HAL open science

A non-linear impedance model for micro-perforated liners

Robin Billard, Gwenael Gabard, Marc Versaevel, Gilles Tissot

To cite this version:

Robin Billard, Gwenael Gabard, Marc Versaevel, Gilles Tissot. A non-linear impedance model for micro-perforated liners. e-FA 2020 - e-Forum Acusticum, Dec 2020, online, France. pp.1537-1542, 10.48465/fa.2020.0230 . hal-03143278

HAL Id: hal-03143278
https://hal.inria.fr/hal-03143278

Submitted on 5 May 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A NON-LINEAR IMPEDANCE MODEL FOR MICRO PERFORATED LINERS

R. Billard[1,2] **G. Gabard**[1] **M. Versaevel**[2]
G. Tissot[3]
[1] LAUM UMR - CNRS 6613, av. Olivier Messiaen, 72085 Le Mans
[2] Safran Nacelles, Route du Pont VIII, 767000 Gonfreville-l'Orcher, France
[3] IRMAR - UMR CNRS 6625, av. Général Leclerc, 35042 Rennes, France
Robin.Billard.Etu@univ-lemans.fr

ABSTRACT

Perforated plate liners are widely used to reduce noise emissions from turbofan engines. Micro-perforated plates (MPP), with a porosity below 5% and apertures diameter below $1\,\mathrm{mm}$, are of particular interest for this purpose since they also help to reduce the flow drag, compared to conventional perforates. In the nonlinear regime without flow, existing models such as those of Guess or Maa are not suitable for MPP liners, *i.e.* when the acoustic velocity becomes high in the perforations. The effective velocity through the perforate is a function of the impedance and conversely. Therefore an iterative procedure is required to obtain accurate impedance predictions in the non-linear regime. In the present study, we propose to modify Guess and Laly impedance models by implementing an iterative procedure to compute the acoustic velocity. Good correspondence with impedance tube measurements is observed for sound pressure level up to $150\,\mathrm{dB}$ for MPP with a porosity and a neck diameter down to 1.4% and $0.3\,\mathrm{mm}$ respectively. The modified model can be adapted to predict the impedance when either a white-noise source or a sine-sweep source is used in the experimental setup. In addition, we illustrate that considering only the absorption coefficient is misleading and that it is important to assess the resistance and reactance to properly evaluate the accuracy of an impedance model.

1. INTRODUCTION

Perforated plate liners are a widespread technology to attenuate noise productions from jet engines, which can reach high Sound Pressure Level (SPL) up $150\,\mathrm{dB}$. The perforations of the liners are in contact with the air flowing through the engine, increasing therefore the flow drag. By reducing the perforations diameter, the equivalent surface roughness of the liner decreases and so does the flow drag. Hence, the micro-perforated plate liners with a porosity below 5% and holes diameter below $1\,\mathrm{mm}$ are of interest for this aeronautic application. Semi-empirical models such as those of Guess [1] or Maa [2] include a non-linear term based on measurement for different acoustic velocity in the perforations of the liner. However, the acoustic velocity depends on the unknown impedance and conversely. In the case of macro-perforated plate liners, the acoustic velocity can be approached correctly by using the air impedance. But in the case of micro-perforated liners, the acoustic velocity is to high to be approximated using this hypothesis. Therefore, it is necessary to determine an impedance/velocity couple with an iteration procedure. This procedure was performed by Beck [3] on a dual resonance acoustic liner and more recently proposed in Laly's impedance model [4] for micro-perforated liners based on a fluid equivalent approach and adding a non-linear term proposed in [5] to the resistivity. In the present paper, we propose to modify Guess model by implementing an iterative procedure on the acoustic velocity. The iterative procedure is also implemented for Laly's model. Comparisons with impedance tube measurements are performed with either a sine-swept source or a white noise source. It is shown that both models can be adapted to the source type.

2. SEMI-EMPIRICAL MODELS

The semi-empirical models from Guess [1] and Laly [6] are introduced in this section.

2.1 Guess model

Without background flow, the explicit expressions provided from Guess for the resistance Θ and the reactance χ are

$$\Theta = \mathrm{Re}\left(z_\nu\right) + \frac{2\pi^2}{\sigma}\left(\frac{R_{\mathrm{neck}}}{\lambda}\right)^2 + \frac{1-\sigma^2}{\sigma}\frac{|u_0|}{c_0}, \quad (1)$$

and

$$\chi = \mathrm{Im}\left(z_\nu\right) - \cot\left(kL\right) + \frac{\omega\delta}{\sigma c_0} \quad (2)$$

such that the impedance of the liner is

$$z = \Theta + \mathrm{j}\chi. \quad (3)$$

In equations (1) and (2) ω is angular frequency, R_{neck} is the radius of the perforation, σ is the porosity, L is the height of the cavity, c_0 is the sound speed, $k = \omega/c_0$ is the wave number, λ is wavelength and u_0 is the acoustic velocity in a perforation. Here, z_ν is the non-dimensional impedance resulting from viscous and mass effects in the apertures derived by Zwikker and Kosten [7]

$$z_\nu = -\frac{\mathrm{j}\omega h}{\sigma c_0}\left(\frac{2\mathrm{J}_1(KR_{\mathrm{neck}})}{KR_{\mathrm{neck}}\mathrm{J}_0(KR_{\mathrm{neck}})} - 1\right)^{-1}. \quad (4)$$

$K = -\mathrm{j}\omega/\nu$ is the Stokes wave number, ν is the kinematic viscosity and h is the thickness of the plate. J_0 and J_1 are the zeroth and first order Bessel functions, respectively, of the first kind of complex argument. The $-\cot(kL)$ term corresponds to the cavity reactance. The end correction δ is defined as

$$\delta = \frac{16R_{\mathrm{neck}}}{3\pi}\left(1-\sqrt{\sigma}\right)\left(\frac{1+5\cdot 10^3\left(\frac{|u_0|}{c_0}\right)^2}{1+10^4\left(\frac{|u_0|}{c_0}\right)^2}\right). \quad (5)$$

The $16R_{\mathrm{neck}}/(3\pi)$ term is derived from Rayleigh's integral [8] and model the radiation effect. The factor $(1-\sqrt{\sigma})$ account for the interaction between multiple perforations. Finally the term featuring the acoustic velocity u_0 account for sound amplitude effects. Guess approximates equation (4) for the high and low frequency regime, namely for $|KR_{\mathrm{neck}} > 10|$

$$z_\nu \simeq z_{\nu H} = \frac{\sqrt{2\nu\omega}h'}{\sigma c_0 R_{\mathrm{neck}}} + \mathrm{j}\left(\frac{\omega h}{\sigma c_0} + \frac{\sqrt{2\nu\omega}h'}{\sigma c_0 R_{\mathrm{neck}}}\right), \quad (6)$$

and for $|KR_{\mathrm{neck}} < 1|$

$$z_\nu \simeq z_{\nu L} = \frac{8\nu h'}{\sigma c_0 (R_{\mathrm{neck}})^2} + \mathrm{j}\frac{4\omega h}{3\sigma c_0}. \quad (7)$$

$h' = h + 2R_{\mathrm{neck}}$ is a corrected length accounting for the viscous effects occurring outside the perforation. This correction is not accounted for in equation (4). It may be noted that equation (6) is suited for macro perforated configuration. In addition, approximations (6) and (7) are not valid for the micro perforated configurations studied further below, for which $1 < |KR_{\mathrm{neck}}| < 10$. Lack of information is given to model the impedance in this range of parameters in Guess [1]. Therefore, in this set of parameters, the model is completed by implementing the following linear interpolation:

$$z_\nu \simeq z_{\nu M} = z_{\nu L}(\omega_L) + \frac{z_{\nu H}(\omega_H) - z_{\nu L}(\omega_L)}{\omega_H - \omega_L}\omega, \quad (8)$$

where $\omega_H = 100\nu/R_{\mathrm{neck}}^2$ and $\omega_L = \nu/R_{\mathrm{neck}}^2$.

2.2 Laly's model

Laly's model [6] is derived using an equivalent fluid approach. The impedance of a micro-perforated plate z_{MPP} is defined as

$$z_{MPP} = \mathrm{j}\frac{\omega h}{\sigma c_0}\alpha_{\infty nl} \quad (9)$$
$$\times\left(1 + \frac{R_t\sigma}{\mathrm{j}\omega\alpha_{\infty nl}\rho_0}\sqrt{1+\frac{4\mathrm{j}\rho_0\omega\mu\alpha_{\infty nl}^2}{\sigma^2 R_t R_{\mathrm{neck}}}}\right).$$

Here, R_t is the flow resistivity which can be expressed directly as a function of u_0 such that

$$R_t = \frac{8\mu}{\sigma R_{\mathrm{neck}}^2} + \beta\frac{\rho_0}{\pi h C_D^2}\sqrt{2}\left(1-\sigma^2\right)u_0. \quad (10)$$

β is a constant value equal to 1.6 and C_D is the discharge coefficient set to 0.76 in the rest of this paper. The tortuosity $\alpha_{\infty nl}$ is expressed as a function of u_0 as well such that

$$\alpha_{\infty nl} = 1 \quad (11)$$
$$+\frac{2\Psi}{h}0.48\sqrt{\pi R_{\mathrm{neck}}}\left[\sum_{n=0}^{8} a_n\left(\sqrt{\sigma}\right)^n\right]\left(1+\frac{\sqrt{2}}{c_0}u_0\right),$$

where Ψ is set to $4/3$ and the coefficients a_n are defined by $a_0 = 1.0$, $a_1 = -1.4092$, $a_2 = 0.0$, $a_3 = 0.33818$, $a_4 = 0.0$, $a_5 = 0.06793$, $a_6 = -0.02287$, $a_7 = 0.003015$, $a_8 = -0.01614$. These coefficients correspond to the Fok's function [9] accounting for hole interaction effect. Finally the impedance including the cavity reactance is given by

$$z_l = z_{MPP} - \mathrm{j}\cot(kL). \quad (12)$$

3. ITERATION PROCEDURE

In the non-linear regime, the impedance z of the liner is strongly dependent on the velocity in the perforations and conversely. Indeed, the velocity u_0 is expressed as

$$u_0 = \frac{p_{\mathrm{ref}}10^{SPL/20}}{\rho_0 c_0 |z|}, \quad (13)$$

in which $p_{\mathrm{ref}} = 2\cdot 10^{-5}$ Pa is the reference pressure and SPL is the sound pressure level in dB. Therefore, the velocity and the impedance need to be determined using iteration. The iteration procedure can be adapted when either a sine-swept or a white noise source is used in impedance tube measurements.

3.1 Sine-swept source

In the case of sine-swept source, the velocity depends on the frequency f. Therefore, the iterations need to be done for each frequency independently. This was performed by [3] and [6] and shows good correspondence with measurements. The iterations are initialized with a first guess of u_0 determined using the normalized air impedance $z_0 = 1$ instead of the liner's impedance in equation (13). The initial velocity is used to compute the initial impedance using (3) for Guess model or (12) for Laly's model. The velocity is computed using the liner's impedance and conversely until the difference between two successive iterated velocities is below $10^{-7}\ \mathrm{m\cdot s^{-1}}$ to ensure the convergence of the resulting impedance.

3.2 White noise source

In the case of a white noise source, the acoustic velocity corresponds to the Root Mean Square (RMS) velocity

$u_{0,RMS}$ which can be calculated using the velocities $u_{0,i}$ corresponding to each frequencies f_i such that

$$u_{0,RMS} = \sqrt{\sum_i u_{0,i}^2}. \quad (14)$$

The iterations are performed on the RMS velocity in a similar manner as for the sine-swept source. An initial RMS velocity is computed with the air impedance and the initial impedance is computed for every frequencies. The iterations stop when the difference between two successive RMS velocities is below $10^{-7}\,\mathrm{m\cdot s^{-1}}$. It is of important to note that the resulting impedance is dependent on the frequency range of interest.

4. COMPARISONS WITH IMPEDANCE TUBE MEASUREMENTS

The results obtained from iterative models are now compared with impedance tube measurements. These measurements are carried out in accordance with the NF EN ISO 10534-2 standard method. The diameter of the tube is $29\,\mathrm{mm}$. The cut-on frequency of the first non plane mode is 6932 Hz. Two $1/4$" microphones with a $20\,\mathrm{mm}$ spacing are used to determine the surface impedance of the liners. The distance between the sample and the closest microphone is $45.2\,\mathrm{mm}$. The frequency range is between $400\,\mathrm{Hz}$ and $6400\,\mathrm{Hz}$. The cavity height is $L = 29\,\mathrm{mm}$. The normalized impedance $\mathrm{Re}(z)/(\rho_0 c_0)$ and the normalized plate reactance $\mathrm{Im}(z)/(\rho_0 c_0) + \cot(kL)$ are plotted against the frequency. The absorption coefficient α defined by

$$\alpha = 1 - \left|\frac{z-1}{z+1}\right|^2, \quad (15)$$

is also shown.

4.1 Sine-swept source

We first use a sine-swept source at $150\,\mathrm{dB}$. Figures 1a to 1d correspond to the results obtained for a macro-perforated configuration. The impedance, the absorption and the velocity are accurately predicted by both models. In Figures 2a to 2d, the results obtained for a micro-perforated configuration are shown. The trend of the resistance is correctly predicted by the models although its value is overestimated, especially for the upper part of the frequency range. The trend of the velocity is correct but its magnitude is underestimated by both models. While discrepancies are observed between the measured and modeled impedance, the absorption coefficient is in good agreement between the models and the measurement. Therefore, the accuracy of the impedance models should not be assessed from the absorption coefficient only.

4.2 White noise source

In this section, the models are compared with measurements performed using a white noise source. The figures 3a to 3c show a very good agreement between measurements and predictions. In this macro-perforated case, the measured RMS velocity is $14.60\,\mathrm{m\cdot s^{-1}}$, the iterated velocity using Laly's model is $14.13\,\mathrm{m\cdot s^{-1}}$ and the iterated velocity using Guess model is $14.48\,\mathrm{m\cdot s^{-1}}$. Figures 4a to 4c correspond to the micro-perforated case. The measured RMS velocity is $20.37\,\mathrm{m\cdot s^{-1}}$, the iterated velocity using Laly's model is $17.80\,\mathrm{m\cdot s^{-1}}$ and the iterated velocity using Guess model is $17.87\,\mathrm{m\cdot s^{-1}}$. Although the impedance is reasonably well predicted, discrepancies are observed with the measurements. In a similar manner that for the sine-swept case, the good absorption correspondence is misleading.

4.3 Discussion

When switching between a sine-swept or a white noise source, the trend of the resistance does not remain the same. The reactance and the absorption remain similar. In the case of a sine-swept source, the resistance is strongly dependent on the velocity in the perforations. It is therefore very important to use iterations in this case. When a white noise source is used, the resistance is almost constant. Both models predict correctly the impedance and the absorption of macro- and micro-perforated liners, although the impedance predictions are less satisfying in the micro case. It seems that the models reach their validity limits when the porosity and the radius of the perforation become small. Only small discrepancies are observed between both models although the theory is different for the linear and non-linear parts of the resistance.

5. CONCLUSION

In order to predict the impedance of perforated liners in the linear regime, an iterative procedure has been implemented on Guess and Laly's model. The model can be adapted to fit impedance measurements carried out with a white noise or a sine-swept source. A very good agreement between the models and the measurements is observed when considering a macro-perforated plate. When a micro-perforated liner is used, the impedance predictions are less accurate. It is important not to assess the accuracy of the models only using the absorption coefficient.

6. ACKNOWLEDGEMENT

The authors gratefully acknowledge the support from ANR Chaire industrielle MACIA (ANR-16-CHIN-0002).

7. REFERENCES

[1] A. W. Guess, "Calculation of perforated plate liner parameters from specified acoustic resistance and reactance," *Journal of Sound and Vibration*, vol. 40(1), pp. 119–137, 1975.

[2] D.-Y. Maa, "Potential of microperforated panel absorber," *The Journal of the Acoustical Society of America*, 1998.

[3] B. S. Beck, N. H. Schiller, and M. G. Jones, "Impedance assessment of a dual-resonance acoustic

liner," *Applied Acoustics*, vol. 93, pp. 15–22, June 2015.

[4] Z. Laly, N. Atalla, and S.-A. Meslioui, "Acoustical modeling of micro-perforated panel at high sound pressure levels using equivalent fluid approach," *Journal of Sound and Vibration*, vol. 427, pp. 134–158, Aug. 2018.

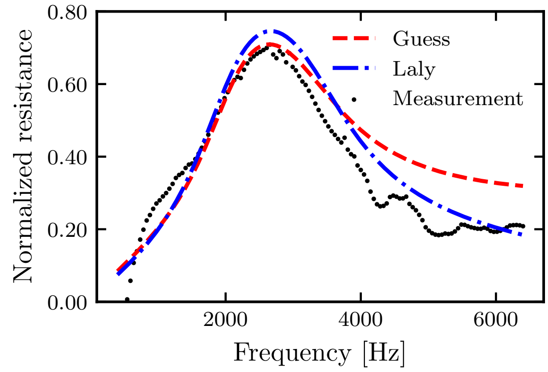
[5] S.-H. Park, "A design method of micro-perforated panel absorber at high sound pressure environment in launcher fairings," *Journal of Sound and Vibration*, vol. 332, pp. 521–535, Feb. 2013.

[6] Z. Laly, N. Atalla, S.-A. Meslioui, and K. EL Bikri, "Modeling of acoustic lined duct with and without grazing air flow by an analytical method," *Noise Control Engineering Journal*, vol. 66, pp. 340–352, July 2018.

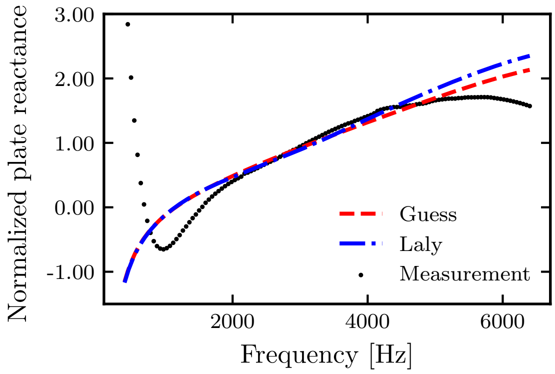
[7] C. Zwikker and C. W. Kosten, *Sound absorbing materials*. New York: Elsevier, 1949.

[8] A. D. Pierce, *Acoustics: An Introduction to its Physical Principles and Applications*. New York: McGraw-Hill Book Company, 1981.

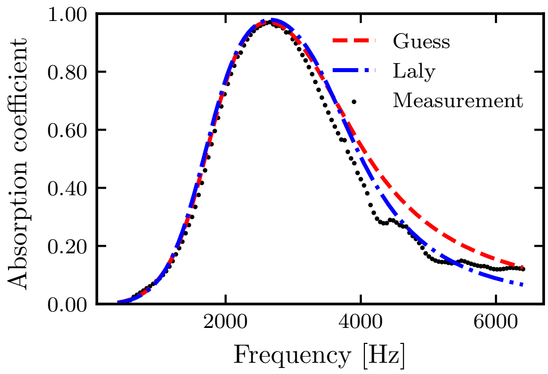
[9] V. A. Fok, "Theoretical research of the conductivity of a circular aperture in a partition across a pipe," *Doklady akademii nauk SSSR*, vol. 31, no. 9, pp. 875–82, 1941.



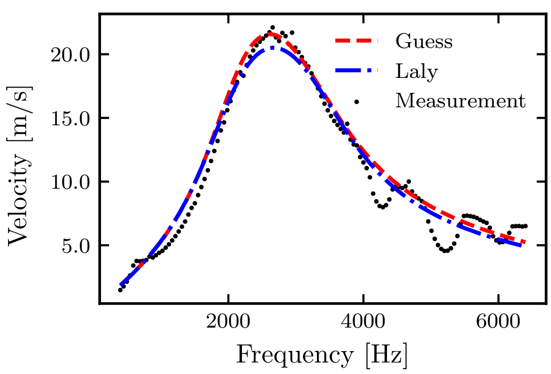
(a) Normalized resistance.



(b) Normalized plate reactance.

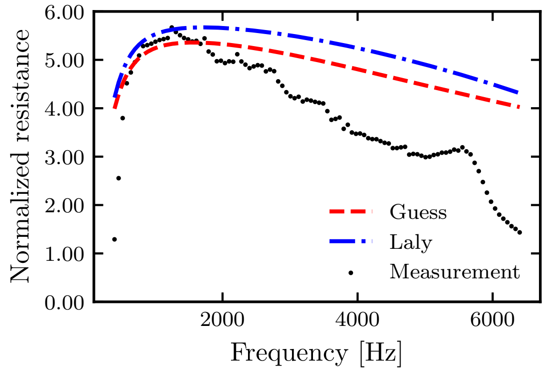


(c) Absorption coefficient.

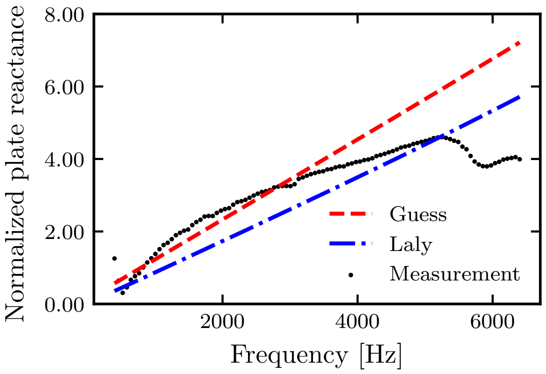


(d) Velocity in a perforation.

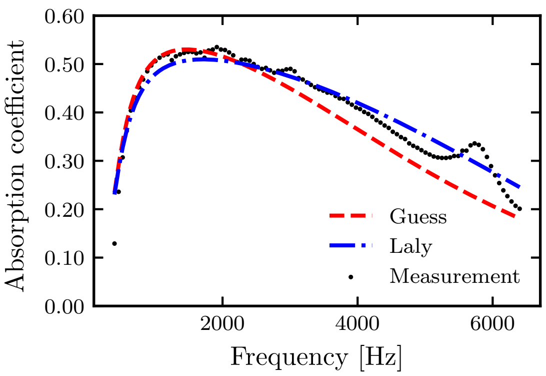
Figure 1: Impedance, absorption and velocity in a perforation of a macro-perforated liner using a sine-swept source at 150 dB with $R_{\text{neck}} = 0.8\,\text{mm}$, $\sigma = 10\,\%$, $h = 1.5\,\text{mm}$ and $L = 19\,\text{mm}$.



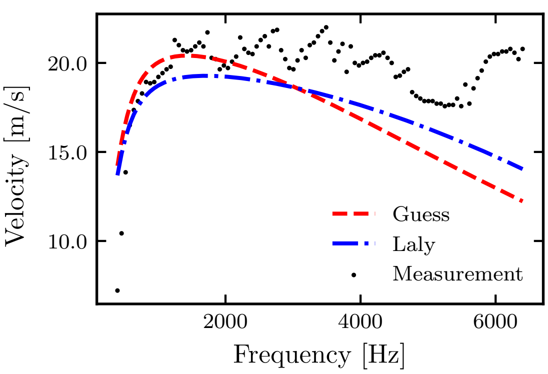
(a) Normalized resistance.



(b) Normalized plate reactance.

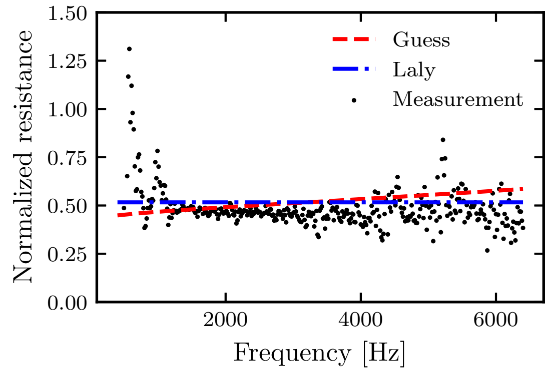


(c) Absorption coefficient.

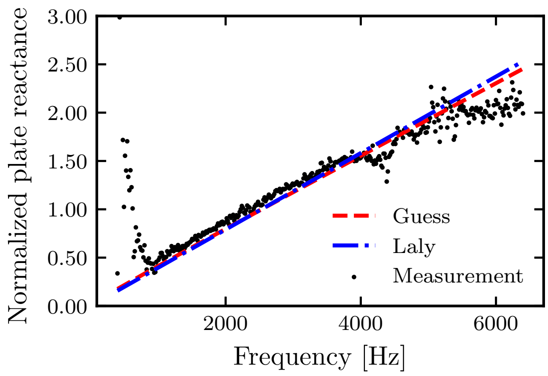


(d) Velocity in a perforation.

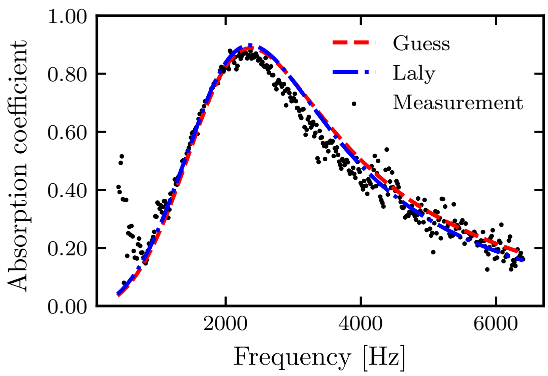
Figure 2: Impedance, absorption and velocity in a perforation of a micro-perforated liner using a sine-swept source at 150 dB with $R_{\mathrm{neck}} = 0.15\,\mathrm{mm}$, $\sigma = 1.4\,\%$, $h = 0.6\,\mathrm{mm}$ and $L = 19\,\mathrm{mm}$.



(a) Normalized resistance.

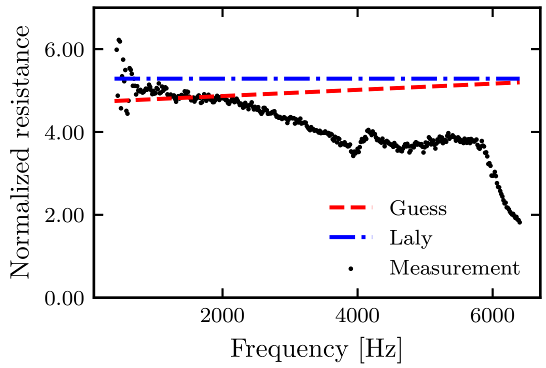


(b) Normalized plate reactance.

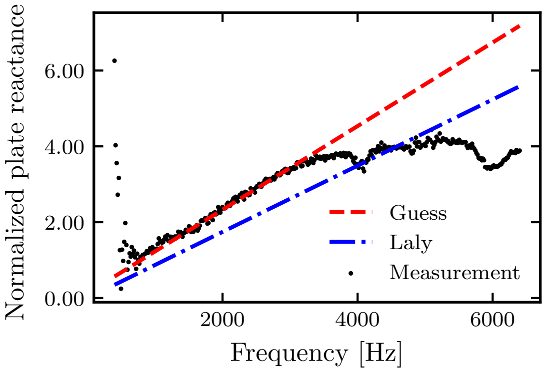


(c) Absorption coefficient.

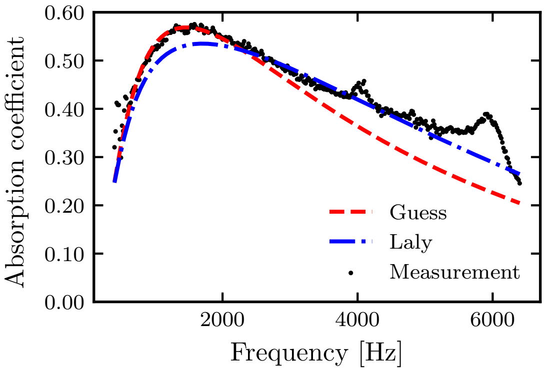
Figure 3: Impedance and absorption of a macro-perforated liner using a white noise source at 150 dB with $R_{\mathrm{neck}} = 0.8\,\mathrm{mm}$, $\sigma = 10\,\%$, $h = 1.5\,\mathrm{mm}$ and $L = 19\,\mathrm{mm}$.



(a) Normalized resistance.



(b) Normalized plate reactance.



(c) Absorption coefficient.

Figure 4: Impedance and absorption of a micro-perforated liner using a white noise source at $150\,\mathrm{dB}$ with $R_{\mathrm{neck}} = 0.15\,\mathrm{mm}$, $\sigma = 1.4\,\%$, $h = 0.6\,\mathrm{mm}$ and $L = 19\,\mathrm{mm}$.