Maximal cable tensions of a N-1 cable-driven parallel robot with elastic or ideal cables

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Abstract. Determining what will be the maximal cable tensions of a cable-driven parallel robot (CDPR) when it moves over a given workspace is an important step in the design phase as it will allow to choose the cable diameter and to provide a requested information for tuning the CDPR actuation. In this paper we consider a suspended N-1 CDPR with *n* cables where all cables are attached at the same point, which leads to a 3-dof robot. We assume a quasi-static behavior of the robot and assume that the cable are either ideal or elastic so that we neglect the sagging effect. Under these assumption we show that the maximum of the cable tensions may be determined in a very fast way by solving a set of second-order polynomials which will lead to the poses at which the maximum of each cable tension will occur. For example for a four-cables CDPR determining the maximal cable tension requires to solve at most 149 second order polynomials.

Keywords: cable-driven parallel robot, static,cable tension

1 Introduction

Determining the largest cable tensions over a given workspace is an essential part of the design of a CDPR. This problem has been addressed for classical parallel robots [1, 2] and has been used for their design [3]. However this problem is somewhat different for CDPR because of their redundancy, configuration changes, cable slackness and influence of the discrete time control [4] that may lead to different tensions for a given pose of the CDPR or on a given trajectory. Most works on static analysis address only indirectly this issue either by computing different types of workspace under the constraint that the tensions lie within given bounds [5–8] or by trying to impose by control an optimal force distribution [9–13]. In the later case the force distribution is difficult to ensure for fully constrained CDPRs because of the difficulty to reliably measure the cable tensions, while for suspended CDPRs this scheme can only be adopted if the cable are elastic (for non-elastic cables executing a given tension control scheme at a given pose requires an **exact** control of the cable lengths, an assumption that is clearly unrealistic). However for safety reasons it is of interest to determine the largest cable tensions in the worst case (i.e. with no assumption on the control) for a given CDPR when the platform moves in a given workspace. Clearly dynamics will play a role on the tensions [14–16] but however a first step is to consider only quasi-static motion.

To the best of the author knowledge the only work having addressed the determination of the largest cable tensions has been presented by Riehl [17] for a 3-1 CDPR with sagging cables. For a given pose the cable tensions are determined assuming ideal cables and then a numerical procedure is used to derive the tensions with sagging cables. Then the given workspace is sampled (i.e. the workspace is approximated by a finite set of poses, called *node poses*) and the maximum cable tensions is derived from the calculation at the node poses. The first limitation of this work is that the inverse kinematics (IK) of the 3-1 CDPR with sagging cable has most probably a single solution while for a N-1 CDPR with $N > 3$ the IK may have multiple solutions so that all solutions of the IK have to be determined in order to determine the largest cable tensions in the worst case. The second limitation is related to the sampling of the workspace which leads to a large computation time for determining an approximation of the largest tensions as the real maximum may possibly not occur at a node pose. For example for a 5×5×5 meters workspace that is sampled with a 5cm step size with a computation time for calculating the cable tensions at a node of 5 ms we will get a computation time of 10mn 25s for determining the maximal cable tensions, which is problematic if this calculation is used in a design process that require a large number of such a calculation. The objective of this paper is to show that the pose at which the maximal tension will occur can be calculated exactly in a very short time.

In this paper we will consider a suspended 3dof N-1 CDPR with $N \geq 4$ cables attached to the same point on the platform so that this robot allows only for translational motions (figure 1). Furthermore we will assume a geometry that is respected for all the existing prototypes: the exit point A_i have the same height z_a while the convex hull of the projection of these points on the ground is a rectangle $\mathscr{R}$. A consequence is that the available workspace for this CDPR in terms of horizontal motions is $\mathscr{R}$. Regarding cable model we will assume ideal cables(no mass, no elasticity). Note that the results we will obtain also cover the case of elastic cables with no mass, whatever the elasticity model is, as elasticity just play a role for determining the cable lengths to reach a given pose but does not influence the cable tensions at this pose.

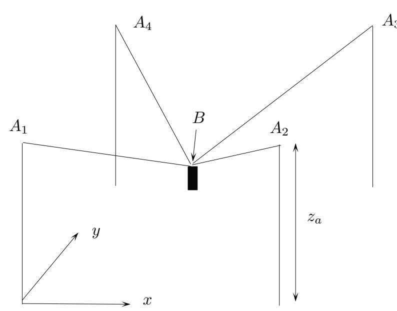

Fig. 1. A example of a $N-1$ CDPR

As mentioned previously the exit point of the cables will be denoted A_i while the common point of the cables will be B with coordinates (x_b, y_b, z_b). We number the A_i point in such a way that the points A_1, A_2, A_3 are three corners of the rectangle $\mathscr{R}$ and we define a reference frame whose origin will be A_1 (whose coordinates are therefore $(0,0,z_a)$) and A_2, A_3 will have as coordinates ($x_{a_2} > 0$,0,z_a), ($x_{a3} \geq 0, ya3 > 0, 0$), while the others A_i coordinates are $(0 \leq x_{a_i} \leq x_{a_2}, 0 \leq ya_i \leq y_{a_3}, z_a)$. We will assume that $x_{a_i} = x_{a_2}$ or $y_{a_i} = y_{a_3}$ so that any A_i is located on one edge of the rectangle $\mathscr{R}$. The point on the ground that results from a vertical projection of a point M will be denoted M^p. We will also assume that the CDPR platform is only submitted to gravity and that its mass is m. In a quasi-static case the center of mass of the platform is located below B so that no torque is exerted at B. We will also assume that the set of poses of the CDPR for which we will calculate the maximal cable tensions (the *desired workspace*) is a rectangle included in $\mathscr{R}$ that is defined by the constraints $0 \leq x_r^1 \leq x \leq x_r^2 \leq x_{a_2}$, $0 \leq y_r^1 \leq y \leq y_r^2 \leq y_{a_3}$. Clearly the maximal height of the desired workspace should be lower than z_a.

2 Calculation of the maximal cable tensions

For ideal cable a CDPR property is that most of the time only 3 of the cables will be under tension. Indeed when 3 cables are under tension their lengths leads to a unique possible pose B_a for B: hence to have any other cable j under tension at B_a may occurs only when its length ρ_j is exactly $||\mathbf{A_j B_a}||$ as if $\rho_j > ||\mathbf{A_j B_a}||$, then cable j is slack and if $\rho_j < ||\mathbf{A_j B_a}||$, then B will move to a pose different from B_a. A consequence is that trying to create by control a configuration in which 4 cables are under tension will require an infinite accuracy on the measurement of the cable lengths together with a exact knowledge of the location of the A_i points. However having more than 3 cables under tension may occur occasionally for example when we initially set the lengths of cable 1,2,3 in such a way that B_a^p lies in the triangle $A_1^p A_2^p A_3^p$ while cable 4 is slack and then coil cable 4 until its lengths is largely lower than $||\mathbf{A_j B_a}||$. Hence for computing exactly the maximum cable tensions we have also to consider that more than 3 cables are under tension as this temporary configuration may occur on any trajectory whatever the control law is and although we are not able to determine when this configuration will occur.

In the next section we will first start to investigate the case where only 3 cables are under tension, the others being slack.

2.1 Maximal tension with only 3 cables under tension

For determining the maximal tension in all the n cables of the CDPR under the assumption that only 3 of them are under tension we will have to consider all the combinations of 3 cables among the n cable i.e. $n(n-1)(n-2)/6$ cases. However the treatment of all these cases is basically the same so that we may consider the example case of cables 1,2,3 being under tension.

If we assume that cable 1,2 and 3 are under tension, while the other cables are slack and the CDPR is in mechanical equilibrium, then the point B_a^p should lie in the triangle $A_1^p A_2^p A_3^p$. If τ_i denotes the tension of cable i and ρ_i its length, then the mechanical equilibrium condition is a linear system of 3 equations in the τ_1, τ_3, τ_3 unknowns so that we get

$$\tau_1 == \frac{mg\rho_1\left(xy_{a_3} + x_{a_2}y - x_{a_2}y_{a_3} - x_{a_3}y\right)}{x_{a_2}y_{a_3}(z-z_a)}$$
$$\tau_2 = -\frac{mg\rho_2\left(xy_{a_3} - x_{a_3}y\right)}{x_{a_2}y_{a_3}(z-z_a)} \qquad \tau_3 = -\frac{mg\rho_3 y}{y_{a_3}(z-z_a)}$$

where $z - z_a$ is negative. We notice that τ_i may be written as $\tau_i = mg\rho_i U_i(x,y)/V_i(z)$ where U_i is linear in x,y, while V_i is always negative.

Regarding τ_1, the term $U_1 = xy_{a_3} + x_{a_2}y - x_{a_2}y_{a_3} - x_{a_3}y$ is the signed distance of B to the line 23 which is negative as soon as B^p lies in the triangle $A_1^pA_2^pA_3^p$, so that $\tau_1 > 0$. Regarding τ_2, the term $xy_{a_3} - x_{a_3}y$ is positive as soon as B^p is inside the triangle $A_1^pA_2^pA_3^p$ so that τ_2 is positive in that case, while τ_3 is positive as soon as $y > 0$.

We may now look at the influence of the altitude z on the value of τ_i: we notice that this influence is related to the variation of the term $Z = \rho_i/(z - z_a)$ as each τ_i is established as $K_i\rho_i/(z - z_a)$ where the $K_i < 0$ are not dependent upon z. As ρ_i may be written as $\sqrt{(x - x_{a_i})^2 + (y - ya_i)^2 + (z - z_a)^2}$ the derivative of Z with respect to z is always negative so that we may state a rather trivial and expected result:

Theorem 1: if only 3 cables are under tension the largest cable tension will be obtained for the highest altitude of the platform

Note that if the A_i are not lying in a horizontal plane the above theorem will still hold although the tension will increase when the distance between B and the plane $A_1A_2A_3$ decreases (implying that it will also hold if pulleys are used at the A_i points).

Now let us look at the derivatives of the τ_i with respect to x,y. There derivatives may be written as

$$\frac{\partial\tau_i}{\partial x} = \frac{\rho_i^2\partial U_i/\partial x + U_i(x - x_{a_i})}{W_i} \qquad \frac{\partial\tau_i}{\partial y} = \frac{\rho_i^2\partial U_i/\partial y + U_i(y - ya_i)}{W_i}$$

where W_i has a constant sign. Looking at the numerator of these derivatives we note that because of the linearity of U_i the term $\partial U_i/\partial x,y$ is a constant so that $\rho_i^2\partial U_i/\partial x,y$ is a second order function in x,y and so are also the terms $U_i(x - xa_{a_i}), U_i(y - y_{a_i})$. Hence the numerators of $\partial\tau_i/\partial x, \partial\tau_i/\partial y$ are two quadratic equations $\mathscr{C}_i^x, \mathscr{C}_i^y$. Note that we get 2 such equations for τ_1, τ_2 but only one for τ_3 as $\partial\tau_3/\partial x$ cancel only for $y = 0$ or $x = x_{a_3}$: hence for the whole set of cables we get a total of 5 quadratic equations. The conics $\mathscr{C}_i^x = 0, \mathscr{C}_i^y = 0$ split the desired workspace in different components in which the derivatives of τ_i with respect to x,y have a constant sign. A consequence is that the extremum of a cable tension may be obtained at:

- the set of poses S_C which satisfy $\mathscr{C}_i^x = \mathscr{C}_i^y = 0$: the resultant with respect to x of these two equations always factor out in either one quadratic equation in y or x (for τ_3) or in two quadratic equations (for τ_1, τ_2). Determining the intersection of the conics leads to up to 4 poses for each τ_i that are obtained by solving two quadratic polynomials so that we have to solve a total of 6 quadratic polynomials

- the set S_E consisting of the three corners of the triangle $A_1^p A_2^p A_3^p$ that are projected at the maximum altitude
- the set S_B for of the poses for which either x or y is extremal (i.e being equal to $x_r^1, x_r^2, y_r^1, y_r^2$ and $\mathscr{C}_i^x = 0$ or $\mathscr{C}_i^y = 0$. As we have 5 conics and four extremal coordinates determining this set require to solve 20 quadratic polynomials
- the set S_U of poses such that $U_j = 0, j \in [2,3]$ (which defines a line in the $x-y$ plane) and $\mathscr{C}_i^x = 0$ or $\mathscr{C}_i^y = 0$ with $i \neq j$. In that case we have either $\tau_2 = 0$ or $\tau_3 = 0$ (only 2 cables are under tension) and S_U is obtained by solving 7 quadratic polynomials

Note that we retain as member of the sets only the one that are inside the desired workspace. Obtaining the sets S_B, S_C, S_E, S_U therefore require to solve a total of 33 second order polynomials. As we have to repeat the process for each triple of cables we have to solve $33n(n-1)(n-2)/6$ quadratic polynomials. For example if we have a 4-cables CDPR we will have to solve 132 second order polynomials.

Examples: The length unit in this paper is the centimeter. For the first example we choose $x_{a_2} = 500$, $x_{a_3} = 50$, $y_{a_3} = 500$, the workspace being restricted to $xi^1 = 10, xr^2 = 490, yr^1 = 10, yr^2 = 490$ and we consider the cable 1, 2 and a cable attached at (50,500). The value of z_a is set to 450, the load weight is 100N, while the height of the workspace is limited to 300. A second example just differs from the first one by having $x_{a_3} = 0$. In figure 2 we present the various geometrical elements that are considered for the calculation are shown, the points that are used to compute the maximal tensions (in solid circles) while table 1 presents the maximal tensions.

	x	y	τ max
cable 1 (1st example)	21.184	211.844	100.06
cable 2 (1st example)	309.962	10	99.82
cable 3 (1st example)	31.02	310.23	100.36
cable 1 (2nd example)	183.379	10	96.94
cable 2 (2nd example)	309.23	10	100.143
cable 3 (2nd example)	10	309.23	100.143

Table 1. Maximal cable tension for the 2 examples

2.2 Maximal tensions with 4 cables under tension

As mentioned previously we cannot by control put a CDPR in a configuration where 4 cables (or more) are under tension and stop the CDPR in that configuration. But during the execution of a trajectory such a configuration may temporarily occur after which the CDPR will move to a configuration with three or two cables only are under tension. Intuitively it may be thought that at a given pose when having four cables under tension instead of three will lead to a decrease of the cable tensions but unfortunately intuition is wrong. Indeed as seen in the 3-cables case the maximum of τ_1 is obtained for a pose that is usually on the line joining A_1 to another of the A_i points. Now if we introduce a fourth cable we have to consider the τ_1 that will be obtained for a pose lying on the line A_1A_4 and it may perfectly occurs that there is pose on that lien such that τ_1 is larger than the maximum obtained in the 3-cables case.

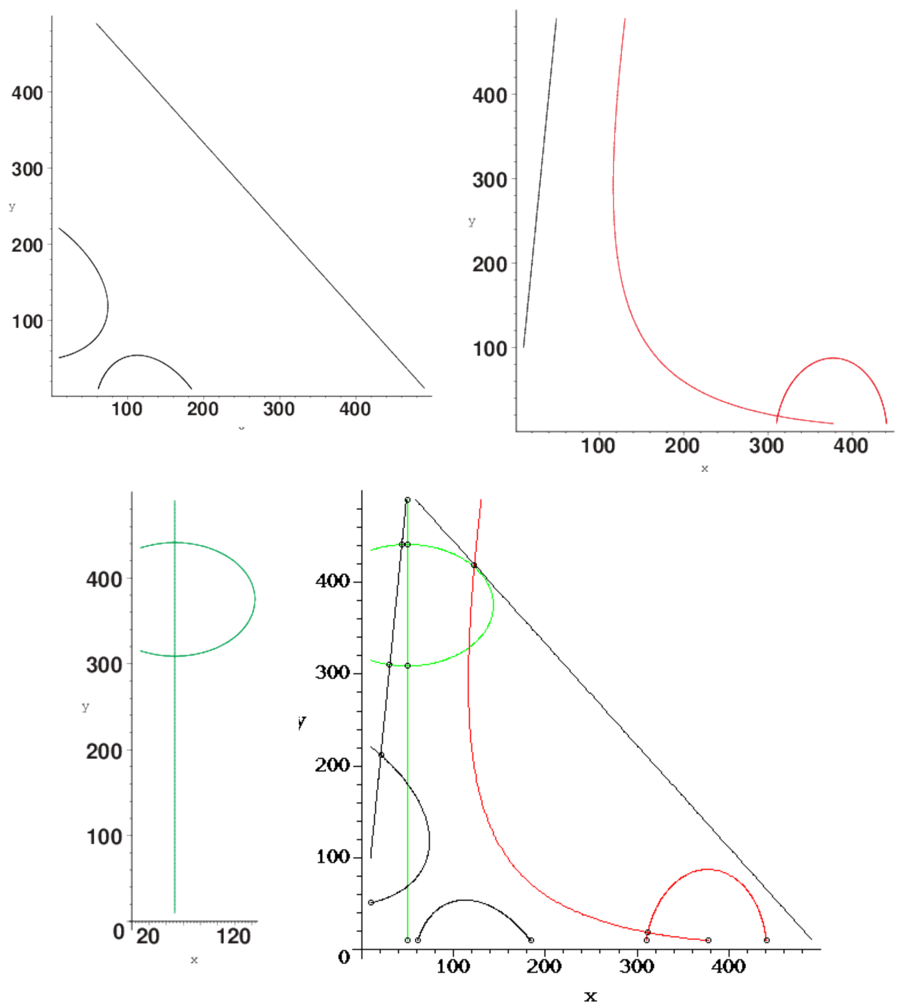

Fig. 2. The elements considered for the calculation of the maximal tensions. Top left we have the 2 conics and line $U_1 = 0$ for cable 1, top right the same elements for cable 2, bottom left for cable 3 (which has a single conic). At the bottom right we have the same elements in a single drawing and In solid circle we have the poses that are considered for the calculation.

Consider for example a four cables CDPR with A_i coordinates $(0,0,z_a),(x_{a_2} > 0,0,z_a),(0,ya_3 > 0,z_a)$ while the A coordinates of a fourth cable are $(0 < x_{a_4} \leq x_{a_2}, y_{a_3}, z_a)$. Using the static equilibrium linear equations we may establish for a given pose $\mathbf{X}$ the tensions in the cables 1, 2, 3 as a function of the tension τ_4 in the fourth cable. These tensions may be written as

$$\tau_i = \tau_i^0 + w_i\tau_4, i \in [1,3] \tag{1}$$

where τ_i^0 is the tension when only the 3 cables 1,2,3 are under tension (which is positive if $\mathbf{X}^p$ lies in the triangle $A_1^pA_2^pA_3^p$) and the w_i coefficients are obtained as:

$$w_1 = \frac{x_{a_4}\rho_1}{\rho_4 x_{a_2}} \quad w_2 = -\frac{x_{a_4}\rho_2}{\rho_4 x_{a_2}} \quad w_3 = -\frac{\rho_3}{\rho_4}$$

so that $w_1 > 0, w_2 < 0, w_3 < 0$. Hence if indeed τ_2, τ_3 decrease when a positive tension is applied on cable 4, at the opposite τ_1 will increase. As we cannot master the tension τ_4 it is therefore necessary to investigate its increasing effect on τ_1. However there is an upper limit for the value of τ_4 which is obtained when either τ_2 or τ_3 is equal to 0 as this implies a change in the CDPR configuration. Note also that the maximum for τ_1 may be obtained at a pose that is different from the one we have obtained when calculating the maximum for the 3-cables case. Indeed the increase of τ_1 due to the presence of τ_4 is dependent upon the pose so that there is no reason that $\tau_1^0 + w_i\tau_4$ may not be larger than the same quantity obtained at the pose where τ_1 has been the maximum for the 3-cables case.

The maximum of τ_1 is obtained at a pose such that either τ_2 or τ_3 (or both) cancels or the derivatives of τ_1 with respect to x,y cancel. The later case is not possible as the numerator of both derivatives is equal to $\rho_1 mg$. We will thus determine τ_1 if $\tau_2 = 0$, $\tau_3 = 0$, $\tau_2 = \tau_3 = 0$ and we will retain the one leading to the largest τ_1.

Case with $\tau_2 = 0$ First we determine the values of τ_4 that cancel τ_2 and report these values in τ_1 so that τ_1 is now a function of only x,y,z. We then consider the derivatives of τ_1 with respect to x,y and determine the constraints that cancel these terms. The derivative of τ_1 with respect to x cancel only for $x = 0$ or $y = y_{a_3}$ while the derivative of τ_1 with respect to y leads to a conic $C_{y_2} = 0$.

Case with $\tau_3 = 0$ First we determine the values of τ_4 that cancel τ_2 and report these values in τ_1 so that τ_1 is now a function of only x,y,z. The derivatives of τ_1 with respect to x,y leads to two conics $C_{x_3} = 0, C_{y_3} = 0$.

Case with $\tau_2 = \tau_3 = 0$ We have also to consider the case where τ_4 cancel both τ_2 and τ_3: this will occur if $x = x_{a_4}y/ya_{a_3}$. We report x and the corresponding value of τ_4 into τ_1 and then calculate the derivative of τ_1 with respect to y leads to a second order polynomial P_y in y.

Hence the maximum of τ_1 will be obtained at a pose such that:

- x or y have one of the extremal value $x_r^1, x_r^2, y_r^1, y_r^2$ and C_{x_3} or C_{y_3} or C_{y_2} cancel. As x or y are fixed the conic leads to a quadratic polynomial in the remaining variable. As we have 3 conics and four extremal values we have to solve 12 quadratic polynomials.

- $C_{x_3} = C_{y_3} = 0$: the resultant of C_{x_3} and C_{y_3} in x factors out in 2 quadratic polynomials. Therefore to deal with this case we have to solve 4 quadratic polynomials
- the roots of P_y

Overall getting the maximum for the 4-cables cases amount to solve 17 quadratic polynomials. In all this cases we get a set of possible poses from which we eliminate the one that are outside the desired workspace. For each of the remaining poses we calculate the value of τ_1 and retain the maximal value. We have to repeat this process by considering all $A = \prod_{j=0}^{n-2}(n-j)$ quadruples of cables among the n cables, thereby having to solve $17A$ quadratics.

Note that in this section we have considered the case where both τ_2 and τ_3 cancel, so that we also consider the case where only $n-2$ cables are under tension.

In summary for a 4-cables CDPR determining the maximal tension for all cables requires at most to solve 17+132=149 second order polynomials, thereby leading to a very fast algorithm.

Example: using the same data than for the second example of the previous section with $x_{a_4} = 250, y_{a_4} = 500$ leads to a maximum of τ_1 of 107.743 at the pose $x = 103.197$,, $y = 206.394$. We note that this value is approximately 10% higher than the value we have obtained for three cables. The pose at which this maximum is obtained is also different from the pose obtained in the three-cables case.

3 Extension to $N > 4$ cables

We may generalize the result obtained in the previous section to a CDPR with an arbitrary number of cables. Again we consider the pose $\mathbf{X}$ whose $\mathbf{X}^p$ is included in the triangle $A_1^p A_2^p A_3^p$ where the coordinates of A_1, A_2, A_3 are $(0,0,z_a), (x_{a_2} > 0,0,z_a), (0, y_{a_3>0}, z_a)$. We will assume that the other cables have as coordinates either (x_{a_2}, y_{a_j}, z_a) with $0 \le y_{a_j} \le y_{a_3}$ or (x_{a_j}, y_{a_3}, z_a) with $0 \le x_{a_j} \le x_{a_2}$ i.e. A_j lies on an edge of the rectangle $\mathscr{R}$. The static equilibrium condition may be written as

$$\mathbf{J_3}(\tau_1, \tau_2, \tau_3)^T = (\sum_{k=4}^{k=n} -\frac{(x - x_{a_k})\tau_k}{\rho_k}, \sum_{k=4}^{k=n} -\frac{(y - y_{a_k})\tau_k}{\rho_k}, -mg - \sum_{k=4}^{k=n} \frac{(z - z_a)\tau_k}{\rho_k})^T \quad (2)$$

where $\mathbf{J_3}$ is the transpose of the inverse kinematic jacobian restricted to cable 1,2,3. Let us define $\mathbf{J_2}$ as the matrix derived from $\mathbf{J_3}$ by substituting the second column of $\mathbf{J_3}$ by (u_1, u_2, u_3). The determinant of $\mathbf{J_2}, \mathbf{J_3}$ are obtained as

$$|\mathbf{J_2}| = -\frac{y_{a_3}(u_1(z - z_a) - u_3 x)}{\rho_1 \rho_3} \quad |\mathbf{J_3}| = \frac{(z - z_a) x_{a_2} y_{a_3}}{\rho_1 \rho_2 \rho_3} \quad (3)$$

so that $-|\mathbf{J_2}|/|\mathbf{J_3}|$ is $(u_1(z - z_a) - u_3 x)\rho_2/((z - z_a)x_{a_2})$. If we set u_1, u_2, u_3 to the components of the right-hand side of equation (2) this ratio will be equal to τ_2. Consequently τ_2 may be written as

$$\tau_2 = a_0 + \sum_{k=4}^{k=n} a_k \tau_k$$

where the a_k are constants that depend only upon the CDPR pose and geometry. The coefficient a_k of τ_k in τ_2 is

$$a_k = \frac{((z-z_a)\frac{(x-x_{a_k})}{\rho_k} - x\frac{(z-z_a)}{\rho_k})\rho_2}{(z-z_a)x_{a_2}} = -\frac{x_{a_k}\rho_2}{x_{a_2}\rho_k}$$

which is always negative. Thus any additional cable having a positive tension will reduce the value of τ_2. The same procedure may be used to determine the coefficient of τ_k in τ_3 whose value is $-y_{a_k}\rho_3/(y_{a_3}\rho_k)$ whose value is also always negative so that τ_3 decreases when additional cables are used. On the other hand the coefficient c_{k1} of τ_k in τ_1 is

$$c_{k1} = -\frac{\rho_1(y_{a_3}(x_{a_2}-x_{a_k}) - x_{a_2}y_{a_k})}{\rho_k y_{a_3} x_{a_2}}$$

as we have either $x_{a_k} = x_{a_2}$ or $y_{a_k} = y_{a_3}$, then we get

$$c_{k1} = \frac{\rho_1 y_{a_k}}{y_{a_3}\rho_k} \quad \text{or} \quad c_{k1} = \frac{\rho_1 x_{a_k}}{x_{a_2}\rho_k}$$

so that c_{k1} is always positive. Hence τ_1 increases whenever a cable with a positive tension is added.

Theorem 2: if we consider a triplet of cables that represent three successive corners i, j, k and two edges of the desired workspace, then for any number of additional cables l whose A lies on one of the two other edges of the workspace a positive tension leads to an increase of one element in the set $\{\tau_i, \tau_j, \tau_k\}$ and to a decrease for the other elements of the set.

Let us assume that τ_i is the cable tension that increases with τ_l. The derivatives of c_{k1} with respect to x, y is always a quadratic equation and consequently determining the value of τ_l that leads to the largest τ_i under the constraint $\tau_j \geq 0, \tau_k \geq 0$ follows the same process than for the 4-cables case.

4 Conclusion

We have proposed in this paper a fast algorithm for computing the maximal cable tensions of a N-1 CDPR over a given workspace that provides exactly the result by solving only a limited set of 2nd order equations, without having to sample the workspace. Furthermore as all these equations are independent the algorithm may be implemented on a multi-core processor so that the computation time will be extremely low. A first possible extension will be to deal with an arbitrary disposition of the winch points together with a more general way to describe the workspace: we believe that as soon as this description involves only linear elements or is parametric this extension will not be difficult. A second possible extension will be to consider an arbitrary number of cables: here again although we will have to manage the combinatorial aspect the principle of the proposed algorithm will remain the same. Finally we may consider uncertainties in the design: as the calculation relies only on simple, low-degree, algebraic equations it will be relatively easy to determine the worst case maximal tension for example by using interval analysis.

Regarding open issues we may consider taking into account the cables sagging: here the open question for the N-1 case is if the maximal cable tensions with sagging will be obtained at the same pose than when considering ideal cables (in which case taking sagging into account will not drastically change the algorithm, including taking into account the uncertainties on the robot geometry) ? We may have also to consider the case where the platform exhibits limited oscillations so that the force exerted at B lie in a cone.

Finally extending this work to 6 dof CDPRs will be quite complex as cable/cable and platform/cable interference and limitation on the cable lengths will have to be taken into account.

Extension to a dynamics analysis for a given trajectory appears to be possible as the platform dynamics may be solved to determine the forces exerted at B but will be more complex as time will be an additional variable so that a temporal-geometrical analysis will have to be performed.

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