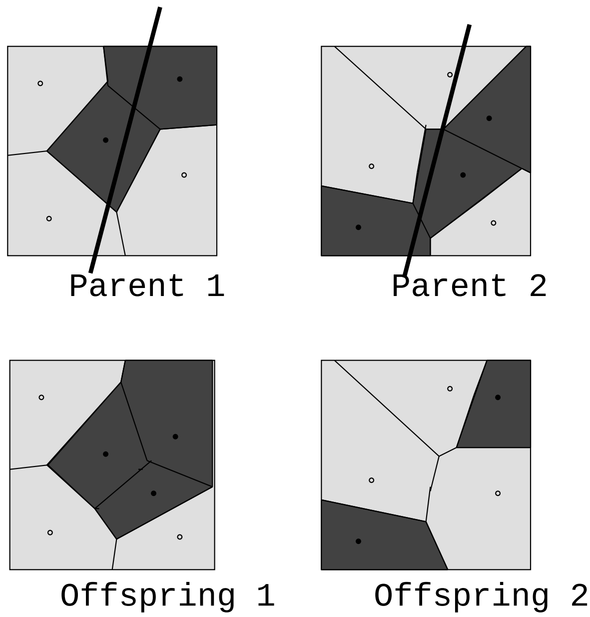

Figure 2: *The Voronoï representation crossover operator. A random line is drawn across both diagrams, and the sites on one side are exchanged*

4.1.2 Evolution operators

The evolution operators on the Voronoï representation are inspired from both the two-dimensional crossover operators designed for the bit-array representation and the usual operators for variable length representations:

- The crossover operators exchange Voronoï sites on the basis of geometrically-based choice. In this respect it is similar to the specific bitarray crossover described in [Kane and Schoenauer 1995]; moreover, this mechanism easily extends to any dimension [Kahng and Moon 1995]. Figure 2 demonstrates an application of this crossover operator.
- a first mutation operator performs a Gaussian mutation on the coordinates of the sites, or randomly flips the boolean attribute of some sites;
- a "standard" mutation for variable-length representations adds or deletes some sites from the list.

An important remark is that this representation presents a high degree of *epistasis* (the influence of one site on the physical shape is modulated by all neighbor sites). This will be discussed in more details in section 5.1.

Practically, the fitness of all shapes is evaluated using the same fixed mesh[2]. A shape described by Voronoï sites is thus mapped on this fixed mesh: the subset (material

[2]This intends to limit the bias due to the unavoidable numerical noise of FEM (the finer the mesh, the lower the numerical error,

or void) an element belongs to is determined from the label of the Voronoï cell in which the center of gravity of that element lies.

4.2 H-representation

Another representation for shapes is based on an old-time heuristic method in TOD: from the initial design domain, remove material where the mechanical stress is minimal, until the constraints are violated. However, the lack of backtracking makes this method useless in most TOD problems. Nevertheless, this idea gave birth to the "holes" representation [Dejonghe 1993], later termed H-representation.

4.2.1 The representation

The design domain is by default made of material, and a (variable length) list of "holes" describes the topology of the structure. These holes are elementary shapes taken from a library of possible simples shapes. Only rectangular holes are considered at the moment. On-going work [Seguin 1995] is concerned with other elementary holes (e.g. triangles, circles).

Example of structures described in the H-representation are presented in Figure 3. The rectangles are taken in a domain larger than the design domain, in order not to bias the boundary parts of the design domain.

4.2.2 Evolution operators

The evolution operators are quite similar to those of the Voronoï representation:

- crossover by geometrical (2D or 3D) exchange of holes (see Figure 3 for an example);
- mutation by Gaussian modification of the characteristics (coordinates of the center, width and length) of some holes;
- mutation by addition or deletion of a hole;

The H-representation, as the Voronoï representation, is independent from any mesh, and hence its complexity does not depend on any required accuracy for the simulation of the underlying physical phenomenon. Its merits and limitations will be discussed in the light of the experimental results presented in next section.

As for the Voronoï representation, the simulated behavior of the shapes is computed on a given fixed mesh, to limit the numerical noise due to re-meshing. The criterion to decide which subset an element does belong to,

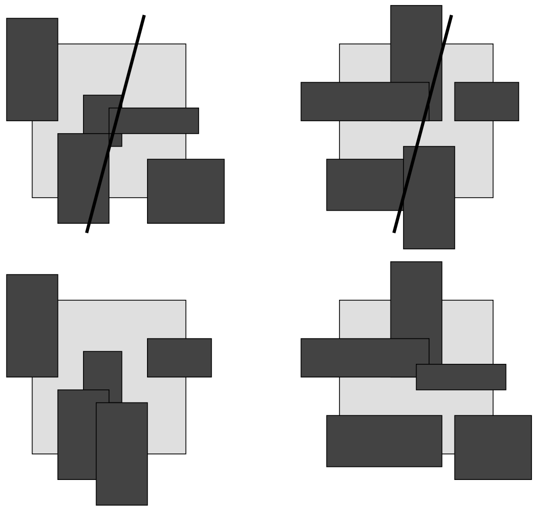

Figure 3: *The H-representation crossover operator. A random line is drawn across both structures, and the holes on one side are exchanged.*

is based on whether its center of gravity belongs to a hole (in which case the whole element is void) or not.

4.3 Preliminary comparative results

The same benchmark problem of Optimal Design has been used for the first comparative results for the three representations of shapes presented in preceding section (see [Schoenauer 1995]). As expected, and even for a rather coarse mesh, the bit-array representation is outperformed by all other representations in terms of computational cost as well as in terms of quality of the solution. Hence, it will not be considered any more in the rest of the paper.

Both the Voronoï representation and the H-representation derive a quasi-optimal solution. But these solutions are more rapidly found as H-based than Voronoï -based shapes.

Tentative explanations for that are proposed in next section.

5 Epistasis and Evolutionary Schemes

5.1 Epistasis and Symmetry of Representations

In the biological context, the epistasis is a measure of how the expression in the phenotype of one single gene is influenced by the other genes of the genotype. In Evolutionary computation, epistasis has a strong influence on the way genetic material is transmitted from parents to offspring, and how it can be modified by evolution operators thereafter applied.

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the higher the computational cost). Hence, the fitnesses of different structures that are to be compared should be performed with the same mesh. Otherwise, numerical noise due to re meshing might hide the actual differences in the mechanical behavior of different structures.

- In the bit-array representation, the contribution of a given bit to the phenotype is clear, and it cannot modified by any other bit. The epistasis is here minimal, all genetic material transmitted to the offspring is expressed, i.e. is dominant. Such a situation can be termed *strong transmission*.
- In the Voronoï representation, the influence of a site on the final phenotype can be greatly modified by other neighbor sites. If one site labeled *void* becomes surrounded by sites labeled *material*, its influence on the final phenotype almost vanishes. Some part of genetic information transmitted to offspring is recessive, and, by contrast to the bit-array situation, this case will be termed *weak transmission*.
- In the H-representation, the situation is even more complex: Only holes are strongly transmitted. When a hole is transmitted from parent to offspring, the whole area will stay a hole. On the opposite, no such strong transmission is achieved for "non-holes". The transmission is *asymmetric*, strong for the hole value, and weak for the default value (i.e. any hole covering the same area changes its value). One can say the hole-value represent dominant genetic material, while the default value is recessive.

The bottom-up approach of GA, trying to recombine *building blocks* to reach the optimal solution should be much more hindered by epistasis than the top-down approach only relying on the competition between phenotypes to reach an optimal point of the search space. In order to check these points, experiments were conducted on the benchmark cantilever problem (section 2.3) using the two different approaches of EC.

5.2 Evolutionary Schemes

Evolutionary Algorithms (EA) crudely mimic the evolution of a population of points of the search space. This population is usually initialized randomly, and undergoes a succession of *generations*. The general outline of a generation can be viewed as

SELECT `parents from the population`

APPLY `evolutionary operators to the selected parents to generate offspring`

REPLACE `some parents by some offspring`

The most widely used EAs, namely Evolutionary Programming [Fogel, Owens and Walsh 1966], Evolutionary Strategies [Schwefel 1981] and Genetic Algorithms [Holland 1975], are instances of this general scheme:

- EP and ES do not use initial selection process while GA uses fitness-based stochastic selection (e.g. roulette wheel).
- EP uses mutation only, each parent generating one offspring, ES uses mostly mutation, each parent generating usually more than one offspring and GA use mostly recombination (crossover), two parents generating two offspring.
- EP replaces the parents using a stochastic tournament among parents and offspring, ES selects the best individuals among parents and offspring in $(\mu + \lambda) - ES$ or among offspring only in $(\mu, \lambda) - ES$ as the new parents, and GA globally replaces all parents with all offspring.

Of course, numerous variations of these canonical algorithms exist and are being used in practical applications on a pragmatic basis [Michalewicz 1992, Fogel 1995a, Bäck 1995]. However, the *a priori* adequation of an evolutionary scheme to a given application is still an open question.

5.3 Experimental results

Experiments were conducted using two evolutionary schemes:

- The standard generational GA, with population size of 100, using ranked-based selection, crossover rate of 0.6, mutation rate per individual of 0.2, with Gaussian mutations of fixed variances.
- A (15+100)-ES (15 parents generate 100 offspring, the best 15 among parents + offspring become the parents of the next generation) with strength of Gaussian mutations depending on the fitness of the individual at hand as in EP [Fogel 1992].

In both cases, the maximum number of generations allowed is set to 100, and the algorithm stops if no improvement is observed during 10 generations. These stopping criterions are severe, and most runs did not reach convergence. But, in contrast with [Kane, Jouve and Schoenauer 1995, Kane 1996], the goal here is to observe the behavior of the algorithms with the perspective of fast convergence, rather than to reach the optimal shape by all means.

As one of the goals is to study the impact of the symmetry of the representations on the evolutionary algorithm, three representations are considered: the Voronoï representation (section 4.1), the "Hole" representation and the "Plate" representation, which are both H-representation (section4.2), where the default value is material in the case of the Hole-representation, and void in the case of the Plate-representation (the structure is here an assembly of small plates).

In order to test the influence of both the epistasis and the symmetry of the representation interacting with the

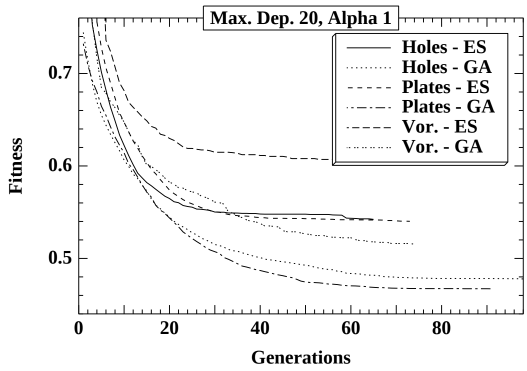

Figure 4: *.Comparative results between combinations of representations and evolutionary schemes: Averaged (over 30 independent runs) best fitness along generations. The constraint on the displacement is very strong (limit value 0.2) and the penalty parameter α is small.*

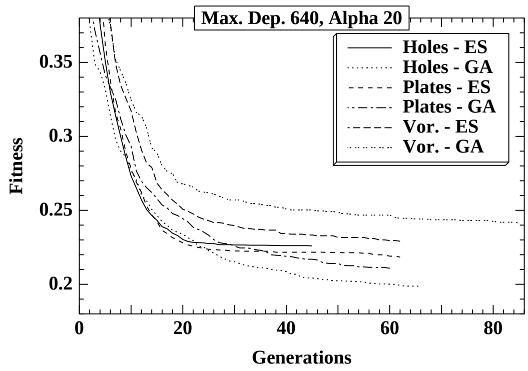

Figure 5: *Comparative results between combinations of representations and evolutionary schemes: Averaged (over 30 independent runs) best fitness along generations. The constraint on the displacement is weak (limit value 6.40) and the penalty parameter α is large.*

evolution scheme, different instances of the cantilever plate problem described in section 2.3 are considered.

For all problems, the design domain is the 2×1 rectangle, the mesh is a 20×10 regular mesh (i.e. all elements are here squares) and the same force is applied at point $(2, 0.5)$ (point M in Figure 1).

The maximum value of the displacement is assigned different values, increasing from 0.2, the actual displacements of the full horizontal beam joining point M and the fixed boundary of thickness 0.8 to 37.10, the actual displacement of the full beam of thickness 0.1. Note that only in this latter case is the optimum known: due to the coarse mesh used here, the minimal structure connecting point M and the fixed boundary is the full beam of thickness 0.1, which does indeed respect the constraint.

Finally, the penalty parameter is varied too: intuitively, the higher the penalty factor, the more difficult the problem (feasible regions are environed by steep slopes).

It is fair to say that only tendencies could be observed, and no absolute conclusion can be drawn.

1. Both H-representations globally outperform the Voronoï representation. So the high degree of epistasis seems to penalize the representation here.

2. ES and GA are hardly distinguishable, except
   – when the limit value for the displacement is small (strong constraint), in which case GA significantly outperforms ES for all representations (see Figure 4). However, this phenomenon decreases when the penalty parameter increases. A possible explanation could be that the elitist ES gets stuck more easily in the first feasible local optimum for small values of the penalty parameter, whereas both algorithms encounter this same difficulty for large values of the penalty parameter.
   – for the Voronoï representation with large limit for the displacement (weak constraint) in which case ES performs better than GA (see Figure 5). This is the only hint meeting the a priori expectation that epistasis should favor the top-down against the bottom-up approach.

3. The “Hole” representation demonstrates slightly better performances than the “Plate” representation in almost all cases (e.g. in Figure 5) except when the optimal solution contains about as much material as void, as can be seen in Figure 4: a value of 0.5 for the fitness of feasible structures corresponds to exactly the same amount of void and material.

6 Further directions for the comparison of representations

Overall, these experiments show that far too many parameters are involved in the TOD problem to make it a good test-bed for shape representation ! A next step will be to design such an adequate test-bed, as an unconstrained problem the solution of which is known, with tunable amount of void and material, and scalable degree of difficulty. On-going work is concerned with the problem of (non-destructive) identification of the repartition of two materials in a structure (e.g. scories in a steel piece) using mechanical experiments [Constantinescu 1994].

Some criterions investigated in the literature will guide systematic experiments:

- The fitness variance theory of Radcliffe [Radcliffe and Surry 1994] studies the variance of the fitness as a function of the order of an extension of schemas called *formae* [Radcliffe 1991], and, simply put, shows that the complexity and difficulties of evolution increases with the average variance of the fitness. But if the formae and their order (or their precision) are well-defined on any binary representation, including the bit-array representation of section 3, it is not straightforward to extend these definitions to variable length representations discussed above.

Moreover, Radcliffe's fitness variance does not take into account the possible evolution operators. Further step in that direction would be to study the variance of the change of fitness with respect to a given evolution operator (e.g. the Gaussian mutation of Voronoï sites for different standard deviations), as in [Fogel 1995b]

- The fitness distance correlation of Jones and Forrest [Jones and Forrest 1995] studies the correlation between the distance to the optimal point and the fitness. Simply put again, the idea is: the stronger this correlation, the narrower the peak the optimum belongs to, and the more difficult the problem. Conjectures based on this remark are experimentally confirmed in the GA-frame. Nevertheless, the difficulty in shape representation is to define a distance which is meaningful for both the representation and the problem at hand. The first important issue is whether the considered distance should be purely genotype-based (i.e. defined on the coded individual only) or partially or totally phenotype-based (i.e. defined on the same space than the fitness function).

7 Conclusion and further work

Different evolutionary approaches for shape optimization have been presented. The emphasis has been put on the representation: the simple bit-array representation allowed significant advances in the domain of topological shape optimization (e.g. the first results in nonlinear elasticity), but hardly scales up, in the sense that the accuracy of the fitness computation is commanded by the size of the individuals.

Two other, variable-length, representations, the Voronoï and the H- representations have been designed to overcome this issue; they demonstrate good results on the Topological Optimum Design problem, significantly and consistently outperforming the bitarray representation.

The main difference between those latter representations a priori appears their respective degree of epistasis, i.e. the way one gene (Voronoï site or hole) may modify the phenotypic traits due to other genes. Systematic experiments have been conducted to see how the degree of epistasis interacts with the evolutionary scheme (bottom-up as in GAs or top-down as in ES) and the symmetry of the representation ... and mainly demonstrate that the classical benchmark in Topological Optimum Design is a problem far too complex to serve as a test-bed for evaluating shape representations.

A number of powerful representations of shapes remain to be investigated in the frame of evolutionary optimization:

- The CAD community uses splines defined from control points to describe and manipulate shapes.
- The L-system paradigm [Prusinkiewicz and Lindenmeyer 1990] uses grammar rules to simulate plant growth. The resulting "plants" can be viewed as shapes. Moreover, it has been demonstrated that such grammar rules can be optimized using Evolutionary Computation algorithms [Simms 1994].
- The fractal theory of Iterated Function Systems [Barnsley 1988] offers another possible representation, for which the inverse problem has been successfully tackled by means of Evolutionary Computation [Garigliano & al. 1993, Lutton and Martinez 1994].

It is emphasized however, that a key problem of evolutionary computation yet is the choice of an adequate representation. Widening this choice would only ask more loudly for judicious choice criterions; and it is our conviction that the first, and maybe the more important step on this way, would be to elaborate a convenient test-bed, of tunable difficulty.

On-going work considers another shape identification problem to this end, namely the identification of inclusions (e.g. scories in a piece of steel) [Constantinescu 1994]. This latter problem presents two main advantages: it is unconstrained and fully tunable (the optimal solution can be fixed a priori).

Acknowledgements

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