Analysis of the stability and solitary waves for the car-following model on two lanes

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Abstract. In this paper, Analysis of the stability and solitary waves for a car-following model on two lanes is carried out. The stability condition of the model is obtained by using the linear stability theory. We study the nonlinear characteristics of the model and obtain the solutions of Burgers equation, KDV equation, and MKDV equation, which can be used to describe density waves in three regions (i.e., stable, metastable and unstable), respectively. The analytical results show that traffic flow can be stabilized further by incorporating the effects come from the leading car of the nearest car on neighbor lane into car-following model.

Keywords: Car-following Model on Two Lanes, Traffic Flow, Density Waves.

1 Introduction

Car-following theory is one of the most important part of modern traffic theory. Since 1953 when Pipes[1] presented the first model, an increasing number of models have been proposed [2-9].In 2002, Jiang et al[6] presented a car-following model called full velocity difference model (FVDM).FVDM revealed the complex dynamic characteristics of traffic flow, therefore, various developed models based FVDM were proposed.

With the development of transportation, study on two-lane traffic has been increasingly necessary. However, early car-following models like FVDM are only subject to single lane traffic, thence, many scholars have made a lot of research on two-lane traffic and proposed a series of new models, which mainly divided into lattice model

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and car-following model. Nagatani[10] proposed lattice model on two lane traffic in 1998. Peng [11-14] extended the two-lane lattice model, and presented a series of new models based lattice model of Nagatani. Tang et al [15] presented a car-following model on two lanes by considering the lateral effects in traffic. They found that vehicle drivers always worry about the lane changing actions from neighbor lane and the consideration of lateral effects could stabilize the traffic flows on both lanes.

A large of traffic accidents are caused by unreasonable lane changing. In order to avoid such accidents, drivers have to worry about the lane changing actions not only of the nearest car in neighbor lane but also of the preceding car of the nearest car on neighbor lane. In this paper, we propose an extended car-following model on two lanes though considering the effects from both the nearest car and its leading car in neighbor lane which is rarely studied by others. Then the stability condition of the new model is derived by using the stability theory. Next, we obtain the solutions of Burgers equation, KDV equation and MKDV equation, which can be used to describe density waves in three regions (i.e.,stable, metastable and unstable) respectively. The analytical results show that traffic flow can be stabilized further by incorporating the effects come from the leading car of the nearest car on neighbor lane into car-following model.

2 Model

In case of two-lane traffic, it is necessary to consider the lateral effects. This is because plenty of surveys show that most drivers have to be ready to take precautions against the near vehicle on neighbor lane due to the suddenly lane changing without any alert message. The 'near vehicle' on neighbor lane is composed of the nearest vehicle and its leading car on neighbor lane. In general ,the distance between one car and it's nearest car on neighbor lane is so small that drivers always judge the lane changing action of his/her nearest-lateral car by observing the distance between his/her leading car and the nearest-lateral car. Hence, the dynamic equation of the car-following model on two lanes is as follows [15]:

$$\frac{d^2 x_{l,n}(t)}{dt^2} = f_{sti}\left(v_{l,n}(t), \Delta x_{l,n}(t), \sum_l \Delta_{l,n}, \Delta v_{l,n}(t) \right) \tag{1}$$

Where $l=0,1$ represent the lane number, $\Delta_{l,n}$ is the distance between the n_l vehicle on lane l and the leading car of its nearest vehicle on neighbor lane.

In this paper , Eq.(1) can be rewritten as:

$$\frac{dx_{l,n}^2(t)}{dt^2} = a_l \left[V_l\left(\Delta x_{l,n}(t), \sum_l \Delta_{l,n} \right) - \frac{dx_{l,n}(t)}{dt} \right] + \lambda_l \Delta v_{l,n}(t) \tag{2}$$

$V_l\left(\Delta x_{l,n}(t), \sum_l \Delta_{l,n} \right)$ is the optimal velocity formulated as

$$V_l\left(\Delta x_{l,n}(t),\sum_l \Delta_{l,n}\right)=V_l\left(\overline{\Delta x_{l,n}}\right)=V_l\left(\alpha_l \Delta x_{l,n}+\beta_{1l}\Delta_{l,n}+\beta_{2l}\Delta_{1-l,n}\right) \tag{3}$$

where $\alpha_l,\beta_{1l},\beta_{2l}$ are the weights of axial headway $\Delta x_{l,n}(t)$ and lateral distance $\sum_l \Delta_{l,n}$ respectively.

$\alpha_l+\beta_{1l}+\beta_{2l}=1$

According to the optimal velocity function presented by Bando [2], the optimal velocity function on two lanes is given by

$$V_l\left(\overline{\Delta x_{l,n}}\right)=\frac{v_{l,\max}}{2}\left[\tanh\left(\overline{\Delta x_{l,n}}-h_{lc}\right)+\tanh\left(h_{lc}\right)\right] \tag{4}$$

This velocity function has a turning point at $\overline{\Delta x_{l,n}} = h_{lc}$

$$V_l^{''}\left(\overline{\Delta x_{l,n}}\right)=\frac{d^2V_l\left(\overline{\Delta x_{l,n}}\right)}{d\overline{\Delta x_{l,n}}^2}\Big|_{\overline{\Delta x_{l,n}}=h_{lc}}=0 \tag{5}$$

3 Linear stability analysis

We apply the linear stability theory to examine the car-following model on two lanes described by Eq.(2).The uniform traffic flow is defined by such a state that all vehicles on lane l move with the optimal velocity $V_l\left(\Delta x_{l,n}(t),\sum_l \Delta_{l,n}\right)$ and the identical headway h_l and the lateral distance $\sum_l \Delta_l$;the relative velocity $\Delta v_{l,n}(t)$ is zero. The solution $x_{l,n}^{(0)}(t)$ is given by

$$x_{l,n}^{(0)}(t)=h_l n_l+V_l\left(h_l,\sum_l \Delta_l\right)t \tag{6}$$

Assuming $y_{l,n}(t)$ be a small deviation from the steady state $x_{l,n}^{(0)}(t)$, we have

$$x_{l,n}(t)=x_{l,n}^{(0)}(t)+y_{l,n}(t) \tag{7}$$

Substituting the Eq. (6) and Eq. (7) into Eq. (2), we rewrite linearized equation as

$$\frac{dy_{l,n}^2(t)}{dt^2}=a_l\left[V_l^{'}(h_l)\left(\alpha_l\Delta y_{l,n}(t)+\beta_{1l}\Delta_{l,n}+\beta_{2l}\Delta_{1-l,n}\right)-\frac{dy_{l,n}(t)}{dt}\right]+\lambda_l\frac{d\Delta y_{l,n}(t)}{dt} \tag{8}$$

Where $V_l^{'}\left(\overline{\Delta x_{l,n}}\right)=dV_l\left(\overline{\Delta x_{l,n}}\right)/d\overline{\Delta x_{l,n}}$, at $\overline{\Delta x_{l,n}} = h_l$, and $\Delta y_{l,n}(t) = y_{l,n+1}(t)-y_{l,n}(t)$ .For a very small perturbation $y_{l,n}(t)$ at $x_{l,n}^{(0)}(t)$, we can let $\Delta_{l,n}=\Delta_{1-l,n}\cong\Delta y_{l,n+1}$.

Expanding $y_{l,n}(t)$ in the Fourier-modes, $y_{l,n}(t) \approx A_l e^{(ik_l n_l + z_l t)}$, we obtain

$$z_l^2 = a_l\left[V_l'(h_l)\times\left(\alpha_l\left(e^{ik_l}-1\right)+\left(\beta_{1l}+\beta_{2l}\right)\left(e^{ik_l}-1\right)e^{ik_l}\right)-z_l\right]+\lambda_l z_l\left(e^{ik_l}-1\right) \tag{9}$$

Substituting $z_l = z_{1l}(ik_l)+z_{2l}(ik_l)^2+......$ into Eq. (9) ,we obtain the first- and second-order terms of coefficients in the expression of z_l as follows:

$$z_{1l} = (\alpha_l+\beta_{1l}+\beta_{2l})V_l'(h_l) = V_l'(h_l)$$

$$z_{2l} = \frac{a_l V_l'(h_l)\left[\alpha_l+3(\beta_{1l}+\beta_{2l})\right]+2\lambda_l z_{1l}-2z_{1l}^2}{2a_l}$$

For small disturbances with long wavelengths, the uniform steady state will become unstable when z_{2l} is negative. Thus the neutral stability curve is given by

$$a_{ls} = \frac{V_l'(h_l)-\lambda_l}{0.5\alpha_l+1.5(\beta_{1l}+\beta_{2l})} \tag{10}$$

The uniform traffic flow will be unstable if $a_l < a_{ls}$

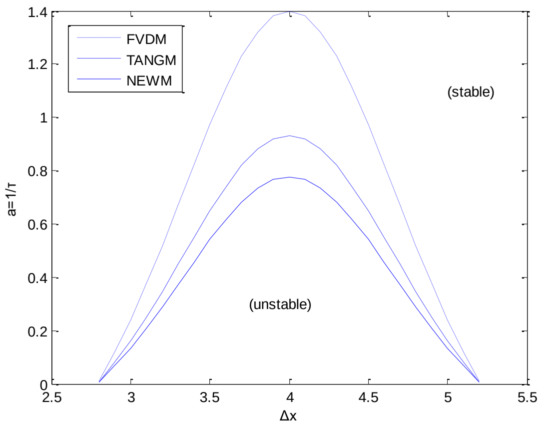

Fig. 1. Phase diagram in the headway-sensitivity space. The parameters related to the models are given in Table 1

The neutral stability curves in parameter space are shown in Fig.1, where the sensitivity $\alpha_l = 1/\tau_l$ .from Fig.1 it can be seen that the stable region of both the new model and Tang model are larger than stable region of the FVDM. It means the uniform traffic flow has been stabilized with taking into account the lateral effects. Furthermore, relative to Tang model, the critical point and neutral stability curve of new model are lower, which shows that the uniform traffic flow has been further strengthened by adjusting the lateral effects from both nearest car and it's leader car in neighbor lane. The traffic jam is thus relieved efficiently.

Table 1. parameters related to the models

	α_l	β_{1l}	β_{2l}	λ_l
FVDM	1	0	0	0.2
TANGM	0.75	0.25	0	0.2
NEWM	0.6	0.25	0.15	0.2

4 Nonlinear analysis

To facilitate the study of the density wave problem in the following three regions below, we rewrite Eq. (2) as follows:

$$\frac{d^2\Delta x_{l,n}(t)}{dt^2}=a_l\left\{V_l\left(\Delta x_{l,n+1}(t),\sum_l\Delta_{l,n+1}\right)-V_l\left(\Delta x_{l,n}(t),\sum_l\Delta_{l,n}\right)-\frac{d\Delta x_{l,n}(t)}{dt}\right\}+\lambda_l\left(\frac{d\Delta x_{l,n+1}(t)}{dt}-\frac{d\Delta x_{l,n}(t)}{dt}\right) \tag{11}$$

Where $V_l\left(\Delta x_{l,n+1}(t),\sum_l\Delta_{l,n+1}\right)=V_l\left(\overline{\Delta x_{l,n}}\right)$, $\overline{\Delta x_{l,n}}=\alpha_l\Delta x_{l,n}+\beta_{1l}\Delta_{l,n}+\beta_{2l}\Delta_{1-l,n}$

4.1 Burger equation

We now consider the slowly varying behaviors for long waves in the three regions (i.e. stable, metastable and unstable).Introduce slow scales for space variable n_l and time variable t. For $0<\varepsilon\le 1$, we define the slow variable X_l and T

$$X_l=\varepsilon(n_l+b_lt), T=\varepsilon^2 t \tag{12}$$

Where b_l is a constant to be determined. Let

$$\Delta x_{l,n}=h_l+\varepsilon R_l(X_l,T) \tag{13}$$

Substituting Eq. (12) and Eq.(13) into Eq.(11) and expanding to the third order of ε ,we obtain the following nonlinear partial differential equation

$$\varepsilon^2 a_l\left(b_l-V^{'}(h_l)\right)\partial_{X_l}R_l+\varepsilon^3\left\{a_l\partial_T R_l-a_lV^{"}(h_l)R_l\partial_{X_l}R_l-\left(\frac{a_l}{2}\left[\alpha_l+3(\beta_{1l}+\beta_{2l})\right]V^{'}(h_l)-b_l^2+\lambda_lb_l\right)\partial_{X_l}^2R_l\right\}=0 \tag{14}$$

Where $V_l^{'}(h_l)=\frac{dV\left(\overline{\Delta x_{l,n}}\right)}{d\overline{\Delta x_{l,n}}}\Big|_{\overline{\Delta x_{l,n}}=h_l}$, $V_l^{"}(h_l)=\frac{d^2V\left(\overline{\Delta x_{l,n}}\right)}{d\overline{\Delta x_{l,n}}^2}\Big|_{\overline{\Delta x_{l,n}}=h_l}$, $\partial_T=\frac{\partial}{\partial T}$, $\partial_{X_l}=\frac{\partial}{\partial X_l}$

By taking $b_l = V^{'}(h_l)$, we eliminate the second-order term of ε from Eq. (14) and have

$$\partial_T R_l - V^{''}(h_l) R_l \partial_{X_l} R_l = \left(\frac{1}{2}\left[\alpha_l + 3(\beta_{1l} + \beta_{2l})\right] V^{'}(h_l) - \frac{b_l^2}{a_l} + \frac{\lambda_l b_l}{a_l} \right) \partial_{X_l}^2 R_l$$
$$= \left(\frac{1}{2}\left[\alpha_l + 3(\beta_{1l} + \beta_{2l})\right] - \frac{V^{'}(h_l)}{a_l} + \frac{\lambda_l}{a_l} \right) V^{'}(h_l) \partial_{X_l}^2 R_l \tag{15}$$

The coefficient $\frac{1}{2}\left[\alpha_l + 3(\beta_{1l} + \beta_{2l})\right] - \frac{V^{'}(h_l)}{a_l} + \frac{\lambda_l}{a_l} > 0$ in the stable region satisfies the stability criterion. Thus, in the stable region Eq.(15) is the Burgers equation. The solution of the Burgers equation is as follow:

$$R(X_l, T) = \frac{1}{\left|V^{''}(h_l)\right| T}\left[X - \frac{\eta_{n+1} - \eta_n}{2} \right] - \frac{\eta_{n+1} - \eta_n}{2\left|V^{''}(h_l)\right| T}$$
$$\times \tanh\left[\left(\frac{1}{2}\left[\alpha_l + 3(\beta_{1l} + \beta_{2l})\right] - \frac{V^{'}(h_l)}{a_l} + \frac{\lambda_l}{a_l} \right) \times V^{'}(h_l) \frac{(\eta_{n+1} - \eta_n)(X_l - \xi_n)}{4\left|V^{''}(h_l)\right| T} \right] \tag{16}$$

Where η_n are the coordinates of the intersections of the slopes with the x-axis and ξ_n are those of the shock fronts. As $T \to +\infty$, $R(X,T) \to 0$, which means in stable region all density waves eventually evolved into a uniform flow with increasing time.

4.2 Kdv equation

We consider the slowly varying behaviors for long waves near the neutral stability point. Slow variable X_l and T are defined as

$$X_l = \varepsilon(n_l + b_l t), T = \varepsilon^3 t \tag{17}$$

We set the headway as

$$\Delta x_{l,n} = h_l + \varepsilon^2 R_l(X_l, T) \tag{18}$$

Subsitituting Eq.(17).and Eq.(18) into Eq.(11) and expanding to the sixth order of ε ,we obtain the following nonlinear partial differential equation

$$\varepsilon^3 a_l\left[b_l - V_l^{'}(h_l)\right]\partial_{X_l} R_l + \varepsilon^4\left[b_l^2 - \frac{a_l}{2}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})V_l^{'}(h_l) - \lambda_l b_l\right]\partial_{X_l}^2 R_l$$
$$+\varepsilon^5 a_l\left\{\partial_T R_l - \left[\frac{1}{6}(\alpha_l + 7\beta_{1l} + 7\beta_{2l})V_l^{'}(h_l) + \frac{1}{2a_l}\lambda_l b_l\right]\partial_{X_l}^3 R_l - \frac{1}{2}V_l^{''}(h_l)\partial_{X_l} R_l^2\right\}$$
$$+\varepsilon^6\left[(2b_l - \lambda_l)\partial_{X_l}\partial_T R_l - \frac{a_l}{24}(\alpha_l + 15\beta_{1l} + 15\beta_{2l})V_l^{'}(h_l)\partial_{X_l}^4 R_l\right. \qquad (19)$$
$$\left. - \frac{a_l}{4}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})V_l^{''}(h_l)\partial_{X_l}^2 R_l^2\right] = 0$$

where $\partial_X \partial_T = \dfrac{\partial^2}{\partial X \partial T}$.

Near the neutral stability point, we set $\dfrac{a_l}{a_{ls}} = 1 + \varepsilon^2$, where $a_{ls} = \dfrac{V_l^{'}(h_l) - \lambda_l}{0.5\alpha_l + 1.5(\beta_{1l} + \beta_{2l})}$

By taking $b_l = V^{'}(h_l)$,we eliminate both the third-order and the forth-order term of ε from Eq.(19) and have

$$\partial_T R_l - f_1 \partial_{X_l}^3 R_l - f_2 R_l \partial_{X_l} R_l + \varepsilon\left[-f_3 \partial_{X_l}^2 R_l + f_4 \partial_{X_l}^4 R_l + f_5 \partial_{X_l}^2 R_l^2\right] = 0 \qquad (20)$$

Where $f_1 = \frac{1}{6}(\alpha_l + 7\beta_{1l} + 7\beta_{2l})V_l^{'}(h_l) + \frac{1}{2a_l}\lambda_l b_l$, $f_2 = V_l^{''}(h_l)$, $f_3 = \frac{1}{2}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})V_l^{'}(h_l)$

$$f_4 = \left[\frac{\lambda\left(2V_l^{'}(h_l) - \lambda\right)}{2a_s^2} + \frac{(\alpha_l + 7\beta_{1l} + 7\beta_{2l})}{6a_s}\left(2V_l^{'}(h_l) - \lambda\right) - \frac{\alpha_l + 15\beta_{1l} + 15\beta_{2l}}{24}\right]V_l^{'}(h_l)$$

$$f_5 = \left(\frac{2V_l^{'}(h_l) - \lambda}{2a_s} - \frac{1}{4}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})\right)V_l^{''}(h_l)$$

In order to drive the regularized equation , we make the transformations as follows:

$T = \sqrt{f_1}T_{kdv}$, $X_l = -\sqrt{f_1}X_{lkdv}$, $R_l = \dfrac{1}{f_2}R_{lkdv}$

Thus, we obtain the KDV equation with a $o(\varepsilon)$ correction term.

$$\partial_{T_{kdv}} R_{lkdv} - f_1 \partial_{X_{lkdv}}^3 R_{lkdv} - f_2 R_{lkdv} \partial_{X_{lkdv}} R_{lkdv} + \varepsilon\left[-f_3 \partial_{X_{lkdv}}^2 R_{lkdv} + f_4 \partial_{X_{lkdv}}^4 R_{lkdv} + f_5 \partial_{X_{lkdv}}^2 R_{lkdv}^2\right] = 0 \qquad (21)$$

We ignore the $o(\varepsilon)$ term and get the KDV equation with the soliton solution

$$R_{l0}(X_{lkdv}, T_{kdv}) = A\operatorname{sec}h^2\left[\sqrt{\frac{A}{12}}\left(X_{lkdv} - \frac{A}{3}T_{kdv}\right)\right] \qquad (22)$$

Where $A = \dfrac{21 f_1 f_2 f_3}{24 f_1 f_5 - 5 f_2 f_4}$,

Hence, we obtain the soliton solution of the KDV equation

$$\Delta x_{l,\mathrm{n}}(t) = h_l + \frac{A}{V_l'''(h_l)}\left(1-\frac{a_{ls}}{a_l}\right)\mathrm{sec}\,h^2\left\{\left[n + V_l'(h_l)t + \frac{A}{3}\left(1-\frac{a_{ls}}{a_l}\right)t\right]\right.$$
$$\left.\times\sqrt{\frac{a_{ls}A}{2a_{ls}(\alpha_l + 7\beta_{1l} + 7\beta_{2l})V_l'(h_l) + 6\lambda_l V_l'(h_l)}\left(1-\frac{a_{ls}}{a_l}\right)}\right\} \tag{23}$$

4.3 Mkdv equation

In unstable region we consider the slowly varying behaviors for long waves. Slow variable X_l and T are defined just as Eq. (17)

We set the headway as

$$\Delta x_{l,n} = h_{lc} + \varepsilon R_l(X_l, T) \tag{24}$$

Subsitituting Eq.(17) and Eq.(24) into Eq.(11) and expanding to the fifth order of ε ,we obtain the following nonlinear partial differential equation

$$\begin{aligned}
&\varepsilon^2 a_l\left[b_l - V_l'(h_{lc})\right]\partial_{X_l}R_l + \varepsilon^3\left[b_l^2 - \frac{a_l}{2}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})V_l'(h_{lc}) - \lambda_l b_l\right]\partial_{X_l}^2 R_l \\
&+\varepsilon^4\left\{a_l\partial_T R_l - \left[\frac{a_l}{6}(\alpha_l + 7\beta_{1l} + 7\beta_{2l})V_l'(h_{lc}) + \frac{1}{2}\lambda_l b_l\right]\partial_{X_l}^3 R_l\right. \\
&\left.-\frac{1}{6}a_l V_l'''(h_{lc})\partial_{X_l}R_l^3\right\} + \varepsilon^5\left\{(2b_l - \lambda_l)\partial_{X_l}\partial_T R_l\right. \\
&-\left[\frac{a_l}{24}(\alpha_l + 15\beta_{1l} + 15\beta_{2l})V_l'(h_{lc}) + \frac{1}{6}\lambda_l b_l\right]\partial_{X_l}^4 R_l \\
&\left.-\frac{1}{12}a_l(\alpha_l + 3\beta_{1l} + 3\beta_{2l})V_l'''(h_c)\partial_{X_l}^2 R_l^3\right\} = 0
\end{aligned} \tag{25}$$

Where $V_l'(h_{lc}) = \left.\frac{dV\left(\overline{\Delta x_{l,n}}\right)}{d\overline{\Delta x_{l,n}}}\right|_{\overline{\Delta x_{l,n}}=h_{lc}}$ $\quad V_l'''(h_{lc}) = \left.\frac{d^3V\left(\overline{\Delta x_{l,n}}\right)}{d\overline{\Delta x_{l,n}}^3}\right|_{\overline{\Delta x_{l,n}}=h_{lc}}$

Near the critical point (h_{lc}, a_{lc}) , taking $\frac{a_l}{a_{lc}} = (1-\varepsilon^2)$, $b_l = V_l'(h_l)$ and eliminating both the second-order and the third-order term of ε ,Eq.(25) can be simplified as

$$\partial_T R_l - \mathrm{g}_1\partial_{X_l}^3 R_l + \mathrm{g}_2 R_l \partial_{X_l} R_l^3 + \varepsilon\left[\mathrm{g}_3\partial_{X_l}^2 R_l + \mathrm{g}_4\partial_{X_l}^4 R_l + \mathrm{g}_5\partial_{X_l}^2 R_l^3\right] = 0 \tag{26}$$

Where

$$g_1 = \left[\frac{1}{6}(\alpha_l + 7\beta_{1l} + 7\beta_{2l}) + \frac{1}{2}\lambda_l\right]V_l^{'}(h_{lc}), \quad g_2 = -\frac{1}{6}V_l^{'''}(h_{lc}), \quad g_3 = \frac{1}{2}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})V_l^{'}(h_{lc})$$

$$g_4 = \left[\frac{\lambda_l\left(2V_l^{'}(h_{lc}) - \lambda_l\right)}{2a_{lc}^2} + \frac{(2b_l - \lambda_l)}{6a_{lc}}(\alpha_l + 7\beta_{1l} + 7\beta_{2l}) - \frac{1}{24}(\alpha_l + 15\beta_{1l} + 15\beta_{2l}) - 6\lambda_l\right]V_l^{'}(h_{lc})$$

$$g_5 = \left[\frac{\left(2V_l^{'}(h_{lc}) - \lambda_l\right)}{6a_{lc}} - \frac{1}{12}(\alpha_l + 3\beta_{1l} + 3\beta_{2l})\right]$$

We make such transformations as $T = \frac{1}{g_1}T_m \quad R_l = \sqrt{\frac{g_1}{g_2}}R_{lm}$

Then we obtain the modified KDV equation with a $o(\varepsilon)$ correction term.

$$\partial_T R_{lm} - \partial_{X_l}^3 R_{lm} + g_2\partial_{X_l} R_{lm}^3 + \frac{\varepsilon}{g_1}\left[g_3\partial_{X_l}^2 R_{lm} + g_4\partial_{X_l}^4 R_{lm} + \frac{g_1 g_5}{g_2}\partial_{X_l}^2 R_{lm}^3\right] = 0 \qquad (27)$$

If we ignore $o(\varepsilon)$ term, this is just the modified KDV equation with a kink solution as the desired solution

$$R_l(X_l, T) = \sqrt{\frac{g_1}{g_2}B}\tanh\left[\sqrt{\frac{B}{2}}(X_l - Bg_1T)\right] \qquad (28)$$

Where $B = \frac{5g_2g_3}{2g_2g_4 - 3g_1g_5}$

Thus , we obtain the kink solution of the headway

$$\begin{aligned}\Delta x_{l,n}(t) &= h_{lc} + \sqrt{\frac{g_1B}{g_2}\left(\frac{a_{lc}}{a_l} - 1\right)}\tan h \times \left\{\sqrt{\frac{B}{2}\left(\frac{a_{lc}}{a_l} - 1\right)} \times \left[n + V_l^{'}(h_{lc})t - Bg_1\left(\frac{a_{lc}}{a_l} - 1\right)t\right]\right\} \\ &= h_{lc} + \sqrt{\frac{(\alpha_l + 7\beta_{1l} + 7\beta_{2l})V_l^{'}(h_{lc}) + 3\lambda_l V_l^{'}(h_{lc})}{-V_l^{'''}(h_{lc})}\left(\frac{a_{lc}}{a_l} - 1\right)B} \\ &\times \tan h\left\{\sqrt{\frac{B}{2}\left(\frac{a_{lc}}{a_l} - 1\right)} \times \left[n + V_l^{'}(h_{lc})t - B\left(\frac{1}{6}(\alpha_l + 7\beta_{1l} + 7\beta_{2l}) + \frac{1}{2}\lambda_l\right)V_l^{'}(h_{lc})\left(\frac{a_{lc}}{a_l} - 1\right)t\right]\right\}\end{aligned} \qquad (29)$$

5 Conclusions

The two-lane car-following model in this paper is the extension of the FVDM in single lane. By considering the lateral effects, the model consists not only of the nearest vehicle on neighbor lane but also of its preceding vehicle. Linear analysis of the model shows that the consideration of lateral effects of the nearest vehicle on neighbor lane could stabilize the traffic flow. The solutions of Burgers equation, KDV equation, and

MKDV equation have been derived to describe density waves in three regions respectively.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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