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Conceptual Model of Debris Flow Information System

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Abstract. Debris flows are natural hazard triggered by intensive rainfall or snowmelt and represent rapid movement of water-saturated colluvial and proluvial earth masses. The propagation of this hazardous event could change ecosystems, increase the solid discharge in the rivers and dam siltation, and affect infrastructure and people. The compound character of debris flows requires collection and analysis of various information and for this purpose, the computer technology and geographic information systems provide great opportunity. The aim of the paper is to present a conceptual model of debris flow information system, which to be used for risk assessment and to support decision making. The study emphasizes to factors and prerequisites, debris flow data, analyses and visualization. A fuzzy logic model for integrated risk assessment of the debris flow due to the multiple natural factors (as rainfall duration, rainfall amount, slope, erosion etc.) is proposed. An example of geoinformation portal is presented.

Keywords: Debris flow, Information System, Fuzzy logic, Risk assessment.

1 Introduction

In the internationally accepted classification proposed by Varnes [1] and developed by Cruden and Varnes [2] debris flows are a type of landslides that consist of a spatially continuous movements of a saturated mass of earth materials, such as debris and mud, mainly controlled by gravity and whose movement mechanics resemble that of a viscous liquid. There is a progression and a change from slides to flows according to water content, mobility and evolution of the movements. As a result of torrential rainfalls or rapid snowmelt in stripped of vegetation mountain slopes, composed mainly of eluvial and deluvial deposits, as well as cracked and rapidly eroding rocks, with landslides and collapses, a water-saturated mobile mud-stone mass is formed. It flows down the slope, drawing in extra bulk material. These masses fall into river beds and temporary streams, where they further destroy the banks and increase the amount of mud and stone material, forming a powerful flow [3]. It has to be taken into account that debris flows can return to slopes where they have already been and start in places they have never been before.

In regard to the above the debris flows are associated with the probability of negative impacts and are considered as limiting factors for people's lives and activities. Rising public awareness about natural hazards could improve the quality of life, save financial resources and even save lives. Development of computer technology and geographic information systems (GIS) allow effective analyses and visualization of the information and helps for better understanding of hazardous event.

Different types of information systems are described in the literature: knowledge information system, management information system (disaster management information system), decision support information system, hazard monitoring information system [4-7].

Disaster information systems use spatial information and disaster attribute information. The systems provide functions necessary for disaster management such as disaster zone management, geological analysis and automatized calculation of damage area, and made the efficient disaster management through the extraction of cadastral information and damage area more than certain scale by damage area [5]. Decision support systems provide data and tools for analyses and considering alternatives to help decision making. Hazard and risk assessment are important part of decision making process and are also closely related to monitoring of hazardous event. In case of compound events like as debris flow a big volume of data about factors and prerequisites, characteristics of the event and damage information should be acquired, sorted, analyzed and visualized in information system to ensure effective activities for risk mitigation and management. Building the information system have to take into account the properties of the hazardous natural phenomena and the purpose of which the system to be used.

Considering the properties of debris flows and their compound character, the aim of the current paper is to present a model of debris flow information system to be used for risk assessment and to support decision making.

2 Structure of the Information System

The information system is considered as an integration of computer hardware, software and data about the hazardous event, in the current case – debris flow. The spatial character of the debris flows allows development of the system in GIS environment. Four main modules determine the system: data acquisition/data entering; processing; analyses and visualization (Figure 1). Hazard and risk assessment are results of the analyses and could be calculated on the base of the data and spatial analyses. Taking into account the complexity of the hazardous event and the importance of multi-criteria decision analysis as a tool to support decision makers this module is presented as a separate module in the system, allowing to choose the best alternative taking into account the results from risk assessment. The visualization is available at each stage of the system work. The data can be presented on maps and/or as a text and tables.

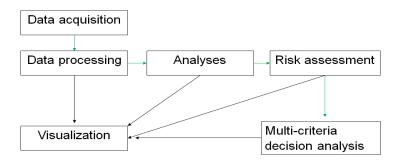


Fig. 1. Structure of the information system

Data acquisition and processing are one of the most important part of the system. The data can be collected by field surveys (using UAVs, LiDAR, sediments sampling), automatic meteorological station (for precipitation and particularly for intensive rain) and literature and maps review. The data processing is mainly related to the consistency of the data, coordinate systems, projections and transformations, vector to raster conversion or vice versa etc. The module "Analyses" allows to make calculations about the debris flows properties, morphometric features of the basins, debris initiation area, transport and accumulation, to analyze the spatial distribution of the hazardous event and to evaluate the debris flow susceptibility and hazard. On the base of the analyses and taking into account the possibility the infrastructure and people to be affected risk assessment can be elaborated. The multi-criteria decision analysis is presented as a separate module of the system, taking into account the importance of the decision making and the need for quick response in case of risk and disaster. This analysis is not a decision making but it is a main tool to support decision making allowing to choose, sort and rank alternatives and solutions according to pre-defined criteria.

3 Main Characteristics of the Data Module

The data module includes the following main groups (Figure 2): baseline data; factors and prerequisites (analysis that is resulted in debris flow susceptibility model); debris flow data; damage information and ecosystem information (used as indicators for calculating hazard and risk assessment).

Baseline data provides details of the regions and places where such events have been occurred for the compilation of the inventory maps. This map is a base for debris flow susceptibility and hazard assessment. Topographic data is of high importance for calculating and analyses of morphometric features of debris flows basins, which can be used as indicators for debris flow susceptibility evaluation. For example many publications consider the basin area, basin relief, relief ratio, Melton's index and slope as indicators for determining and analyses of debris flows initiation areas and propagation [8-14]. For the purpose of the analyses high resolution DEMs are needed.

Regarding the debris flows factors main triggering forces are intensive rainfall and rapid snowmelt. In this relation, the information system should contain data about daily precipitation, mean daily temperature and maximum daily temperature.

Prerequisites for debris flows occurrence and propagation that have to be taken into account are slope, lithology, active faults and vegetation. Slope is the most significant topographic parameter in the debris flow initiation analysis [12]. Most of publications show that debris flow is initiated in an area steeper than 15 ° or 20° [11, 12, 15]. Existence of an unstable hillside in a steep slope is a prerequisite for landslides and debris flows. Having regard the results in the above publications and our investigations of debris flows areas in the Eastern Rhodopes (Bulgaria) we suggest the following classes of slope gradients (in degrees): 0-2; 2-5; 5-15; 15-30; 30-45 and >45, where slopes in a range 30-45° are most susceptible for debris flows initiation. Longitudinal gradient of riverbed, plane and longitudinal configurations of river course also have to be taken into account. The changes in the longitudinal profile of the slope or in the acceleration or deceleration of the flow can be presented by profile curvature of the topographic surface. Plan curvature indicates the changes in flow line divergence.

Baseline Data: Locations of debris flows; **Debris Flow Data:** Topographic data - DEM; topographic maps Arrival time Flow depth Flow velocity Factors and prerequisites: Peak discharge Rainfall Event duration Snowmelt Debris flow volume Slopes Inundated area Lithology Vegetation Indicators for Hazard and risk assessment Debris flow susceptibility

Fig. 2. Data module – main characteristics

A data about lithological variations of the rocks, the presence of weathered rocks, sandy-clay sediments with inclusions of boulders and gravel, cover of altered soils should be entered in the information system for the purpose of debris flows susceptibility and hazard assessment. A special attention have to be given to unstable sediments and the following parameters: thickness of weathered soil layer in a hillside slope, thickness and amount of riverbed sediment, volumetric concentration and grain size distribution of the sediments, accumulated sediments due to slope failure.

Vegetation has a controlling role for debris flows occurrence. Usually forest areas are less prone to debris flows while bare soils and arable lands are considered as more prone to debris flows. The data about land use and land cover should be considered in the analysis of debris flow susceptibility and hazard. Fires and land cover changes also have to be taken into account. When a wildfire burns wooded mountain slopes, it increases the chance of debris flows for several years. Uncontrolled deforestation exposes

slopes to rainfalls and erosion, and makes them vulnerable to rapid movement of slope materials.

Debris flows arrival time, flow depth, flow velocity, peak discharge, event duration, debris flow volume and inundated area are indicators used for hazard and risk assessment. The results of the assessment can be visualized as debris flow hazard maps and debris flow risk assessment.

Taking into account the possible impacts of debris flow on infrastructure and ecosystem, in the study it is suggested two separate subsets of "Data" module: damage information and eco-system information (Figure 3) which provide information for risk assessment.

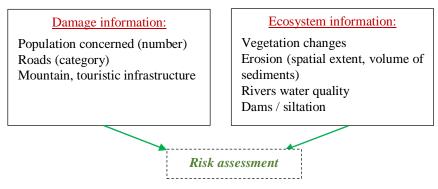


Fig. 3. Data module - indicators for risk assessment

4 Main Characteristics of the Analyses Module

The module "Analyses" of the system has to provide statistical and spatial analyses. Statistical analyses are needed to investigate the climate parameters and debris flows data like as frequency, peak discharge, flow velocity and debris volume.

The spatial character of the data concerned to debris flows allows application of spatial analyses like as spatial overlay, data reclassification, surface analyses etc. For example in result of spatial overlay of the data about factors and prerequisites for debris flows (Figure 4) a susceptibility assessment is derived which allows to determine the debris flow prone areas. Of high importance in debris flow susceptibility assessment is assigning weights of different factors and prerequisites which to be applied in weighted overlay analysis. For this purpose Analytic Hierarchy Process can be done [16, 17].

5 Main Characteristics of the Risk Assessment Module

Risk assessment module is an important part of the proposed debris flow information system. Accepting that risk assessment is a function of hazard and following the concept of risk [18-20] the assessment can be considered in natural phenomena context (Figure 5a) and in industrial context (Figure 5b).

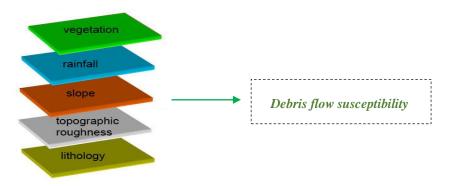
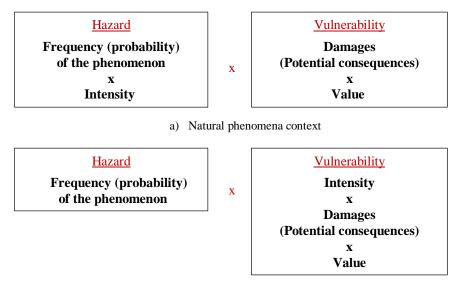


Fig. 4. Spatial overlay



b) Industrial context

Fig. 5. Risk assessment

For the assessment of debris flow susceptibility, hazard and vulnerability a fuzzy logic can be applied. The risk assessment using fuzzy logic is particularly recommended in case of data limitation and unclear, gradual boundaries of the distribution of natural factors (indicators) [17].

In this study as a part of the risk assessment module is proposed to be included a fuzzy logic model for integrated risk assessment from the debris flow due to the multiple natural factors (as rainfall duration, rainfall amount, slope, erosion etc.). The idea is this fuzzy logic model to be designed as a hierarchical system with several inputs and one output. Each level of the hierarchical system is consisted from one fuzzy logical

subsystem with several inputs. The fuzzy logic system output gives the integrated assessment of risk degree from the investigated debris flow in certain area regarding to the multiple natural factors. A scheme of the three-level hierarchical fuzzy system is presented on Figure 6.

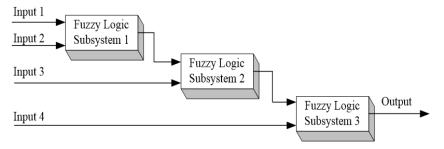


Fig. 6. Three-level hierarchical fuzzy system with four inputs

The fuzzy system inputs are defined on the basis of expert knowledge, debris flow data and current analyses, for example:

- Input 1 "Flow depth";
- Input 2 "Flow velocity";
- Input 3 "Debris flow duration";
- Input 4 "Inundated area".

Two intermediate linguistic variables are defined in the design of the model:

Intermediate variable 1 "Debris flow characteristics";

Intermediate variable 2 "Debris flow hazard".

In this case, the inputs of the first fuzzy logic subsystem are Input 1 "Flow depth" and Input 2 "Flow velocity", and the output variable is the Intermediate variable 1 "Debris flow characteristics".

The inputs of the second fuzzy logic subsystem are Intermediate variable 1 "Debris flow characteristics" and Input 3 "Debris flow duration", the output variable is the Intermediate variable 2 "Debris flow hazard".

The inputs of the third fuzzy logic subsystem are Intermediate variable 2 "Debris flow hazard" and Input 4 "Inundated area". The output of third fuzzy logic subsystem is output of the proposed fuzzy logic system – the integrated assessment of risk degree from the investigated debris flow in certain area regarding to the selected four factors. The higher value corresponds to the higher risk degree.

In the proposed fuzzy logic model, the all input linguistic variables, corresponding to the defined four inputs and two intermediate variables, are represented by five fuzzy membership functions, as follow: "Very low (VL)", "Low (L)", "Moderate (M)", "High (H)", and "Very high (VH)". The all input variables are assessed in the interval [0, 5] using trapezoidal membership functions (Figure 7).

The output of the fuzzy logic model (Integrated assessment of risk degree) is described by five fuzzy membership functions: "Very low (VL)", "Low (L)", "Moderate

(M)", "High (H)", and "Very high (VH)". The output variables is assessed in the interval [0, 100] using trapezoidal membership functions (Figure 8).

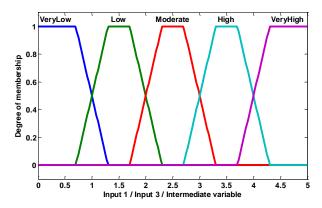


Fig. 7. Membership functions of the Input 1, Input 3 and Intermediate variable

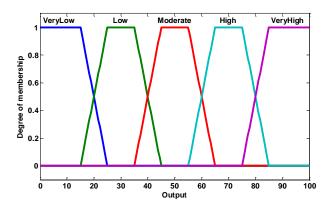


Fig. 8. Membership functions of the Fuzzy logic system output

The inference rules in the three fuzzy logic subsystem are defined as "If - then" - clause. The number of rules in the knowledge base for each of the fuzzy logic subsystems is 25. Some of the inference rules are defined as follow:

If Flow depth is VL and Flow velocity is H then Debris flow characteristics is M;

If Flow depth is L and Flow velocity is M then Debris flow characteristics is M;

If Flow depth is M and Flow velocity is VH then Debris flow characteristics is H;

If *Debris flow characteristics* is M and *Debris flow duration* is L then *Debris flow hazard* is M;

If Debris flow characteristics is M and Debris flow duration is VH then Debris flow hazard is H;

If Debris flow hazard is L and Inundated area is H then Integrated assessment of risk degree is M;

If *Debris flow hazard* is H and *Inundated area* is VH then *Integrated assessment of risk degree* is VH.

The outputs of the three fuzzy logic subsystems are calculated as an average weighted of all the inference rules, included in the fuzzy logic matrix. The three fuzzy logic models are based on Mamdani's inference machines, max/min operations and center of gravity defuzzification [21].

The inference surface of the third fuzzy logic subsystems is shown on Figure 9.

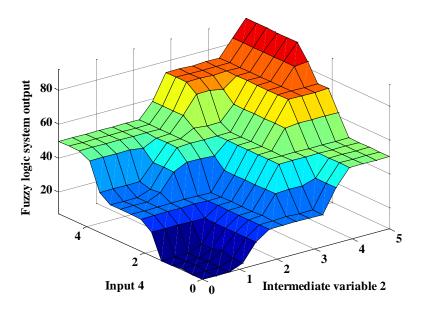


Fig. 9. Inference surfaces of the third fuzzy logic subsystem

6 Main Characteristics of the Multi-criteria Decision Analysis Module

The proposed information system needs to provide potential for multi-criteria decision analysis to support decision making. The Analytic Hierarchy Process is an easy-to-use multi-criteria decision-making method. This method is expressed in arranging the factors considered to be important for a decision in a hierarchic structure descending from an overall goal to criteria, sub-criteria, and finally alternatives at successive levels [19]. The main question that needs to be answered in case of occurrence of hazardous event is "where to take actions first?". For this purpose, specific indicators have to be considered. For example, in case of debris flow the following indicators have to be

taken into account: importance of the affected roads, number of cars, affected people, connections to other roads, other infrastructure – buildings, dams, danger of flooding.

The multi-criteria decision analysis allows decision makers to take a motivated decision based on scientific and expert analysis of the data. In this regard the decision support information system contain three main components: data; scientific analyses, and risk assessment and decision-making analyses.

7 Presentation of a Geoinformation Portal

For optimal data management, mapping, visualization and sharing of the debris flow data and results of the analyses, a dedicated server-based enterprise GIS can be used. It is integrated through the entire organization so that all project users can manage, share, and use spatial data and related information to address a variety of needs, including data creation, modification, visualization, analysis, and dissemination. The GIS architecture of the project was developed using ArcGIS Enterprise 10.7.1 products and consists of the several software components (Figure 10):

- ArcGIS Server a software feature that enables publishing of GIS data via web services and sharing across the organization and through an Internet connection. It also provides a client-server connection for geospatial data queries and analytical operations. Server management is available to authorized users at https://maps.mgu.bg:6443/arcgis/manager;
- ArcGIS Web Adapter (available at https://maps.mgu.bg/arcgis) is registered to the server and provides access to its resources to anyone with an Internet connection.
- ArcGIS Portal a corporate cloud (Software-as-a-Service) service for easy access to
 data publishing, web mapping, sharing, and performing of analytical tasks. The portal access is organized in hierarchical model with different task-specific roles to the
 users. Administrative access to the server and portal is only allowed for the project
 GIS administrator. The ArcGIS server is federated to the Portal, thus the publishing
 services are managed entirely by the latter component. The Portal's home page is
 publicly available at https://maps.mgu.bg/arcgis/home/.
- ArcGIS Data Store an application for easy configuration of databases for storage and operation of hosted GIS layers, created through the hosting server from ArcGIS Portal. Two databases (relational and cached) are registered in the GIS server.
- ArcGIS Desktop a standalone GIS platform for managing databases, mapping, performing analytical tasks, with the help of which hosted GIS layers are created in the Portal.
- Web App Builder an HTML/JavaScript web application development platform integrated into the GIS Portal. It is used to create mapping web applications. The project's web app is available at: https://maps.mgu.bg/arcgis/apps/View/index.html?appid=3836f023c6b34beeb0bd2422e1a777a6 (Figure 11).



Fig. 10. Architecture of the geoinformation portal of University of Mining and Geology "St. Ivan Rilski" (Sofia, Bulgaria), [Source (with modifications): https://enterprise.arcgis.com/en/get-started/latest/windows/additional-server-deployment.htm]

A gully induced debris flow basin is presented on the Figure 11. The basin is depicted as a polygon layer and morphometric parameters and debris flow data are entered in the polygon attribute table.



Fig. 11. The main interface of the web mapping application

The machine which runs the software components has the following technical parameters: Microsoft Windows 10 Enterprise (with web server component enabled - IIS); Intel (R) Core (TM) i5-6600 CPU @ 3.30GHz, 3301 MHz, 4 cores; 32 Gb RAM; graphics card - NVIDIA GeForce GTX 1060 6GB. The machine is named MIGGEOSERVER, and is part of the internal LAN of UMG "St. Ivan Rilski" (192.168.2.108 at mgu.bg), with external IP 78.90.247.26 and DNS maps.mgu.bg.

The implementation of the geoinformation portal for the needs of the debris flow study and monitoring proved successful and enabled seamless data management and public sharing of the results.

8 Conclusion

The presented model of debris flow information system can be used in debris flow monitoring and to support decision making in case of occurrence of this hazardous event. The quantity and quality of the initial data are the most important elements of the system, of which the results depend on. The possibility of development and using geoinformation portal allow better communication of the information, analyses and data management, and increases the opportunity for taking a motivated decision. The public access to the portal has a great contribution to rising awareness about the debris flows, increasing the preparedness and mitigating the hazard.

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