



**HAL**  
open science

# A Bayesian Framework for Poisson Process Characterization of Extremes with Objective Prior

Théo Moins, Julyan Arbel, Anne Dutfoy, Stéphane Girard

► **To cite this version:**

Théo Moins, Julyan Arbel, Anne Dutfoy, Stéphane Girard. A Bayesian Framework for Poisson Process Characterization of Extremes with Objective Prior. ISBA 2021 - World Meeting of the International Society for Bayesian Analysis, Jun 2021, Virtual, France. hal-03347871

**HAL Id: hal-03347871**

**<https://hal.inria.fr/hal-03347871>**

Submitted on 17 Sep 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A Bayesian Framework for Poisson Process Characterization of Extremes with Objective Prior

Théo Moins<sup>1</sup>, Julyan Arbel<sup>1</sup>, Anne Dutfoy<sup>2</sup>, Stéphane Girard<sup>1</sup>

<sup>1</sup>Statify, Inria Grenoble Rhône-Alpes

<sup>2</sup>EDF R&D dept. Périclès

2021 World Meeting of the International Society for Bayesian Analysis

July 01, 2021

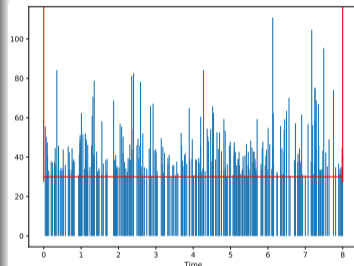
# Motivations

Let  $(X_1, \dots, X_n)$  be i.i.d. r.v. and the associated point process  $N_n$ , evaluated on  $I_u = [u, +\infty)$ .

## Theorem 1 (S. Coles 2001)

Under mild conditions and for a sufficiently large  $u$ ,  $N_n$  can be approximated by a non-homogeneous Poisson process  $N$  of intensity measure  $\Lambda$  with parameters  $\theta = (\mu, \sigma, \xi) \in \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}$ , such that for all  $x > u$ ,

$$\Lambda(I_x) = \int_x^{+\infty} \lambda(t) dt = \begin{cases} \left\{ 1 + \xi \left( \frac{x-\mu}{\sigma} \right) \right\}_+^{-\frac{1}{\xi}} & \text{if } \xi \neq 0, \\ \exp\left(-\frac{x-\mu}{\sigma}\right) & \text{if } \xi = 0. \end{cases}$$



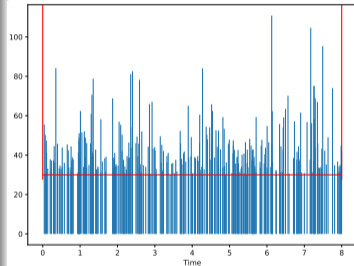
# Motivations

Let  $(X_1, \dots, X_n)$  be i.i.d. r.v. and the associated point process  $N_n$ , evaluated on  $I_u = [u, +\infty)$ .

## Theorem 1 (S. Coles 2001)

Under mild conditions and for a sufficiently large  $u$ ,  $N_n$  can be approximated by a non-homogeneous Poisson process  $N$  of intensity measure  $\Lambda$  with parameters  $\theta = (\mu, \sigma, \xi) \in \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}$ , such that for all  $x > u$ ,

$$\Lambda(I_x) = \int_x^{+\infty} \lambda(t) dt = \begin{cases} \left\{ 1 + \xi \left( \frac{x-\mu}{\sigma} \right) \right\}_+^{-\frac{1}{\xi}} & \text{if } \xi \neq 0, \\ \exp\left(-\frac{x-\mu}{\sigma}\right) & \text{if } \xi = 0. \end{cases}$$



$$\text{Likelihood: } p_{PPP}(\mathbf{x} \mid \mu, \sigma, \xi) = \exp \left\{ -m \left( 1 + \xi \left( \frac{u-\mu}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right\} \sigma^{-n} \prod_{i=1}^n \left( 1 + \xi \left( \frac{x_i - \mu}{\sigma} \right) \right)^{-\frac{1}{\xi} - 1}$$

# The Bayesian Point of view

Inference on  $\theta = (\mu, \sigma, \xi)$  using Bayesian algorithms such as MCMC.

The main advantages are (S. G. Coles et al. 1996):

- Consideration of expert information with informative prior,
- Can be used in any case, even when the likelihood is irregular (for  $\xi < -1$ ),
- Access to the posterior predictive distribution:

$$p(\tilde{x} | \mathbf{x}) = \int \underbrace{p(\tilde{x} | \theta)}_{\text{randomness of future obs.}} \underbrace{p(\theta | \mathbf{x})}_{\text{parameter uncertainty}} d\theta.$$

# The Bayesian Point of view

Inference on  $\theta = (\mu, \sigma, \xi)$  using Bayesian algorithms such as MCMC.

The main advantages are (S. G. Coles et al. 1996):

- Consideration of expert information with informative prior,
- Can be used in any case, even when the likelihood is irregular (for  $\xi < -1$ ),
- Access to the posterior predictive distribution:

$$p(\tilde{x} | \mathbf{x}) = \int \underbrace{p(\tilde{x} | \theta)}_{\text{randomness of future obs.}} \underbrace{p(\theta | \mathbf{x})}_{\text{parameter uncertainty}} d\theta.$$

**Computational Bayesian issues:** prior elicitation, reparameterization, etc.

# Uninformative prior for extremes

# Uninformative prior for extremes

## Theorem 2 (Moins et al.)

*The uniform prior on  $(\mu, \log \sigma, \xi)$  leads to a proper posterior for the Poisson process model for extremes as soon as  $n \geq 3$ .*



# Uninformative prior for extremes

## Theorem 2 (Moins et al.)

*The uniform prior on  $(\mu, \log \sigma, \xi)$  leads to a proper posterior for the Poisson process model for extremes as soon as  $n \geq 3$ .*

## Theorem 3 (Moins et al.)

*Jeffreys prior for a Poisson process for extremes with parameters  $\theta = (\mu, \sigma, \xi)$  exists provided  $\xi > -1/2$ , and can be written as*

$$\pi_{J,PP}(\mu, \sigma, \xi) \propto \frac{(1 + \xi (\frac{u-\mu}{\sigma}))^{-\frac{3}{2\xi}-1}}{\sigma^2(1 + \xi)(1 + 2\xi)^{\frac{1}{2}}}.$$

*It is improper and leads to a proper posterior as soon as  $\xi > -1/2$ .*

# An orthogonal parameterisation

**Alternative parameterisation:** instead of  $(\mu, \sigma, \xi)$ , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

$$(r, \nu, \xi) = \left( m \left( 1 + \xi \left( \frac{u - \mu}{\sigma} \right) \right)^{-1/\xi}, (1 + \xi)(\sigma + \xi(u - \mu)), \xi \right).$$

$r$ : intensity of PP, expected number of exceedances.

This ensures full orthogonality, which means that  $I(r, \nu, \xi)$  is diagonal.

# An orthogonal parameterisation

**Alternative parameterisation:** instead of  $(\mu, \sigma, \xi)$ , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

$$(r, \nu, \xi) = \left( m \left( 1 + \xi \left( \frac{u - \mu}{\sigma} \right) \right)^{-1/\xi}, (1 + \xi)(\sigma + \xi(u - \mu)), \xi \right).$$

$r$ : intensity of PP, expected number of exceedances.

This ensures full orthogonality, which means that  $I(r, \nu, \xi)$  is diagonal.

**Jeffreys prior:**

$$\pi_{J,PP}(r, \nu, \xi) \propto \frac{r^{1/2}}{\nu(1 + \xi)(1 + 2\xi)^{1/2}}.$$

# An orthogonal parameterisation

**Alternative parameterisation:** instead of  $(\mu, \sigma, \xi)$ , run MCMC on an orthogonal parameterisation suggested for another purpose by (Chavez-Demoulin et al. 2005):

$$(r, \nu, \xi) = \left( m \left( 1 + \xi \left( \frac{u - \mu}{\sigma} \right) \right)^{-1/\xi}, (1 + \xi)(\sigma + \xi(u - \mu)), \xi \right).$$

$r$ : intensity of PP, expected number of exceedances.

This ensures full orthogonality, which means that  $I(r, \nu, \xi)$  is diagonal.

**Jeffreys prior:**

$$\pi_{J,PP}(r, \nu, \xi) \propto \frac{r^{1/2}}{\nu(1 + \xi)(1 + 2\xi)^{1/2}}.$$

**Our experiments show better convergence with orthogonal parameters.**

# References

- Chavez-Demoulin, V. and A.C. Davison (2005). “Generalized additive modelling of sample extremes”. In: *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 54.1, pp. 207–222.
- Coles, S. G. and E. A. Powell (1996). “Bayesian Methods in Extreme Value Modelling: A Review and New Developments”. In: *International Statistical Review / Revue Internationale de Statistique* 64.1, pp. 119–136.
- Coles, S.G. (2001). *An introduction to statistical modeling of extreme values*. Springer Series in Statistics. London: Springer-Verlag. ISBN: 1-85233-459-2.