



HAL
open science

Security of the STARK-friendly hash functions

Anne Canteaut, Tim Beyne, Itai Dinur, Maria Eichlseder, Gregor Leander, Gaetan Leurent, Maria Naya Plasencia, Léo Perrin, Yu Sasaki, Yosuke Todo, et al.

► **To cite this version:**

Anne Canteaut, Tim Beyne, Itai Dinur, Maria Eichlseder, Gregor Leander, et al.. Security of the STARK-friendly hash functions. Dagstuhl Seminar 20041 - Symmetric Cryptography, Jan 2020, Dagstuhl, Germany. hal-03143904

HAL Id: hal-03143904

<https://hal.inria.fr/hal-03143904>

Submitted on 17 Feb 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Security of the STARK-friendly hash functions

Tim Beyne, Anne Canteaut, Itai Dinur, Maria Eichlseder, Gregor Leander,
Gaëtan Leurent, Léo Perrin, María Naya Plasencia, Yu Sasaki,
Yosuke Todo, Friedrich Wiemer

Dagstuhl seminar - January 20, 2020

Motivation

ZK-STARK protocol is expected to be deployed on top of the Ethereum blockchain within the next year

→ its security and performance highly depend on the underlying **hash function**.

Performance. SFH are specified as sequences of low-degree polynomials or low-degree rational maps over a finite field.

Security.

- algebraic attacks based on Gröbner basis [Albrecht et al. 19]...
- **all other cryptanalytic techniques.**

MPC-friendly, Snark-friendly and Stark-friendly primitives

Objectives:

- minimize the number of multiplications **in large fields**
- minimize the size of the **polynomial relations** representing the execution trace over a finite field.

Examples:

- Cradic [Knudsen Nyberg 92], Misty [Matsui 97]
- MiMC [Albrecht et al. 16]

SFH contenders

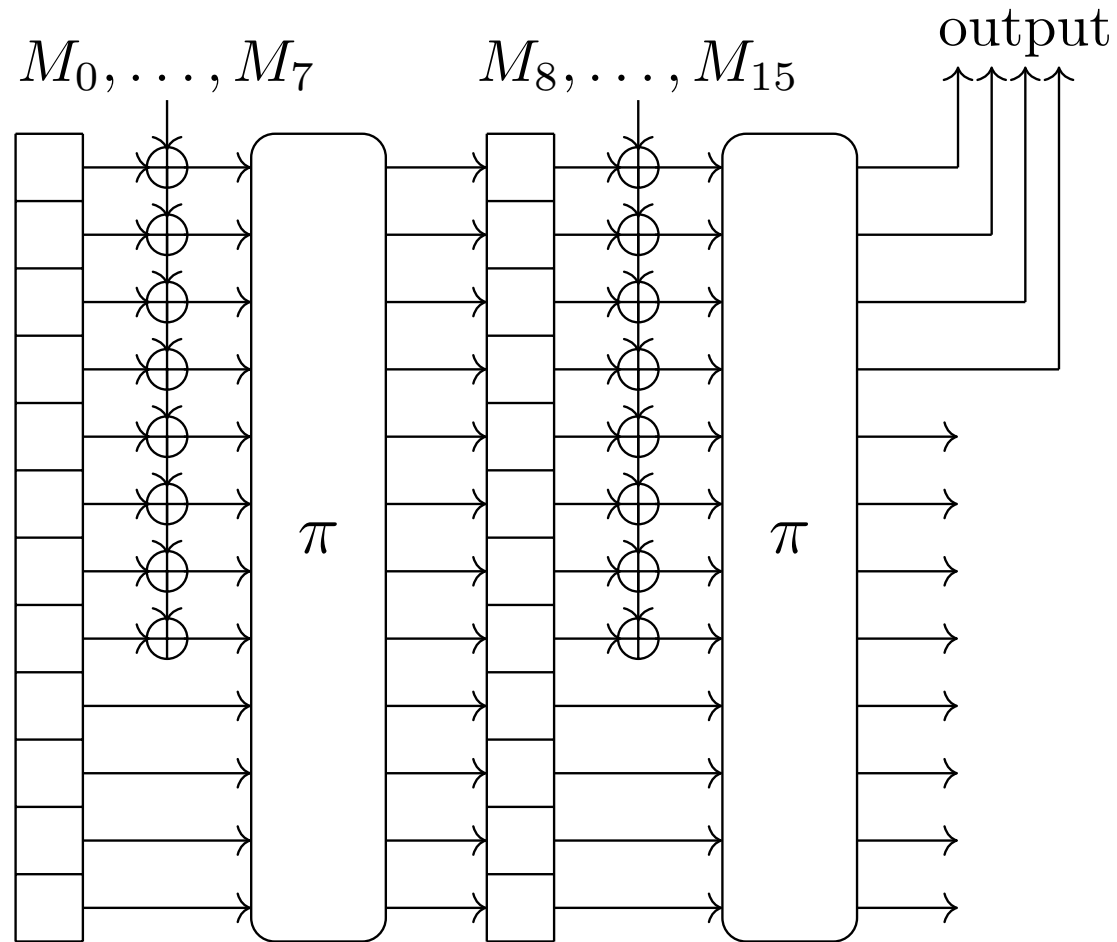
StarkWare challenges <https://starkware.co/hash-challenge/>

Three families of sponges with different permutations

- SPN with large blocks: **Vision** (\mathbb{F}_{2^n}) and **Rescue** (\mathbb{F}_p) [Aly et al. 19]
- **HadesMiMC** permutation: **Starkad** (\mathbb{F}_{2^n}) and **Poseidon** (\mathbb{F}_p) [Grassi et al. 19]
- **GMiMC** i.e. $\text{GMiMC}_{\text{erf}}$ over \mathbb{F}_p [Albrecht et al. 19]

Sponge construction

All candidates follow the same sponge construction with **blocksize t** and **capacity c** .



Parameters of the sponge

security level	$\log_2 q$	c	t	
128 bits	64	4	12	variant 128-d
	128	2	4	variant 128-a
	128	2	12	variant 128-c
	256	1	3	variant 128-b
	256	1	11	variant 128-e
256 bits	128	4	8	variant 256-a
	128	4	14	variant 256-b

Performance for 128-bit security

Best candidate:

Variant 128-d:

$t = 12$ and $c = 4$ over \mathbb{F}_q

$$q = \begin{cases} 2^{63} \\ 2^{61} + 20 \times 2^{32} + 1 \end{cases}$$

Compared performance for these parameters

prime fields are more STARK-friendly than binary fields

Prime field:

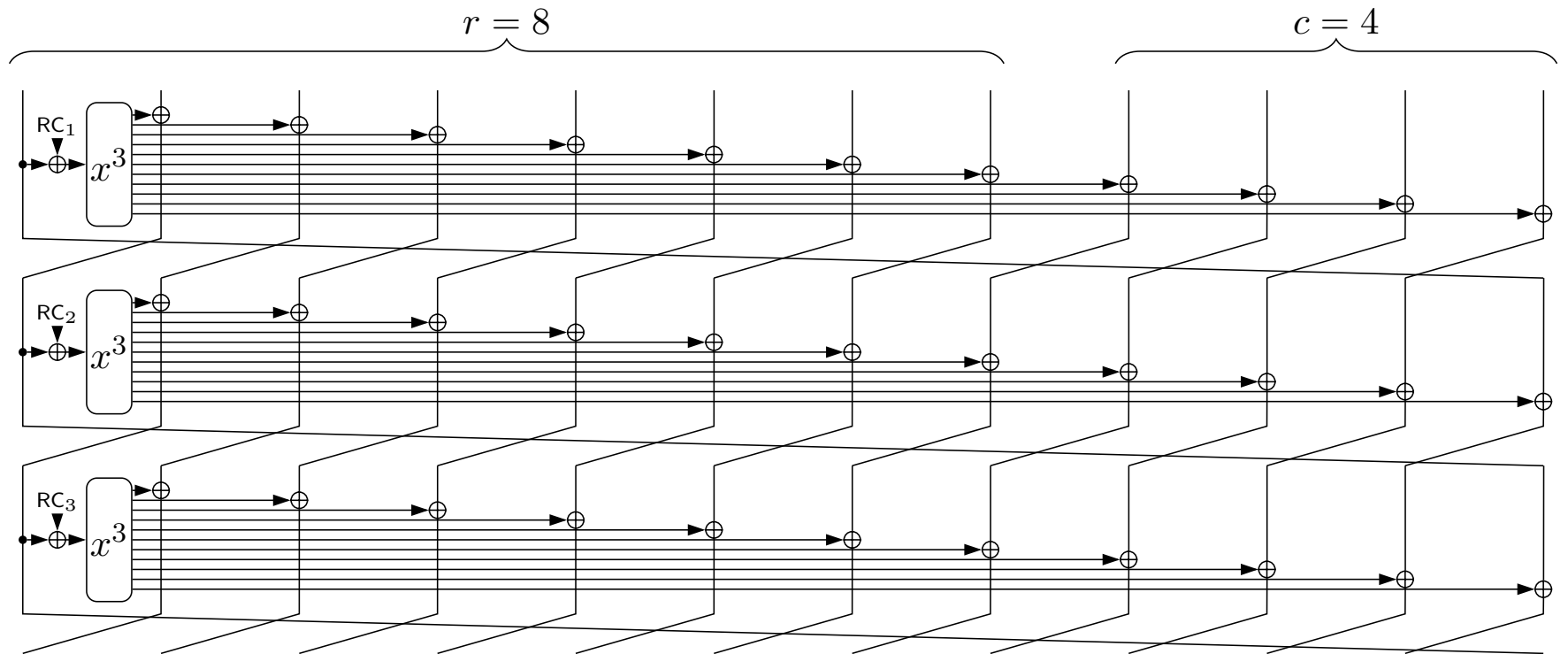
1. GMiMC
2. Rescue
3. Poseidon

Binary field:

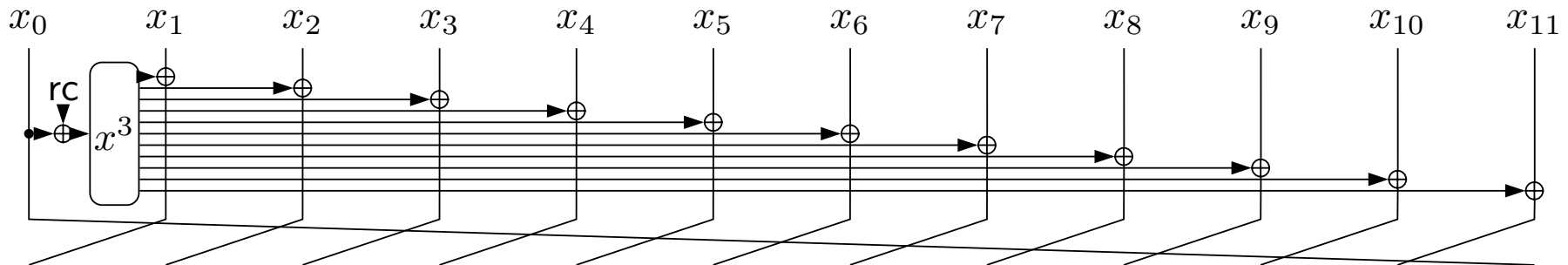
1. Vision
2. Starkad

GMiMC

GMiMC with 101 rounds



A differential distinguisher



Original analysis:

best attack with a characteristic over $(t + 1)$ rounds with probability $(2q^{-1})^2$.

A better differential:

$$(0 \dots 0, \alpha, \alpha') \xrightarrow{\mathcal{R}^{t-2}} (\alpha, \alpha', 0 \dots 0) \xrightarrow{\mathcal{R}} (\alpha' + \beta, \beta \dots \beta, \alpha) \xrightarrow{\mathcal{R}} (\beta + \beta' \dots \beta + \beta', \alpha + \beta', \alpha' + \beta)$$

For $\beta' = -\beta$, we get an iterative differential

$$(0 \dots 0, \alpha, \alpha') \xrightarrow{\mathcal{R}^t} (0, \dots, 0, \alpha + \beta, \alpha' + \beta) \text{ with probability } 2q^{-1}$$

A differential distinguisher

With this t -round differential with proba $2q^{-1}$

- A differential characteristic over the **101** rounds with probability 2^{-480} for a **732**-bit blocksize.
- With structures, we get valid pairs with **complexity 2^{359}** (full permutation) and valid pairs with **complexity less than 2^{128}** for **58** rounds.
- With a rebound-like technique, we expect to get valid pairs conforming with the differential over **58** rounds with complexity close to 2^{64} (on-going work).

Impossible differentials

Original analysis: best impossible differential over $(2t - 2)$ rounds

A better impossible differential over $(3t - 4)$:

$$(0, \dots, 0, \alpha) \xrightarrow{\mathcal{R}^{3t-4}} (\beta, 0, \dots, 0)$$

for any nonzero $\alpha \neq \beta$.

Integral attacks over \mathbb{F}_q

When $q = 2^n$.

For any (affine) subspace $V \subset \mathbb{F}_2^n$ with $\dim V > \deg F$,

$$\sum_{x \in V} F(x) = 0.$$

Integral attacks over \mathbb{F}_q

When $q = 2^n$.

For any (affine) subspace $V \subset \mathbb{F}_2^n$ with $\dim V > \deg F$,

$$\sum_{x \in V} F(x) = 0.$$

Because, for $V = b + \langle a_1, \dots, a_v \rangle$,

$$D_{a_1} D_{a_2} \dots D_{a_v} F(b) = \sum_{x \in V} F(x)$$

Not valid in odd characteristic.

But for any q

For any exponent k with $0 \leq k \leq q - 2$,

$$\sum_{x \in \mathbb{F}_q} x^k = 0$$

General result.

For any $F : \mathbb{F}_q \rightarrow \mathbb{F}_q$ with $\deg(F) \leq q - 2$,

$$\sum_{x \in \mathbb{F}_q} F(x) = 0 .$$

But for any q

For any exponent k with $0 \leq k \leq q - 2$,

$$\sum_{x \in \mathbb{F}_q} x^k = 0$$

General result.

For any $F : \mathbb{F}_q \rightarrow \mathbb{F}_q$ with $\deg(F) \leq q - 2$,

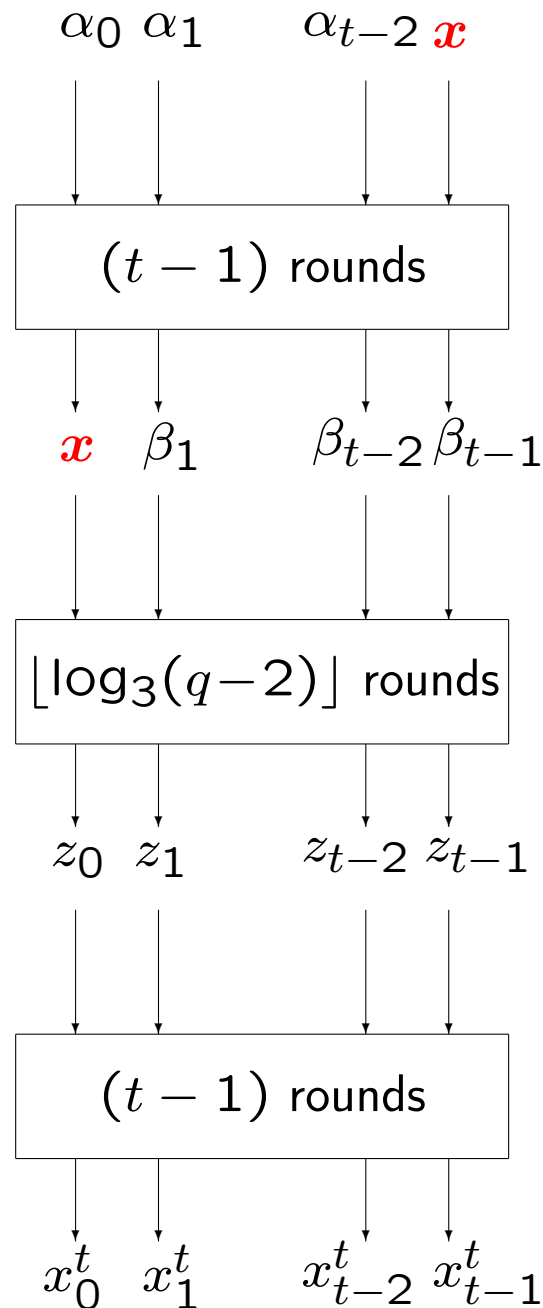
$$\sum_{x \in \mathbb{F}_q} F(x) = 0 .$$

Less general than the property over \mathbb{F}_{2^n} :

For any (affine) subspace $V \subset \mathbb{F}_2^n$ with $\dim V > \deg F$,

$$\sum_{x \in V} F(x) = 0$$

Integral distinguisher on GMiMC



polynomial in \mathbf{x}
of degree $\leq q - 2$

$$Q(\mathbf{x}) = \sum_{i=1}^{t-1} x_i^t - (t - 2)x_0^t$$

Until the degree does not exceed $(q - 2)$

Input set.

$$\mathcal{X} = \{(\alpha_0, \dots, \alpha_{t-2}, \mathbf{x}), \mathbf{x} \in \mathbb{F}_q\}$$

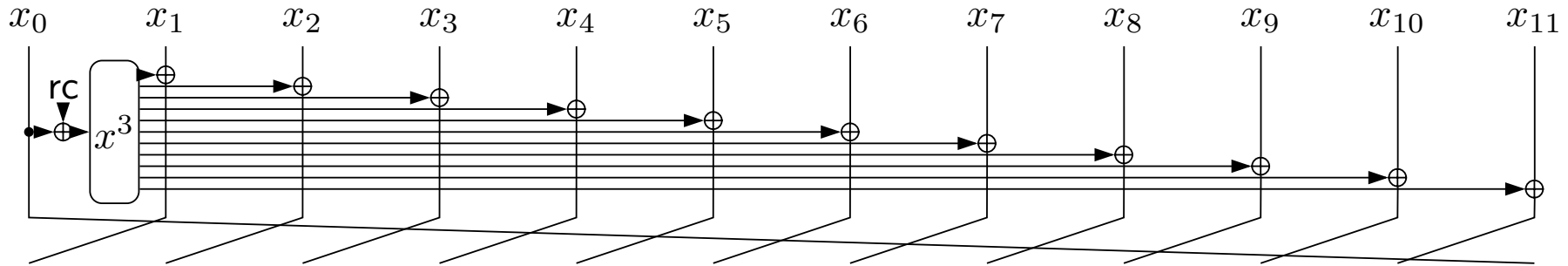
After $(t - 1)$ rounds.

$$\mathcal{X}' = \{(\mathbf{x}, \beta_1, \dots, \beta_{t-1}), \mathbf{x} \in \mathbb{F}_q\}$$

After r rounds, the degree in \mathbf{x} of each branch is at most 3^r .

\Rightarrow all branches are balanced if $3^r \leq q - 2$.

Adding $(t - 2)$ rounds



The inputs and outputs of Round ℓ satisfy

$$x_i^\ell - x_{i+1}^\ell = x_{i+1}^{\ell-1} - x_{i+2}^{\ell-1}, \quad \forall i \leq t - 3$$

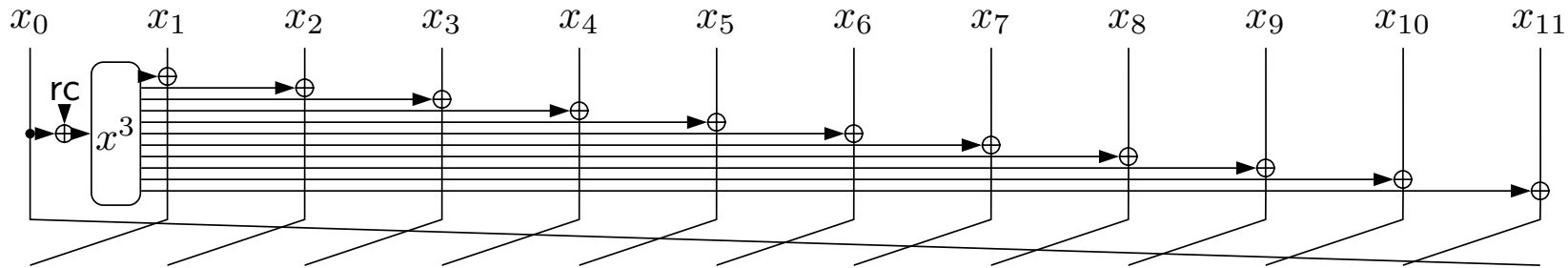
Over $(t - 2)$ rounds,

$$x_0^{t-1} - x_1^{t-1} = x_{t-2}^1 - x_{t-1}^1$$

is a polynomial in \mathbf{x} of degree $\leq (q - 2)$.

\Rightarrow Distinguisher with complexity q on $(2t - 3 + \lfloor \log_3(q - 2) \rfloor)$ rounds (59 rounds)

Adding one more round



The inputs and outputs of Round ℓ satisfy

$$x_i^\ell = x_{i+1}^{\ell-1} + (x_j^\ell - x_{j+1}^{\ell-1}) \text{ and } x_{t-1}^\ell = x_0^{\ell-1}$$

$$\Rightarrow \sum_{i=0}^{t-1} x_i^{\ell-1} - (t-1)x_j^t = \sum_{i=0}^{t-1} x_i^\ell - (t-1)x_{j-1}^\ell$$

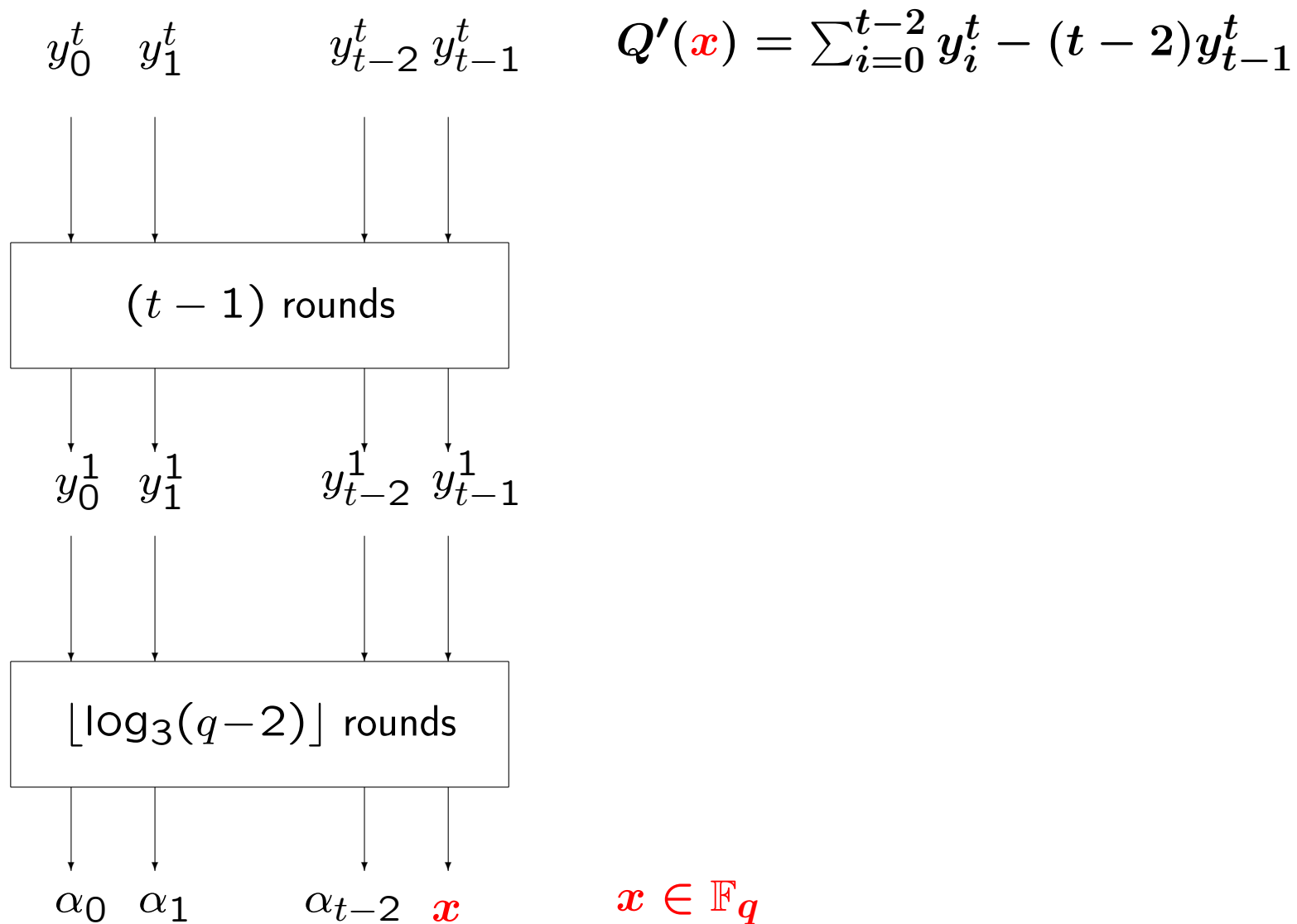
Over $(t-1)$ rounds,

$$\sum_{i=0}^{t-1} x_i^1 - (t-1)x_{t-1}^1 = \sum_{i=0}^{t-1} x_i^t - (t-1)x_0^t$$

\Rightarrow Distinguisher with complexity q on $(2t-2 + \lfloor \log_3(q-2) \rfloor)$ rounds (60 rounds)

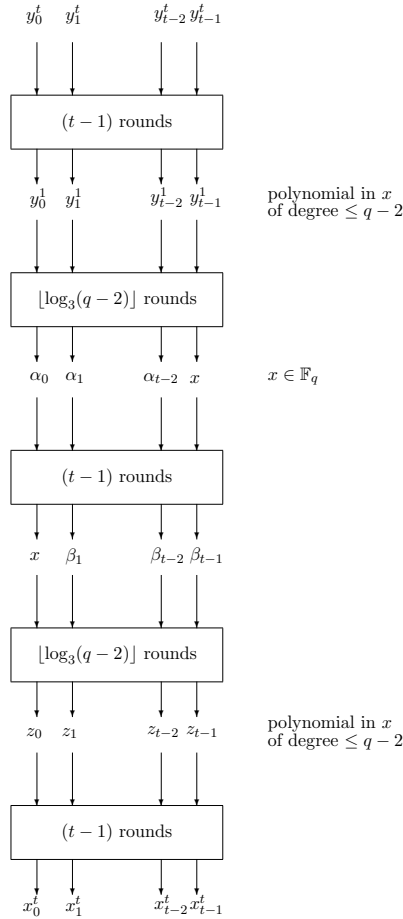
A few more rounds with two active branches (on-going work).

Computing backwards



Zero-sum partition on GMiMC on $(3t - 3 + 2\lfloor \log_3(q - 2) \rfloor)$ rounds (109)

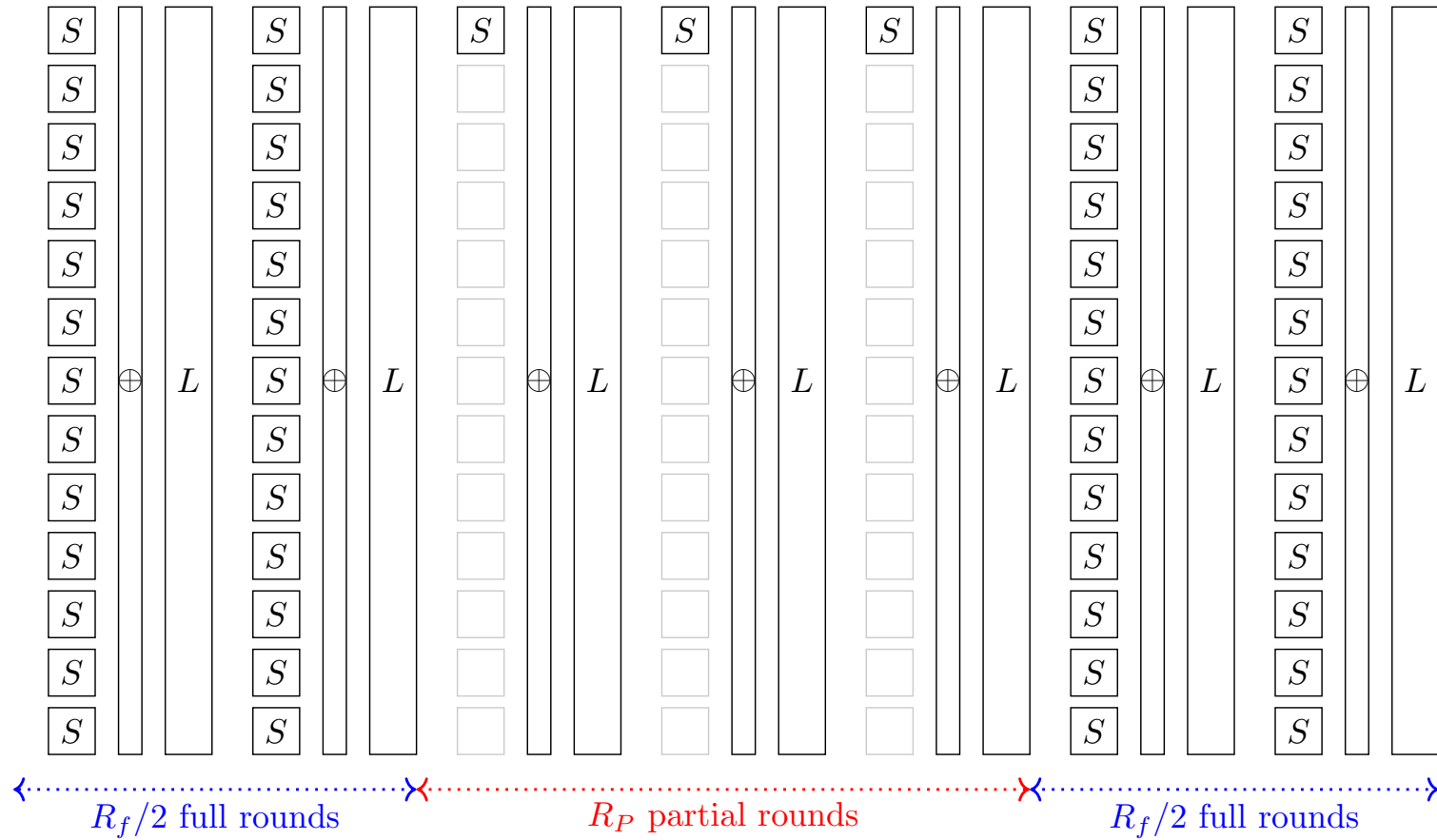
$$\ell(x_0, \dots, x_{t-1}) = \sum_{i=1}^{t-1} x_i - (t - 2)x_0 \text{ sum to } 0$$



$$\ell'(y_0, \dots, y_{t-1}) = \sum_{i=0}^{t-2} y_i - (t - 2)y_{t-1} \text{ sum to } 0$$

HadesMiMC

HadesMiMC



$R_f = 8$ full rounds and $R_P = 43$ (binary) and $R_P = 40$ (prime)

Resistance against statistical attacks

Analysed without the partial rounds.

Differential cryptanalysis:

x^3 has differential uniformity **2** over \mathbb{F}_q .

The best differential characteristic satisfies

$$\mathbf{EDP} \leq \binom{2}{-}^{(t+1)R_f/2} / q$$

→ $R_f = 6$ are enough.

Degree of the permutation over \mathbb{F}_q

Each coordinate is seen as a **multivariate polynomial over \mathbb{F}_q**

After r rounds:

$$\sum_{u=(u_1, \dots, u_t)} \lambda_u \left(\prod_{i=1}^t x_i^{u_i} \right) \text{ where } u_i \leq 3^r$$

\Rightarrow 39 rounds are enough for Poseidon (40 for Starkad) to reach degree $(q - 1)$ in each variable

$\Rightarrow \lceil \log_3(t) \rceil$ more rounds are enough to get total degree $(q - 1)t$.

Remark: StarkWare challenges with $q \simeq 2^{256}$ and 96 rounds have degree at most 2^{152} in each variable.

Zero-sum partition over \mathbb{F}_q

State after the last full Sbox layer before the partial rounds.

$$\mathcal{X} = \{(\alpha_0, \dots, \alpha_{t-2}, \mathbf{x}), \mathbf{x} \in \mathbb{F}_q\}$$

After 38 rounds forwards.

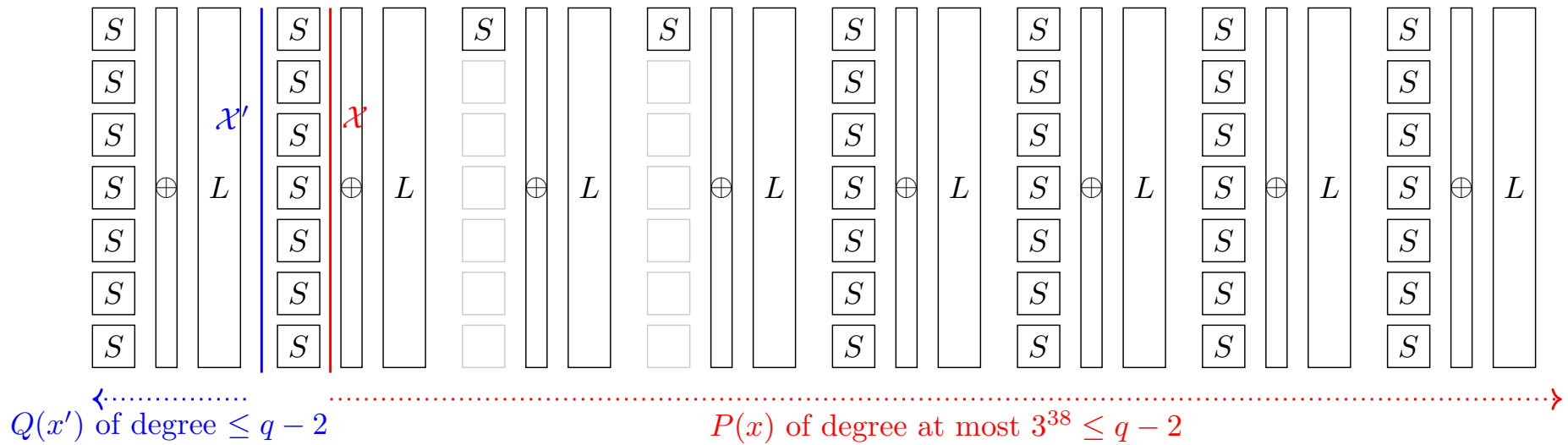
each coordinate has degree at most $(q - 2)$.

Computing backwards.

$$S^{-1} : x \mapsto x^s \text{ with } s = \frac{2q - 1}{3}$$

\Rightarrow Zero-sum for $R_f = 2 + 4$ and $R_P = 34$ (35 for Starkad).

Zero-sum partition over \mathbb{F}_q



Improvement when $q = 2^n$

Each Boolean coordinate is seen as a **multivariate polynomial in nt variables over \mathbb{F}_2**

Degree over \mathbb{F}_{2^n} vs. binary degree.

$$P(x) = \sum_{u \leq 2^n - 1} \lambda_u x^u$$

has binary degree

$$\max\{wt(u) : 0 \leq u < 2^n \text{ and } \lambda_u \neq 0\}$$

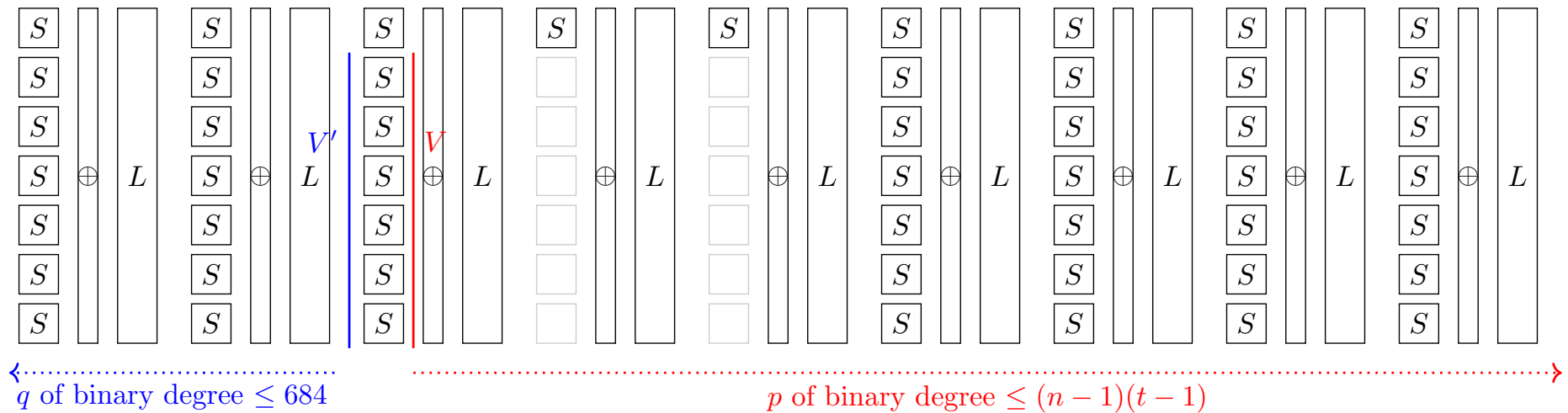
\Rightarrow The inverse Sbox has binary degree $\frac{n+1}{2}$.

Several rounds backwards [Boura, C. 13].

- Two rounds backwards have binary degree ≤ 684
- Three rounds backwards have binary degree ≤ 748

Zero-sum partition over \mathbb{F}_2 with $R_f = 3 + 4$ and $R_P = 35$

$$V = \{(0, x_1, \dots, x_{t-1}), x_i \in \mathbb{F}_{2^n}\}.$$



When the MDS matrix has a small order

How to propagate a subspace through all partial rounds?

Choose V such that all elements in each coset of $L(V)$ have the same value on the first coordinate.

$$L(V) \subset H_0 = \{(0, x_1, \dots, x_{t-1}), x_i \in \mathbb{F}_q\}$$

or equivalently

$$V \subset \langle M_0 \rangle^\perp .$$

We can iterate this R_P times if

$$\mathcal{V} = H_0 \cap \bigcap_{r=0}^{R_P-1} L^r \left(\langle M_0 \rangle^\perp \right) \neq \{0\}$$

This holds if $L^r = \mathbf{Id}$ for some $r \leq t - 2$.

When the MDS matrix is an involution

The internal states after each partial layer form a coset of V or of $W = L(V)$.

Special choice for V .

$$V = \{(\mathbf{x}v_0, \dots, \mathbf{x}v_{t-1}), \mathbf{x} \in \mathbb{F}_q\}$$

with $v \in \mathcal{V}$.

\Rightarrow The outputs of the partial rounds vary in a coset of

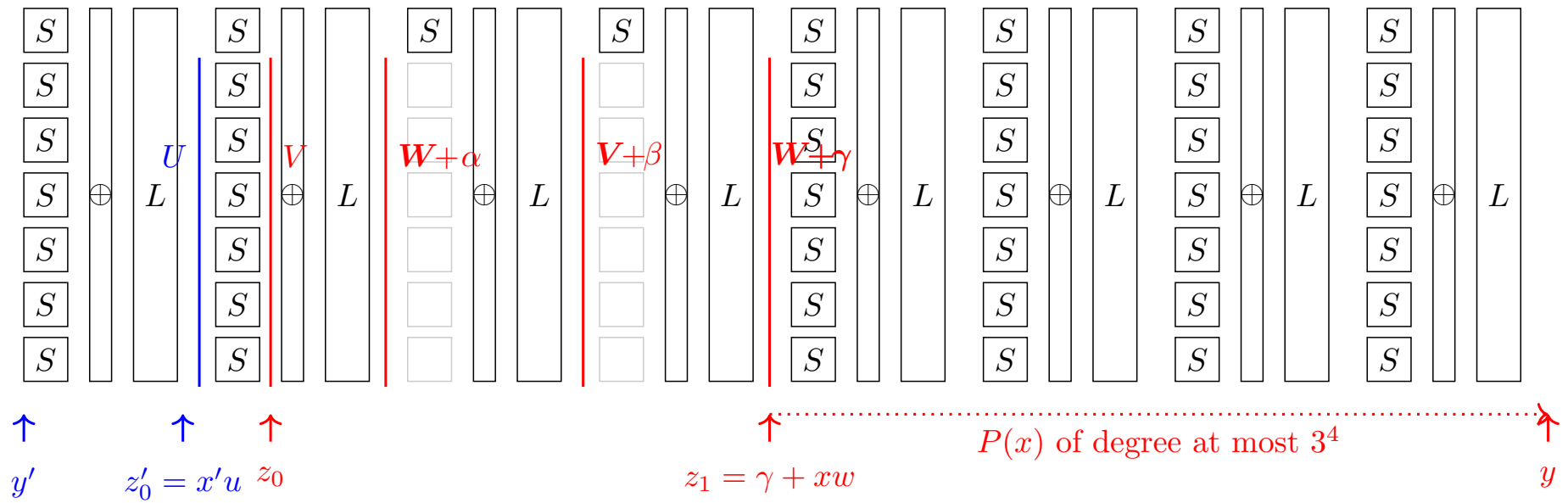
$$\{(\mathbf{x}w_0, \dots, \mathbf{x}w_{t-1}), \mathbf{x} \in \mathbb{F}_q\}$$

Forward direction.

Each output coordinate is a polynomial in \mathbf{x} of degree at most $3^{R_f/2} \leq q - 2$.

\Rightarrow The output coordinates sum to zero.

Zero-sum partition with $R_f = 2 + 4$ and any R_P with complexity q



Open question on the complexity of algebraic attacks

Input: $(a_1, \dots, a_{t-k}) \in \mathbb{F}_q^{t-k}$ and $(b_1, \dots, b_k) \in \mathbb{F}_q^k$

Find $x_1, \dots, x_k \in \mathbb{F}_q^k$ such that

$$\pi(a_1, \dots, a_{t-k}, x_1, \dots, x_k) = (b_1, \dots, b_k, y_1, \dots, y_{t-k}) \text{ for some } y_1, \dots, y_{t-k}$$

Degree of the univariate polynomial of the lexicographical Gröbner basis [Faugère-Perret].

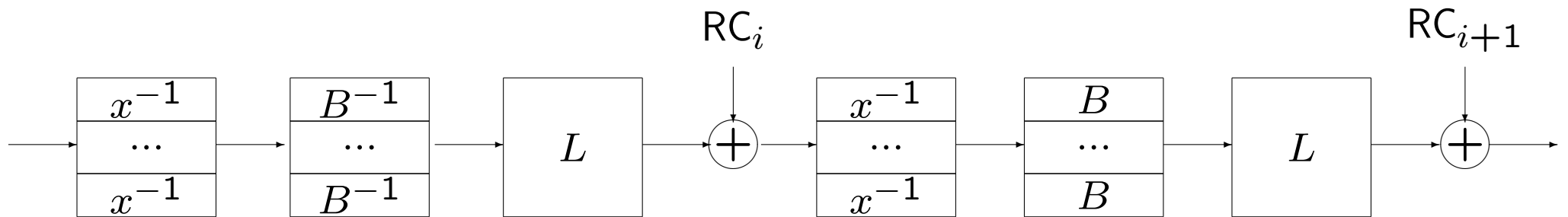
$$D = 3^{kR_f + R_P - 2k + 1}$$

Complexity for solving the system = D^2 .

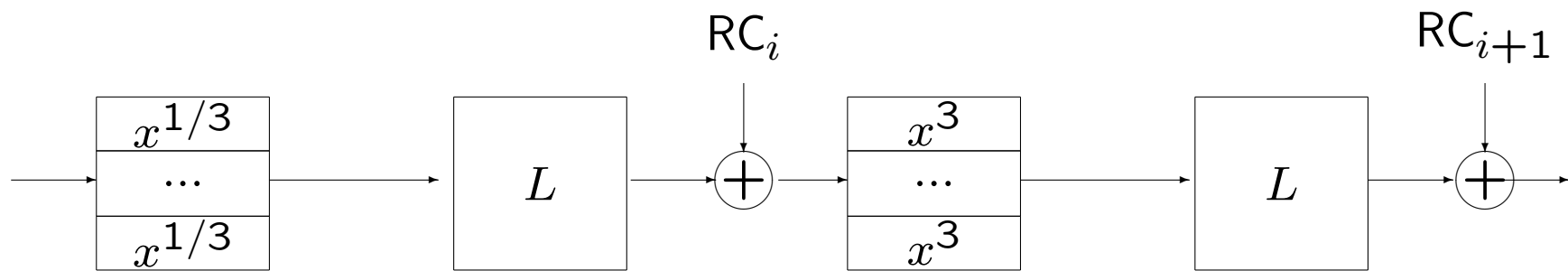
Variants aiming at 256-bit security have $D \simeq 2^{170}$.

Vision and Rescue

Vision (20 rounds)



Rescue (20 rounds)



Degree of Rescue

Activate one input coordinate $x \in \mathbb{F}_p$

After one round.

$$\lambda x^{1/3} + \mu$$

$$\Rightarrow \text{degree } \frac{2p-1}{3}$$

After the second Sbox layer.

$$(\lambda x^{1/3} + \mu)^3$$

which contains only monomials $x^{1/3}$, $x^{2/3}$, x and a constant term.

$x^{2/3}$ has degree $\frac{p+1}{3}$.

\Rightarrow The degree does not increase between the first and second round.

But even by activating more inputs, we cannot find an integral attack on more than 4 rounds.

Conclusions

We need to **find the right tools** for analyzing symmetric primitives over non-binary fields:

- linear attacks and their variants?
- more general integral attacks?

Open question:

Does the **form of q** affect the security?

For instance, if $p = 2^{2^n} + 1$?