Strategic Resource Management in 5G Network Slicing. (Invited paper)

Mandar Datar*†, Eitan Altman*†§
*INRIA Sophia Antipolis - Méditerranée; †CERI/LIA, University of Avignon, Avignon, France;
§LINCS Lab, Paris;

***Abstract*—Network Slicing is one of the essential concepts that has been introduced in 5G networks design to support demand expressed by next generation services. Network slicing will also bring new business opportunities for service providers (SPs) and virtual network operators, allowing them to run their virtual, independent business operations on shared physical infrastructure. We consider a marketplace where service providers (SPs) *i.e.,* slice tenants lease the resources from an infrastructure provider (InP) through a network slicing mechanism. They compete to offer a certain communication service to end-users. We show that the competition between SPs can be model using the multi-resource Tullock contest (TC) framework, where SPs exert effort by expending costly resource to attract users. We study the competition between the SPs under a static and dynamic resource sharing scheme. In a dynamic resource sharing scheme, SPs are pre-assigned with fixed shares (budgets) of infrastructure, and they are allowed to redistribute their shares and customise their allocation to maximise their profit. The decision problem of SPs is analysed using non-cooperative game theory, and it is shown that the resultant game admits a unique Nash Equilibrium (NE). Furthermore, a distributed reinforcement algorithm is proposed that allows each SP to reach the game's unique Nash equilibrium. Finally, simulations results are conducted to analyse the interaction between market players and the economic efficacy of the network sharing mechanism.**

***Index Terms*—Communication service market, game theory, tullock contest, trading post mechanism, 5G network slicing, resource allocation.**

I. Introduction

Next-generation wireless network is expected to spread its applicability and deliver support to emerging sectors like Virtual Reality (VR) live broadcast, automotive, healthcare, manufacturing etc. Critical challenges in mobile network applicability to the sectors mentioned above are their heterogeneity and conflicting communications needs that the current monolithic network is insufficient to meet. Several new concepts have been proposed for the upcoming 5G network design to satisfy these critical needs. Out of those, probably one of the most important concepts in 5G network design is "network slicing".

Network slicing is the concept of running multiple independent logical networks (slice) on top of the common shared physical infrastructure. Each independent logical network (slice) is then explicitly dedicated to meeting each slice tenant's needs, contrary to the approach "one-size-fits-all" witnessed until previous mobile generations [1].

This work was supported by Nokia Bell Labs and MAESTRO-5G-ANR

The implication of network slicing brings a paradigm shift towards a multitenancy ecosystem where multiple tenants owning individual slices negotiate with multiple InPs to request the resources for their service provision. In this scenario, the SPs or slice tenets generally express a demand for a dedicated isolated (that may need dedicated fixed resources allocation) and independent virtual network with full ownership of their service level agreement (SLA). On the contrary, InPs aim to maximize their return on investment by enabling the dynamic sharing of infrastructure, as this lowers the operational and capital costs and allows InPs to monetize their infrastructure to its fullest potential. However, the sharing of infrastructure may expose the tenants to the risk of violating their SLAs. Hence, one of the fundamental issues in network slicing is an efficient sharing of the network resources, which regulates the trade-off between two conflicting interests, *i.e.,* interslice isolation and efficient network resource utilization.

In order to balance the interslice isolation and efficient resource utilization, authors of [2] suggested the 'share-constrained proportional allocation' (SCPA) scheme where each slice is pre-assigned with a fixed share (budget) of infrastructure; slices are allowed to redistribute their shares and customize their allocation according to dynamic load. In turn, InP allocates each resource to slices in proportion to their shares on that resource. This approach allows a dynamic sharing, where tenants can redistribute their network share based on the dynamic load; at the same time, it provides the slice tenants degree of protection by keeping the pre-assigned share intact.

In the context of the above resource sharing mechanism, we consider a market scenario where a set of SPs lease their respective networks from InP and employ the network slicing mechanism to request the resources required for their service provision. We consider the SPs offer a particular service to users, and the resources inventory available with SPs characterizes their service performance. The users are free to choose their SP; their decisions are made based on the service satisfaction attained from SPs. Furthermore, we consider that the SP collects revenue by providing the service to its customer. Under the combined effect of a dynamic resource sharing mechanism and profit-oriented nature of SPs, it is highly expected that selfish SPs may exhibit strategic behaviour. For example, they might strategically distribute their shares on the resources conditioned on the tradeoff between quality of service (QoS) they want to offer and the congestion perceived at the resources. Such selfish behaviour could hamper the market's economic efficiency or cause instability in the network slicing mechanism. In this work, we focus on (1) building a business model representing the communication service market where SPs negotiate with InP to request resources and compete to serve a pool of end-users. (2) with the help of the proposed model, analyzing the effect of the network slicing mechanism (*i.e.,*SPCA based dynamic resource sharing mechanism) in terms of the economic efficiency and stability of the network.

Related work: There is an enormous amount of related work on the communication service market, broadly, the communication service market has been studied as a two-level market where three types of participants: Infrastructure provider (InP), Service Provider (SP[1]) and (EU) End Users, are generally considered. In the first level market, SPs (buyers) leases the resources from the InPs (sellers), negotiating for resource prices and resource quantity. In the second level, SPs (buyers) utilize the acquired resources from InPs to offer a certain service to their end users (buyers). At this level, SPs decide on their service price and scheduling of resources, while EUs make their subscription decisions. In [3], SPs' strategic decision over their service pricing scheme has been analyzed as Cournot game. In [4], authors considered that the Qos achieved by the user from SP depends on the number of subscribers associated with that SP, and users' choice behaviour can be analyzed by evolutionary game theory (EGT). The authors in [5] integrated both the users' choice evolution and the SPs pricing scheme and analyzed it with the Stackelberg game approach. The SPs, the leaders, strategically decide the price to attract the users and the users the followers choose the SPs to maximize their service satisfaction level. Also, the number of subscribers associated with the SPs depends on QoS and consequently the resources available with them; bearing in mind the competition among the SPs, resource demand by SPs can be analyzed with the non-cooperative game [6]. In [7], authors considered that competition between SPs takes place in pricing and quality of service SPs offer. In practice, SPs may not have complete information about the other SPs resources. Keeping this in mind, authors of [8] studied SPs' pricing behaviour with the bayesian game, where SPs decide the pricing schemes based on their belief about the resources available with others. Furthermore, the authors also considered the possibility of cooperation between the SPs and analyzed its impact on the pricing scheme. In all the above work, the SPs lease the resources from the InP and compete to serve end-users, which is also the case in our work. However, our work departs from these works in that resources are shared using a slice-based dynamic sharing mechanism. Moreover, in our case, resources are spatially distributed, and service offered in a particular cellular cell can only be supported by the resources available within that cell. In communication networks, one of the well-known scheme for resource allocation is the auction-based allocation [9] *e.g.,* kelly mechanism. Authors of [10], [11] proposed multi-bidding kelly mechanism-based resource allocation for 5G slicing. They showed that mechanism leads to a fair and efficient resource allocation on the level of both the slices and their end-users. Our work departs from the auction-based mechanism like [10]-[11], where agents' bids are unbounded.

In follow up work to [2], authors in [12] considered the network slicing under stochastic loads and applied SPCA based resource sharing scheme; they modeled resource sharing scheme as a game and showed that slices achieve efficient statistical multiplexing at the Nash equilibrium. The authors of [13] studied the communication service market where SPs employ the SPCA mechanism to request the resources from InP; they analyzed the economic impact of network slicing on the market. In [14], authors designed an automated negotiation mechanism based on the aggregate game that enables the slice tenants to dynamically trade the radio resources and customize their slices on instantaneous demands, which help tenants achieve higher profits. Our work is closely related to [13] however main novelty of our work lies in considering multi-resource service provisioning; at the best of our knowledge, the works related to the communication service market only deals with radio resource.

In this work, we leveraged the TC [15] framework to model the competition between the slices. This framework has been extensively used before in the communication field to model the interaction between competitive agents. To mention a few, in the paper, [16], the competition between social media users for visibility over the timeline has been model as a TC. The authors of [17] proposed the TC based incentive mechanism for crowdsourcing. The Tullock contest framework has been applied to the multipath TCP network utility maximization problem [18]. In the paper [19], authors studied the multi-cryptocurrency blockchain from a game-theoretic perspective, where the competition between the miners is framed as a TC.

Main Contribution: The key contributions of this work are the following 1)We proposed the business model for the service providers, where the SPs deploy the network slices for their business and leases their respective resources through network slicing mechanism (*i.e,* dynamic sharing). The SPs compete to serve end-users in terms of QoS. 2)We model the competition between the SPs as a multi-resource Tullock contest. To the best of our knowledge, this is the first paper where the framework of the multi-resource TC is used. 3)We show that the game induced through competition between the SP *i.e,* multi-resource Tullock rent-seeking game admits a unique Nash equilibrium (NE). Thus our theoretical results also contribute to the study of the tullock rent-seeking game. 4)We consider that the InP faces with challenge of deciding the resource pricing and we propose the trading post mechanism as its pricing solution 6) For some special case, we show that game induces by trading post mechanism admits unique Nash equilibrium. 7)We also provide the distributed reinforcement learning algorithms that provably converge to the game's unique NE.

The rest of the paper is organised as follows; Section II introduces the system model, Section III present the game-theoretic model of competition between the SPs. In Section IV, we study the existence and uniqueness properties of NE. Section V introduces resource pricing and market equilibrium; in section VI, we provide the distributed learning scheme. In

[1]In many works term Mobile virtual network operator (MVNO), tenate, slice, Mobile service provider (MSP) has been used for SP

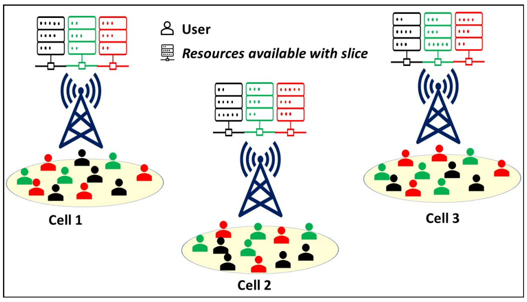

Fig. 1: service providers *i.e.,* (slices) compete to offer a certain service to geographically distributed pool of users

Section VII, we report on numerical results. A concluding section ends the paper.

TABLE I: Main notations used throughout the paper

$\mathcal{C} := \{1, \dots, C\}$	$\triangleq$	set of base stations or cells
$\mathcal{S} := \{1, \dots, S\}$	$\triangleq$	set of slices (tenants)
$\mathcal{M}^c$	$\triangleq$	set of resources at base station c
N^c	$\triangleq$	number of users in cell c
n_s^c	$\triangleq$	number (subscribers) users associated with slice s in cell c
$d_s^c := (d_{sm}^c, \dots, d_{sM}^c)$	$\triangleq$	bundle of resources available with slice s in cell c
d_{sm}^c	$\triangleq$	amount of resource type m available with slice s in cell c
D_m^c	$\triangleq$	the capacity of resource type m at base station c
q_s^c	$\triangleq$	the quality of service of slice s in cell c
ω_m^c	$\triangleq$	the price per unit resource of type m at base station c
p_s	$\triangleq$	service fees charge by slice s to users
B_s	$\triangleq$	budget available with slice s

II. System Model

We consider a market scenario, where in the first stage, a set of SPs $\mathcal{S}$ lease their respective networks from InP and employ the network slicing mechanism to request the resources required for their service provision. In stage two, the SPs (sellers) use the leased resources and compete to serve the large set of end-users (buyers). Specifically, as described in Figure 1, we consider InP owns a network that consists of a set of base stations or cells $\mathcal{C}$. Each base station at different locations accommodates multiple types of resources such as bandwidth, CPU, memory, etc. Users are spread across the network, let N^c number of users present in the cell c and service offered by SP in a particular cell can only be supported by the resources available within that cell.

A. User Model

We assume all the users need the same type of service, and they achieve their demand by subscribing to one of the SPs. We consider each user is opportunistic and free to change its SP, i.e., a slice from available slices at its base station. The user chooses a slice as its SP that offers a better deal, i.e., higher QoS at a lower price. We model the utility of each user served by SP $s \in \mathcal{S}$ in cell c as [8]

$$U_s^c(n_s^c, q_s^c, p_s) = \log\left(\frac{q_s^c}{n_s^c}\right) - p_s \tag{1}$$

Where q_s^c is the quality of service of SP s in cell c, n_s^c is number of users connected to SP s while p_s is the fees charged by SP for its service. Here the use of a logarithmic[2] function as the user's utility in QoS signifies that the users' satisfaction level saturates as the QoS increases, which is coherent with the economic principal of diminishing marginal returns. The SP's QoS depends on the resources inventory available with it. We assume each SP applies a scheduling policy to distribute its resources among users that achieve equal QoS among users in the long run. From (1) we observe that the utility of each user depends on the total number of users associated with the same SP, as the number of users connected to the same service increases the utility of the user decreases.

Assumption 1. *We assume that users revise their choice occasionally. As the users' selection process evolves, the market reaches equilibrium states where none of the users alters their SP choice, and the SPs provide equal utilities to operate with each other.*

This type of assumption is generally used in game theory while analyzing the strategic behaviour of a large number of selfish decision-makers, where for each decision-maker, exact information about all other decision-makers is rarely possible *e.g.,* Evolutionary game theory [20].

Lemma 1. *Under assumption* 1*, the number of users associated with each SP at equilibrium is given by*

$$n_s^c = \frac{N^c q_s^c e^{-p_s}}{\sum\limits_{s' \in \mathcal{S}} q_{s'}^c e^{-p_{s'}}} \tag{2}$$

Proof. AppendixA □

B. Service provider Model

We assume that the service providers aim at maximizing their number of subscribers (n_s^c) by attracting users with better QoS and lower price. In (2), the number of users joining a particular SP depends on QoS and the price offered by that SP and QoS and price offered by other SPs. Notice that expression for the number of users associated with SP, in the long run, resembles a contest success function from well know Tullock contest framework [21]. The TC framework is commonly used in economics literature for modeling economic or social interactions between two or more competing agents. The basic contest framework consists of competing agents who expend costly resources to win a prize (a contest); given the efforts exerted by all agents, the probability of an agent winning a prize is defined by the contest success function (CSF). Typically, the CSF function is defined as $\rho(x) = \frac{(x_i)^r}{\sum_{i'}(x_{i'})^r}$ where x_i is the effort of agent i and r is a parameter, for

[2]The logarithm function also signifies that the SPs achieve the proportional fair allocation between the user in the long run

example $r = 1$ is the well know lottery and $r \to \infty$ defines the all-pay auction.

In the slicing context, we consider that SPs compete to attract users to their services. SPs exert effort by expending costly resources; the resources acquired by SPs further reflect their service quality and help SPs to attract users. Thus, in our case, the contest success function represents the probability that any SP successfully attracts an end-user to its service. Keeping in mind the context of this work, we prefer to call contest success function as *slice association probability function* A_s, representing the probability that given resources expended by all SPs; a user will associate with a SP s. For our model, we defined a more general and multi-resource CSF function or slice association probability function

$$A_s^c(d^c, p) = \frac{f_s^c\left(d_s^c, p_s\right)}{\sum\limits_{s' \in \mathcal{S}} f_{s'}^c\left(d_{s'}^c, p_{s'}\right)} \tag{3}$$

Where function $f_s^c\left(d_s^c, p_s\right)$ is concave non decreasing in d_s^c and convex and decreasing in p_s. We assume that the QoS provided by SP depends on the resources inventory available to slice and its relation is defined as $q_s^c := q_s^c(d_s^c)$ where $d_s^c = (d_{sm}^c, \dots, d_{sM}^c)$ denotes a bundle of resources available with SP s and element d_{sm}^c shows amount of resource type m acquired by SP s at cell c. We assume that $\forall c \in \mathcal{C}$ and $\forall s \in \mathcal{S}$ function $q_s^c\left(d_s^c\right)$ is concave non decreasing in d_s^c, this type of assumption is widly use in economics signifying principle of diminishing marginal returns. In this work, we consider $f_s^c(d_s^c, p_s)$ as $q_s^c(d_s^c)e^{-p_s}$. In (3) the number of potential users in each cell as well as the slice association probability for each slice, might vary from cell to cell. The expected number of users associated with SP s throughout the network is defined as.

$$\sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) = \sum_{c \in \mathcal{C}} \frac{N^c f_s^c\left(d_s^c, p_s\right)}{\sum_{s'} f_{s'}^c\left(d_{s'}^c, p_{s'}\right)} \tag{4}$$

Each service provider collects the revenue from the fees paid by its subscribers. The expected revenue generated by SP s by its subscriber over the network is defined as.

$$R_s(d, p) = p_s \left(\sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) \right) \tag{5}$$

On the other hand, each SP needs to pay for the resources it leased from the InP. Let ω_m^c be the price per unit resource of type m charge by InP at base station c. Thus total cost each SP s needs to pay for its resources is $\sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c$. The profit gained by SPs is defined as

$$U_s(d, p) = p_s \left(\sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) \right) - \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c \tag{6}$$

We assume that each SP s is pre-assigned with a finite budget B_s, which depends on its service level agreement (SLA) with the InP, and this budget represents the SP's priority or a fixed share of the available resources pool, such that $\sum_{s \in \mathcal{S}} B_s = 1$. We observe that the profit gain by the SPs depends not only on their own decisions but also on decisions made by other SPs; in such a scenario, SPs might exhibit strategic behaviour and face the non-cooperative game.

III. Game Model

In this section, we model the interaction between the service providers as a non-cooperative game; we assume that the SPs are selfish, and each SP aims at maximizing its profit. We study the competition between the SPs in term of their quality of service, that is, how SPs strategically spend their budget on the resources to attract the users and, in turn, maximize their profits. The profit gain by the SPs depends on both their individual decision and the decision taken by their counterpart. The decision problem of each SP s is defined as.

$$Q_s \qquad \underset{d_s \in \mathcal{B}_s}{\text{maximize}} \quad U_s(d_s, d_{-s})$$

We assume that the service providers are strategic while making a decision; they also take into account the decision of other SPs. To theoretically analyze this strategic interaction, we define the non-cooperative game $\mathcal{G} := \left\langle \mathcal{S}, \left(\mathcal{B}_s\right)_{s \in \mathcal{S}}, \left(U_s\right)_{s \in \mathcal{S}} \right\rangle$ as follows:

- Player set: the set of service providers $\mathcal{S}$
- Strategy: the vector of resource demand $d_s = \left(d_s^1, \dots, d_s^C\right)$ where d_c^s is the amount of resource to be requested to the each base station c. The strategy set for each SP s is $\mathcal{B}_s$
- Utility: The utility of each SP s is equal to the U_s

To study the outcome of the defined game, we consider the standard notion of a Nash equilibrium,

Definition 1. *A strategy profile $d^* = (d_1{}^*, \dots, d_S{}^*)$ is called a NE of the game $\mathcal{G}$ if*

$$\forall s \in \mathcal{S},\, U_s(d_s{}^*, d_{-s}{}^*) \geq U_s(d_s, d_{-s}{}^*), d_s \in \mathcal{B}_s \tag{7}$$

Here, $(d_s, d_{-s}{}^*)$ denotes the strategy profile with s^{th} element equals d_s and all other elements equal $d_{s'}{}^*$ (for any $s' \neq s$).

In the next section, we analyze the existence and uniqueness of Nash equilibrium for the game $\mathcal{G}$

IV. Existence and Uniqueness of the Nash Equilibrium

In this section, we establish the existence and the uniqueness of Nash equilibrium of game $\mathcal{G}$; for the proof of the uniqueness of NE, we rely on the concept of diagonally strict concavity (DSC) introduced by Rosen [22]. Intuitively, DSC is a generalization of the idea of convexity to a setting of games.

Definition 2 (Diagonal strict concavity [22])**.** *A game with strategy vectors d and utility function U is called diagonally strict concave (DSC) for a given vector r if for every distinct $\bar{d}$ and $\hat{d}$,*

$$\left[g(\bar{d}, r) - g(\hat{d}, r)\right] (\bar{d} - \hat{d})' < 0 \tag{8}$$

with

$$g(d, r) = \left[r_1 \nabla_1 U_1(d), r_2 \nabla_2 U_2(d), \dots, r_S \nabla_S U_S(d)\right]. \tag{9}$$

where $\nabla_s U_s(d)$ denotes the gradient of utility of player s with respect its won strategy d_s

Theorem 1. *The game $\mathcal{G}$ always admits a unique NE.*

Proof. The utility of each SP in-game $\mathcal{G}$ is continuous, increasing, and concave, while the action space for each SP is convex and compact. Therefore the existence of an equilibrium for the game is followed by (Theorem 1 [22]). Now for the uniqueness of Nash equilibrium, If the utilities of players in the game $\mathcal{G}$ satisfies the DSC property, then the uniqueness of NE to game $\mathcal{G}$ follows by (Theorem 2 [22])

Let $G(d, r)$ be the Jacobian of $g(d, r)$ with respect to d, where d is any multistrategy of the game. In order to prove strict diagonal concavity of $g(d, r)$, by (Theorem 6 [22]), it is sufficient to prove that the symmetrized version of the pseudo-jacobian, i.e., $\widehat{G}(d, r) = G(d, r)+G(d, r)'$, is negative definite for all the domain of interest. To show that the $\widehat{G}(d, r)$ is negative definite it must be shown that following three conditions are satisfied:

C 1. *each $U_s(d)$ is a regular strictly concave function of d_s (i.e., its Hessian is negative definite)*

C 2. *each $U_s(d)$ is convex in d_{-s}*

C 3. *there is some $r > 0$ such that function $\sigma(d, r) = \sum_s r_s U_s(d)$ is concave in d*

then negative definiteness of $[G(d, r) + G'(d, r)]$ follows from Lemma 1 [23]. We first consider a case of single base station c and show that $\widehat{G}^c(d, r)$ is negative definite for this case. We calculate the Hessian $(H_s U_s^c)$ of utility of any SP s with respect to SP s owns strategy.

$$H_s U_s^c = -2\frac{p_s \sum\limits_{s'\in\mathcal{S}, s'\neq s} f_{s'}^c}{\left(\sum\limits_{s'\in\mathcal{S}} f_{s'}^c\right)^3}\left[(\nabla_s f_s^c)^T \nabla f_s^c - H_s(f_s^c)\sum_{s'\in\mathcal{S}} f_{s'}^c\right] \tag{10}$$

on the right hand side of (10) matrix $(\nabla_s f_s^c)^T \nabla f_s^c$ is positive semi-definite, where $\nabla_s f_s^c$ is gradient row vector of f_s^c with respect to its own strategy d_s^c, $H_s(f_s^c)$ is the Hessian of f_s^c with respect to d_s^c and its negative definite as f_s^c is concave. Thus the Hessian of utility $H_s U_s^c$ is negative definite and satisfies the first condition **C1**. Now we will show that the utility of each SP s is convex in the strategy of all other SPs, for that purpose consider the Hessian of utility of SP s with respect to strategy of all other SPs

$$H_{-s}U_s = 2\frac{f_s^c}{\left(\sum\limits_{s'\in\mathcal{S}} f_{s'}^c\right)^3}\left[M_s^c - diag_{-s}\left\{H(f_u^c)\right\}\right] \tag{11}$$

where is M_s^c block matrix and uv^{th} block is defined as

$$M_{suv}^c = (\nabla_u f_u^c)^T \nabla_v f_v^c \text{ where } u, v \neq s,\ u, v, s \in \mathcal{S} \tag{12}$$

$\nabla_u f_u^c$ is gradient row vector of f_u^c with respect to its own strategy and $diag_{-s}\{H(f_u^c)\}$ is block diagonal matrix with block u is $H(f_u^c)$ hessian of f_u^c with respect strategy vector of u itself $\forall u, u \neq s, u \in \mathcal{S}$. In right hand side of equation (11) matrix M_s^c is positive definite and the block diagonal matrix $diag_{-s}\{H(f_u^c)\}$ is negative definite as the each diagonal matrix element $H(f_u^c)$ is negative definite thus $H_{-s}U_s$ is positive definite, which satisfies the condition **C2**. Now we take the $r_s = \frac{1}{p_s}\ \forall s \in \mathcal{S}$ and then $\sigma(d, r) = \sum_s r_s U_s(d)$ is concave in d

Now we will extend the proof for multi-base station case; we have already shown $(\widehat{G}^c)$ is negative definite for any single base station c. For C base stations, consider a $\widehat{G}$ symmetrized version of the pseudo-Jacobian; after arranging columns and rows, we get (see Corollary 2 in [24])

$$(\widehat{G}) = diag\left\{\widehat{G}^1, \ldots, \widehat{G}^c, \ldots \widehat{G}^C\right\}$$

The above $\widehat{G}$ matrix is negative definite as each diagonal matrix is negative definite, which proves the DSC property holds for the multi-cell scenario. Then by Theorem 2 [22] the equilibrium point d^* for the game $\mathcal{G}$ is unique. □

V. Resource pricing and equilibrium

We have shown in the previous section that there exists a unique NE to game $\mathcal{G}$. We assume that the physical resources available with the InP in each cell are finite. Given per-unit prices for resources decided by the infrasture provider, the total resource demanded by SPs at NE of game $\mathcal{G}$ may violate the Infrastructure capacity. Thus, InP's primary concern is how to efficiently allocate the limited physical resources to competing SPs with diverse characteristics and preferences. The desired allocation must satisfy all the SPs and simultaneously maintain high resource utilization. In this regard, we assume that InP seeks the prizing scheme (per-unit prices) for each resource such that at the Nash equlibrium of game $\mathcal{G}$ each SP utilizes its entire budget and no resources remain leftover *i.e,* the total demand of resources matches the available infrastructure capacity. In economics, such a pricing decision problem has been often studied as a market equilibrium problem *e.g,* Fisher market [25]; market equilibrium is a solution concept where market prices are settled in such a way that the amount of resources requested by buyers is equal to the amount of resources produced or supplied by sellers.

One way to find market equilibrium or pricing scheme is through a tatonnement process, *i.e.,* if the demand for resources exceeds its capacity, increase the resource's price. Contrarily decreases resource's price when the demand is smaller than the capacity. The disadvantage of the above approach is that it does not always guarantee the ability to satisfy the resource capacity while applying such a process. To overcome this limitation, we use the approach introduced by Shapley and Shubik in their pioneer work [26], also known by the various names like Trading Post, share-constrained proportional allocation (SCPA) scheme [2]. Now we formally define the Trading post mechanism.

A. Trading post mechanism

In the trading post mechanism, each player (*i.e,* SP) places a bid on each type of resource. Once all SPs place the bids, each resource type's price is determined by the total bids placed for that resource. Precisely, let SP s submits a bid b_{sm}^c to resource m at cell c. The price per unit of resource m at cell c is then set to $\frac{\sum_{i\in\mathcal{N}} b^c_{sm}}{D^c_m}$, accordingly SP s receives a fraction of d^c_{sm} in return to his spending of b^c_{sm}.

$$d^c_{sm} = \begin{cases} \frac{b^c_{sm} D^c_m}{\sum_{u\in\mathcal{S}} b^c_{um}} & \text{if } b^c_{sm} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

After replacing d^c_{sm} in (6) in terms of bids and the decision problem of each SP s is written as below.

$$\hat{Q}_s \quad \underset{b_s}{\text{maximize}} \quad U_s\left(b_s, b_{-s}\right)$$
$$\text{subject to} \quad \sum_{c\in\mathcal{C}} \sum_{m\in\mathcal{M}^c} b^c_{s,m} \leq B_s, b^c_{s,m} \geq 0.$$

Here we consider two possible nature of the service providers; first, they are price takers. *i.e.,* they accept the price decided by the market, and they only act strategically in terms of demand for the resources. Second, SPs are price anticipating; they expect the effect of their demand on the price of the resources. Hence they act strategically in term of resource and the congestion on the resources. When SP are strategic in both, the trading post mechanism induces a new non-cooperative game. We define the non-cooperative game $\hat{\mathcal{G}}$ as follows:

- Player set: the set of SPs $\mathcal{S}$
- Strategy: the vector of bids $b_s = \left[b^1_s, \dots, b^C_s\right]$ where b^c_s is the bid to be submitted to the resource cell c. The strategy set for each SP s is $\mathcal{B}_s = \left\{b_s | \sum_{c\in\mathcal{C}} \sum_{m\in\mathcal{M}^c} b^c_{s,m} = B_s, \right\}$
- Utility: The utility of each SP s is equal to the U_s

To study the outcome of the mechanism, we consider the standard notion of NE,

Definition 3. *A multi-bid strategy $b^* = (b^*_1, \dots, b^*_S)$ is called a NE of the game $\hat{\mathcal{G}}$ if*

$$\forall i \in \mathcal{N},\, U_s(b^*_s, b^*_{-s}) \geq U_s(b_s, b^*_{-s}), b_s \in \mathcal{B}_s \tag{14}$$

Here, (b_s, b^*_{-s}) denotes the strategy vector with s^{th} element equals b and all other elements equal b^*_v (for any $v \neq s$).

For the proposed mechanism, interpretation of NE of game $\hat{\mathcal{G}}$ constitutes a stable bidding policy where each SP is satisfied with its individual utility characteristics and the existing resource allocation mechanism. Now, we investigate the existence and uniqueness properties of NE; showing the uniqueness and existence of multi-resource $\hat{\mathcal{G}}$ game requires complex calculations; thus, we keep our theoretical analysis of game $\hat{\mathcal{G}}$ limited to a single resource (radio resource). We assume that the QoS provided by SP s in cell c is given by $q^c_s = (d^c_s)^{\rho^c_s}$ where ρ^c_s is a sensitivity parameter and $0 < \rho^c_s \leq 1$, such type of function has been used in [13] to model the effect of users sensity towards their service provider selection. We replace $q^c_s = (d^c_s)^{\rho^c_s}$ in (1) and from (3) we get

$$A^c_s(d^c, p) = \frac{(d^c_s)^{\rho^c_s}\, e^{-p_s}}{\sum\limits_{s'\in\mathcal{S}} (d^c_{s'})^{\rho^c_{s'}}\, e^{-p_{s'}}} \tag{15}$$

Proposition 1. *If for single resource case, the QoS provided by SP s in cell c is defined by $q^c_s = (d^c_s)^{\rho^c_s}$ and $0 < \rho^c_s \leq 1$ then game $\hat{\mathcal{G}}$ admits unique NE.*

Proof. If the the QoS provided by SP s in cell c is defined by $q^c_s = (d^c_s)^{\rho^c_s}$ and $0 < \rho^c_s \leq 1$ then utilities of SPs satisfies the conditions **C1**,**C2** and **C3**, rest of proof is same as the proof of theorem 1. □

Moving ahead, now we compare the profit gain by service providers at the Nash equilibrium of the game with baseline static proportional allocation scheme (SS)*i.e.* allocation where each resource is allocated to a service provider s in proportion to its budget $\frac{B_s}{\sum_{s'\in\mathcal{S}} B_{s'}}$

Proposition 2. *For two service providers, the revenue gain under a dynamic resource sharing scheme at least equal to the revenue gain under a proportional sharing scheme*

Proof. Appendix C □

In the next section, we provide the distributed learning algorithm, which provable converge to both $\mathcal{G}$ and $\hat{\mathcal{G}}$ games' unique Nash equilibrium.

VI. Distributed Learning Algorithm

We have already proved in the previous section that the Game $\mathcal{G}$ admits a unique equilibrium for any price vector decided by the Infrasture provider. However, we still need to verify whether tenants can reach this equilibrium in a distributed fashion. In this regard, we propose an exponential learning algorithm that allows the tenants to converge to the game's unique NE. The proposed learning algorithm is a special case of dual averaging or mirror–descent method suggested for continuous action convex games [27]. Now, we proceed by describing the dual averaging method; in the dual averaging method, each player *i.e.,* SP s estimates its marginal utility or utility gradient with respect to its own strategy. To increase their utilities, players need to take action along the direction of their utility gradient while maintaining their action in feasible action space. In order to achieve this, each player s at each time step n accumulates its discounted utility gradient in some auxiliary variable y_s,

$$y_s(n+1) = \left[y_s(n) + \alpha_n\, \nabla_{b_s} U_s(b_s(n), b_{-s}(n))\right]. \tag{$\mathcal{A}1$}$$

In the above equation α_n denotes the discount factor or step size. Once the discounted gradient has been accumulated , every SP s utilize its own updated value of the auxiliary variable y_s to take the next feasible action.

$$b_s(n+1) = Q_s(y_s). \tag{16}$$

In turn, each SP s maps the recent value of auxiliary variable y_s to its decision space $\mathcal{B}_s$ using the some mapping $Q_s(y_s)$, *e.g.,* Q_s can be projection map. The map $Q_s(y_s)$ is defined in more general as

$$Q_s(y_s) = \underset{b_s\in\mathcal{B}_s}{\operatorname{argmax}} \left\{\langle y_s(n), b_s\rangle - h_s(b_s)\right\}, \tag{$\mathcal{A}2$}$$

where $h_s(b)$ is regularization function or a penalty function over the feasible action set $\mathcal{B}_s$. Here penalty $h_s(b)$ helps the convergence of algorithm within the interior of the feasible domain set. The different value regularization functions induce different maps. We propose using the Gibbs entropy function as a regularization function

$$h_s(b_s) = \sum_{c\in\mathcal{C}} \sum_{m\in\mathcal{M}} b^c_{sm} \log(b^c_{sm}). \tag{17}$$

We replace $h_s(b_s)$ in equation ($\mathcal{A}2$) by the entropic regularization function and after some calculation we get exponential mapping

$$b^c_{sm} = \frac{B_s \exp(y^c_{sm})}{\sum_{c\in\mathcal{C}} \sum_{k\in\mathcal{M}} \exp(y^c_{sk})}. \tag{18}$$

The induced map $Q_s(y_s)$ is similar to well know Logit map, where each player distributes his budget (weights) to different resources depending on exponential of accumulated discounted gradients.

Algorithm 1 On-line Distributed Learning Algorithm

Require: $\sum_{n=0}^{\infty} \alpha_n = \infty, \alpha_n \to 0$ as $n \to \infty$
```
repeat n = 1, 2, . . . ,
    for each SP s ∈ S
        Observe gradient of utility and update
        y_s = [y_s + α_n ∇_{b_s} U_s(b_s, b_{-s})]
    end for
    for each player s ∈ S
        for each cell x ∈ C and resource m ∈ M^c
            Play b^c_sm ← B_s exp(y^c_sm) / (Σ_{c∈C} Σ_{k∈M} exp(y^c_sk)).
        end for
    end for
until ‖(b(n) − b(n − 1)‖ ≤ ϵ
```

Theorem 2. *If the Algorithm* 1 *satisfies the required conditions for step size sequence,* $\sum_{n=0}^{\infty} \alpha_n = \infty, \alpha_n \to 0$ *as* $n \to \infty$ *then distributed Algorithm 1 converges to the unique NE of the Game* $\mathcal{G}$

Proof. As we have already discussed, the proposed exponential algorithm is the special case of the dual averaging algorithm. If the NE of the any continuous action convex game is strictly r-variationally stable, then the converges of the dual averaging algorithm to a unique NE of the game is guaranteed by Theorem 4.6 [27]. Hence to prove the convergence of the proposed algorithm, it is sufficient to show that the unique NE of game $\mathcal{G}$ is strictly r-variationally stable. The unique NE $\hat{b}$ to the any convex game is strictly r-variationally stable if $\forall b_s \in \mathcal{B}_s$

$$\sum_{s\in\mathcal{S}} r_s \nabla_s U_s(b)(b_s - \hat{b}_s) < 0 \tag{19}$$

As we have already shown in section IV that utility of SPs in game $\mathcal{G}$ satisfies the diagonal strict concavity for $r_s = \frac{1}{p_s}$, $\forall s \in \mathcal{S}$

$$\sum_{s\in\mathcal{S}} r_s \left[\nabla_s U_s(b) - \nabla_s U_s(\hat{b})\right](b_s - \hat{b}_s) < 0 \tag{20}$$

Now we know that for any continuous action convex game, a feasible point $\hat{b}$ is a Nash equilibrium of the game if and only if

$$\sum_{s\in\mathcal{S}} r_s \nabla_s U_s(\hat{b})(b_s - \hat{b}_s) \le 0 \tag{21}$$

From inequality (21) and (20) implies (19), which proves that the unique NE of game $\mathcal{G}$ is strictly r-variationally stable and then by Theorem 4.6 [27] Algorithm1 converges to unique NE of game $\mathcal{G}$ □

VII. NUMERICAL EXPERIMENTS

In this section, we illustrate an analysis of the dynamic resource allocation scheme with the support of numerical results. Our simulation primarily focuses on a network with two cells, CI and CII, and two service providers SP1 and SP2, who request the resources for their service provision. This setting allows us to efficiently study the dynamics of interaction between users and SPs and the effect of different system parameters on the outcome of the game $\mathcal{G}$. We assume there are 200 and 300 users present in the cell CI and CII, respectively. First, we consider the simple case of a single resource where the quality of service offered by the slices only depends on the radio resource (bandwidth). The plot in Figure 2 (c) illustrates the impact of the price parameter on the number of users associated with the slices at the NE of $\mathcal{G}$. For this simulation, we assume that the price applied by the SP1 is constant 5, and we vary the fee applied by SP2 in the range of 0 to 10. Figure 2(c) shows the change in the distribution of users associated with the SPs as a function of price applied by SP1. In the same figure, we also analyze the effect of slices shares on the distribution of users at the outcome of the game. The regular lines in red and blue show the distribution of users with SP1 and SP2 as a function of price provided by SP1 and when SP1 and SP2 are assigned with 10% share and 90% share of the infrastructure, respectively. The plots with the dashed line, dot line and dot-dash line are the outcome when 30%, 70% and 90% of share are assigned to SP1. With the same settings, the simulations in Figure 2(d) illustrate the impact of price applied by the slices and their infrastructure share on the revenue gain by them. From Figure 2(c) we can observe that the SPs' subscribers decrease with their offered price, while the rate in the fall in the subscriber's is reducing in their budgets. The Figure 2(d) shows that the revenue gain of SPs is increasing in their budgets. As second case, we consider QoS provided by SP s in cell c is given by $q^c_s = (d^c_s)^{\rho^c_s}$ where ρ^c_s is sensitivity parameter and $0 < \rho^c_s \le 1$, we vary the ρ^2_2 *i.e.* the sensitivity parameter for SP2 in cell C2 form 0.1 to 1, the Figure 2(b) shows the comparison of profit gain by SPs at Nash equilibrium with the profit gain under static resource allocation scheme. For the multi-resource case, we consider that the quality of service provided by the SPs depends on their bandwidth as well as power allocation. To be precise, we assume that the QoS is the maximum possible data rate that SP can achieve, given by

$$q_s = B_s \log_2\left(1 + \frac{h^2 P_s}{N_0}\right) \tag{22}$$

Where B_s and P_s is bandwidth and power allocated to SP s respectively, while h is channel gain and N_0 noise, For simulations purpose, we assume that the availability of maximum bandwidth and transmitting power at each base station is 30Mhz and 47dBm, respectively. The prices applied

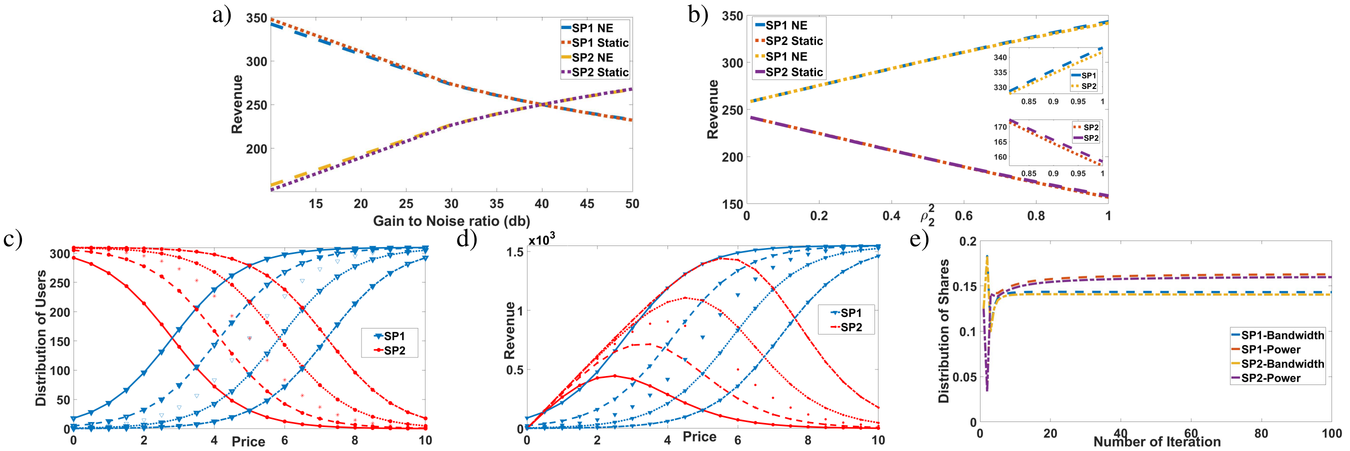

Fig. 2: a) Comparison between the revenue gain by the SPs at NE of game vs the revenue gain under SS for the different value of power to noise ratio of SP2 at C2. b) Comparison between the revenue gain by the SPs at NE of game vs the revenue gain under SS for the different value of the sensitivity parameter ρ_2^2. c) The distribution of users at NE wrt fees charged by SPs d) The revenue gain by SPs wrt fees charged by them e) Converges of the distributed algorithm 1 to NE

by each SP is constant 1, and each SP is assigned with half of the infrastructure share. For the numerical experiments, we vary the channel gain to noise ratio for SP2 at cell C2 from 10db to 50db; for each value of the channel gain to noise ratio, we compute the Nash equilibrium. The Figure 2(a) shows a comparison between the profit gain by SPs at Nash equilibrium with profit gain by SPs under a static resource allocation scheme. Figures 2(a) and 2(b) show a negligible difference between the revenue gain under the dynamic resource sharing scheme and static resource allocation and dynamic resource sharing scheme. It allows service providers with efficient resource sharing while keeping the revenue of SPs coherent with their SLA.

VIII. Conclusion

In this work, we have considered a communication market scenario where service providers lease resource from infrastructure provider through a network slicing mechanism (SPCA) and compete to serve a large pool of end-users. We have modeled the competition between the service provider as the muti resource tullock rent-seeking game. We have proved that the resultant game admits a unique Nash equilibrium. We have considered that InP faces the challenge of finding the pricing scheme (per-unit prices) for each resource such that at the Nash equilibrium of the game, total demand satisfies the capacity of the infrastructure. In this regard, we have proposed the trading post-mechanism-based resource allocation. For some limited cases, we have shown that game induced by the trading post mechanism admits a unique Nash equilibrium; thus, resource allocation through the slicing mechanism is provably stable. We have provided the distributed exponential learning algorithm, which allows service providers to reach the unique Nash equilibrium of the game. Our numerical results confirm that under SPCA, the network slicing mechanism enables service providers with stable and economically efficient resource utilization. In the future, we will consider different types of SLA models for SPs and develop a general resource sharing and resource pricing scheme based on the concept of normalized Nash equilibrium and coupled constrained game [22][15].

Appendix

A. Proof Of Lemma I

To find the equilibrium of above dynamics consider

$$\log\left(\frac{q_s^c}{n_s^c}\right) - p_s = \log\left(\frac{q_{s'}^c}{n_{s'}^c}\right) - p_{s'} \tag{23}$$

taking exponential of both sides

$$\frac{q_s^c}{n_s^c}\frac{n_{s'}^c}{q_{s'}^c} = e^{p_s - p_{s'}} \tag{24}$$

$$\frac{q_s^c}{n_s^c} n_{s'}^c = q_{s'}^c e^{p_s - p_{s'}} \tag{25}$$

summing over $\forall s' \in \mathcal{S}$

$$\sum_{s'} \frac{q_s^c}{n_s^c} n_{s'}^c = \sum_{s'} q_{s'}^c e^{p_s - p_{s'}} \tag{26}$$

$$n_s^c = \frac{N^c q_s^c e^{-p_s}}{\sum_{s'} q_{s'}^c e^{-p_{s'}}} \tag{27}$$

B. Proof of Preposition 1

$\frac{\partial^2 U_s}{\partial {d_1}^2} = \frac{A+B}{C} < 0$

$$A = -\left(\frac{b_2^c}{b_1^c + b_2^c}\right)^{\rho_2}\left(\frac{b_1^c}{b_1^c + b_2^c}\right)^{2\rho_1}\Big(\left({\rho_2}^2 + \rho_2\right){b_1^c}^2 + 2\rho_1 b_2^c(\rho_2 + 1)b_1^c + \rho_1 {b_2^c}^2(\rho_1 + 1)\Big) \tag{28a}$$

$$B = \left(\frac{b_2^c}{b_1^c + b_2^c}\right)^{2\rho_2} \left(\frac{b_1^c}{b_1^c + b_2^c}\right)^{\rho_1} \Big(\left(\rho_2{}^2 - \rho_2\right) b_1^{c\,2} + 2\rho_1 d_2^c(\rho_2 - 1) b_1^c + \rho_1 b_2^{c\,2}(\rho_1 - 1) \Big) \tag{28b}$$

$$C = (b_1^c + b_2^c)^2 b_1^{c\,2} \left(\left(\frac{b_1^c}{b_1^c + b_2^c}\right)^{\rho_1} + \left(\frac{b_2^c}{b_1^c + b_2^c}\right)^{\rho_2} \right)^3 \tag{28c}$$

$\frac{\partial^2 U_s}{\partial {d_1}^2} = \frac{G+H}{I} > 0$

$$G = -\left(\frac{b_2^c}{b_1^c + b_2^c}\right)^{\rho_2} \left(\frac{b_1^c}{b_1^c + b_2^c}\right)^{2\rho_1} \Big(\left(\rho_1{}^2 - \rho_1\right) b_2^{c\,2} + 2\rho_2 b_1^c(\rho_1 - 1) b_2^c + \rho_2 b_1^{c\,2}(\rho_2 - 1) \Big) \tag{29a}$$

$$H = \left(\frac{b_2^c}{b_1^c + b_2^c}\right)^{2\rho_2} \left(\frac{b_1^c}{b_1^c + b_2^c}\right)^{\rho_1} \Big(\left(\rho_1{}^2 + \rho_1\right) b_2^{c\,2} + 2\rho_2 b_1^c(\rho_1 + 1) b_2^c + \rho_2 b_1^{c\,2}(\rho_2 + 1) \Big) \tag{29b}$$

$$I = (b_1^c + b_2^c)^2 b_2^{c\,2} \left(\left(\frac{b_1^c}{b_1^c + b_2^c}\right)^{\rho_1} + \left(\frac{b_2^c}{b_1^c + b_2^c}\right)^{\rho_2} \right)^3 \tag{29c}$$

C. Proof of Preposition 2

Consider that for any bid $b_2^c > 0$ submitted by SP 2 at cell c SP 1 place a bid of $b_1^c = B_1 \frac{b_2^c}{B_2}$ at cell c then quantity of resource received by SP1 at cell c $d_1^c = \frac{B_1 \frac{b_2^c}{B_2}}{B_1 \frac{b_2^c}{B_2} + b_2^c} = \frac{B_1}{B_1 + B_2}$ this proves that for any strategy played by service provider there exist strategy for opponent SP such that it receives the resources in proportion to its budget

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